VELOCITY DISTRIBUTION IN ALLUVIAL CHANNELS

BY

R. J. GARDE *

AND

A. S. PAINTAL **

* Reader in Civil Engineering, University of Roorkee (India).

** Lecturer in Civil Engineering, University of Roorkee (India).

Introduction

In the study of hydraulics of alluvial channels the engineer is often interested in finding the quantity of water and the sediment load carried by the stream under given conditions of flow. The mean velocity of flow is obtained by integrating the velocity distribution over the depth and dividing by the depth of flow. In order to find the total sediment load, the curve of (sediment concentration × velocity) must be integrated over the depth. In studies of scour, design of channels, silt excluders and extractors etc., the knowledge of velocity distribution is also required.

Review and theoretical considerations

Two types of formulae have been suggested to describe the velocity distribution for turbulent flow over rigid boundaries; they are exponential type and logarithmic type. The logarithmic law rather than the exponential law is more frequently used by hydraulic engineers. The logarithmic law can be obtained from Prandtl's hypothesis of mixing length [13]* by assuming that in the vicinity of wall, the mixing length is linearly proportional to the distance from the wall, and the shear stress is assumed to be constant. It can also be obtained from Kármán's similarity law [13] by supposing that mixing length is only a function of velocity distribution and shear stress is constant at the boundary. The basic logarithmic law can be stated as :

$$\frac{u}{u_*} = \frac{1}{\mathcal{K}} \log_e \frac{y}{y'} \qquad (1)$$

in which

$\mathcal{K}$ is the Kármán constant;
u is the velocity at distance y from bed;
u_* is the shear velocity

$$\left(= \sqrt{\frac{\tau_0}{\rho_f}} = \sqrt{(g\,RS)} \right);$$

τ_0 is the average shear stress at the bed ($= \gamma_f$ RS);
γ_f is the unit weight of fluid;
ρ_f is the mass density of fluid;
R is the mean hydraulic radius;
S is the slope; and,
y' is some length at which $u = 0$.

Nikuradse [13] conducted experiments with pipes which were artificially roughened by coating their inside surface with sand grains of uniform size. He found that if $u_* k_s/\nu$ is less than 3.5, the boundary acts as hydrodynamically smooth, and equation (1) takes the form:

$$\frac{u}{u_*} = \frac{2.3}{\mathcal{K}} \log_{10} \frac{u_* k_s}{\nu} + 5.5 \qquad (2)$$

in which ν is the kinematic viscosity of fluid and k_s is the size of sand grains.

* Numbers in [ ] refer to numbers in the bibliography.

If $u_* k_s/\nu$ is greater than 67, the boundary acts as hydrodynamically rough boundary and equation (1) takes the form:

$$\frac{u}{u_*} = \frac{2.3}{\mathcal{K}} \log_{10} \frac{y}{k_s} + 8.5 \qquad (3)$$

Keulegan [8] 1938, applied Nikuradse's results to open channel flow. He found that when hydraulic radius is used as the characteristic length, Nikuradse's formula for pipe flow can be applied to open channel flow.

In the past, attempts have been made to formulate the laws of velocity distribution for alluvial channels and it has been found that the velocity distribution in alluvial channels follows the logarithmic law.

Investigations carried out by Iwagaki [7] 1954, Einstein and Chien [3] 1955, and Tsubaki [15] 1955, have shown that the length parameter k_s for alluvial channels should depend on flow, fluid and sediment characteristics. When the bed is plane, the length parameter k_s may be correlated with bed material size d. But, when the ripples and dunes are formed on the bed, the length parameter may be several times greater than the bed material size. Therefore, this length parameter should in general depend on flow, fluid and sediment characteristics. With this idea the following equation of velocity distribution was proposed (4):

$$\frac{u}{u_*} = \frac{2.3}{\mathcal{K}} \log_{10} \frac{y}{k'_s} \qquad (4)$$

in which $\mathcal{K}$ is dimensionless number called Kármán constant and k'_s is variable length parameter. The advantage of equation (4) over use of equation (3) is that the numerical constant is grouped with length parameter in equation (4). Hence the analysis becomes rather simpler.

Variation of Kármán constant

In connection with flow over artificial roughnesses in open channels, it was found by Rand [11] 1952, that $\mathcal{K}$ is a function of roughness size, geometry and relative roughness h/D (where h is the height of roughness and D is the depth of flow). From this it can be stated that variation of $\mathcal{K}$ in alluvial channel can be attributed at least partly, to the undulations of the bed.

Vanoni (17) 1946, Einstein and Chien [3] 1955 and Tusbaki [14] 1956 showed that presence of suspended sediment affects the value of $\mathcal{K}$. The turbulence is damped by the presence of suspended sediment and Kármán constant is dependent on it being an index of turbulence.

From the point of view if dimensional analysis it has been shown [4] that for duned bed, the relation:

$$\frac{h}{l} = f(\tau_*, \mathcal{F}) \qquad (5)$$

holds good. Here h is the average height and l is the average spacing of bed undulations, τ_* is dimensionless shear $\{ = \gamma_f \mathrm{RS}/g(\rho_s - \rho_f)\, d \}$ and $\mathcal{F}$ is Froude number $(= \mathrm{U}/(g\mathrm{R})^{1/2})$.

Further it is found that the rate of sediment transport is primarily a function of dimensionless shear on the bed τ_*. Hence the value of $\mathcal{K}$ should depend upon τ_* and $\mathcal{F}$, if the sediment concentration and bed configuration govern the value of $\mathcal{K}$.

Variation of k'_s

It was shown [4] that following functional relationship can be written for variation of k'_s in alluvial channels:

$$\frac{k'_s}{d} = f\left(\frac{u_* d}{\nu}, \mathcal{F}, \tau_*\right) \qquad (6)$$

The term $u_* d/\nu$ can be interpreted as a parameter proportional to the ratio of sediment size to thickness of laminar sublayer when the bed is plane, the resistance to flow is governed by this ratio. However, when ripples and dunes are formed on the bed, the height of irregularities and their spacing governs the resistance to flow in part. Therefore the ratio of thickness of laminar sublayer to bed material size loses its full significance. Hence $u_* d/v$ can be omitted from equation (6).

Recent investigations show that Froude number defined either as $\mathrm{U}/(g\mathrm{R})^{1/2}$ or

$$\frac{\mathrm{U}}{[(\Delta\gamma_s/\rho_f)\,.\,d]^{1/2}}$$

is important in several aspects of flow over alluvial beds. The scale of undulations is proportional to $\mathrm{U}/(g\mathrm{R})^{1/2}$. On the other hand in problems of resistance and sediment transport,

$$\frac{\mathrm{U}}{[(\Delta\gamma_s/\rho_f)\,.\,d]^{1/2}}$$

seems to be more important. Hence the inclusion of Froude number in equation (6) is justified. Preliminary analysis indicated that for velocity distribution studies

$$\frac{\mathrm{U}}{[(\Delta\gamma_s/\rho_f)\,.\,d]^{1/2}}$$

is preferable to $\mathrm{U}/(g\mathrm{R})^{1/2}$.

The parameter τ_* is dimensionless shear which can be called as index of movability of sediment. Hence in case where ripples have grown into dunes (as it happens in most of alluvial channels), it seems that τ_* and

$$\frac{\mathrm{U}}{[(\Delta\gamma_s/\rho_f)\,.\,d]^{1/2}}$$

should be adequate to study the variation of k'_s/d.

Variation of k'_s with manning's coefficient n

The variable length parameter k'_s and Manning's coefficient n are dependent on resistance to flow in alluvial channels, Einstein and Chien [7] 1955, correlated the length parameter k_s of following equation:

$$\frac{u}{u_*} = 17.4 + \frac{2.3}{\mathcal{K}} \log_{10} \frac{y}{35.4\, k_s} \qquad (7)$$

with Manning coefficient in the following form:

$$k_s = (29.3\, n)^6 \qquad (8)$$

There may exist similar relationship between

Manning coefficient n and variable length parameter k'_s. As n has got the dimensions of $L^{1/6}$, the parameter $n/d^{1/6}$ is dimensionless. It may be correlated to k'_s/d another dimensionless parameter by the equation:

$$\frac{k'_s}{d} = f\left(\frac{n}{d^{1/6}}\right) \qquad (9)$$

Collection of data

Since reasonably sufficient field and flume data were already available to the writers, no additional data were collected. A summary of field and flume data used in present analysis, is given in table 1 and 2.

Presentation and analysis of data

Computation of parameters:

The velocity profiles were first drawn. A plot between u/u_* and $\log_{10} y$ was made for each set of observations (see Fig.1). The points near the surface and near the movable boundary showed some deviation from the logarithmic law but rest of the points appeared to lie on a line. An average line was drawn representing the trend. It showed that velocity distribution in alluvial channels could be described by a logarithmic law.

1. From the velocity profile the value of Kármán constant and variable length parameter k'_s can be computed as follows:

(i) *Kármán Constants:* The variation of u/u_* in one cycle interval gives the value of $2.3/\mathcal{K}$, from which $\mathcal{K}$ can be computed.

(ii) *Variable length parameter k'_s:*

a) From the plot of u/u_* versus $\log_{10} y$, the value of u/u_* is read at $y = 1$. The value of $\mathcal{K}$ is already known. Substituting the value of y, $\mathcal{K}$ and u/u_* in equation (4), k'_s may be computed.

b) Knowing the hydraulic mean radius R and mean velocity of flow U, the value of k'_s can be computed from the following equation :

$$\frac{U}{u_*} = \frac{2.3}{\mathcal{K}} \log_{10} \frac{R}{e k'_s} \qquad (10)$$

This equation is obtained by integration of equation 4, over the depth of flow.

The value of k'_s was computed from both the methods and the mean value of it has been used in this analysis.

2. Dimensionless shear parameter:

It is defined as follows:

$$\tau_* = \frac{\gamma_f RS}{(\gamma_s - \gamma_f)\, d} \qquad (11)$$

Since the wall effects could be easily computed for flume data, hydraulic radius with respect to bed was used instead of R in the above expression. In case of canals and rivers, side or bank effects were not known; hence R was used. Where the width of canal or river was large; D, the depth of flow, has been used instead of R.

3. Froude number: It is defined as follows:

$$\mathcal{F} = \frac{U}{[(\gamma_s - \gamma_f/\rho_f)\,.\,d]^{1/2}}$$

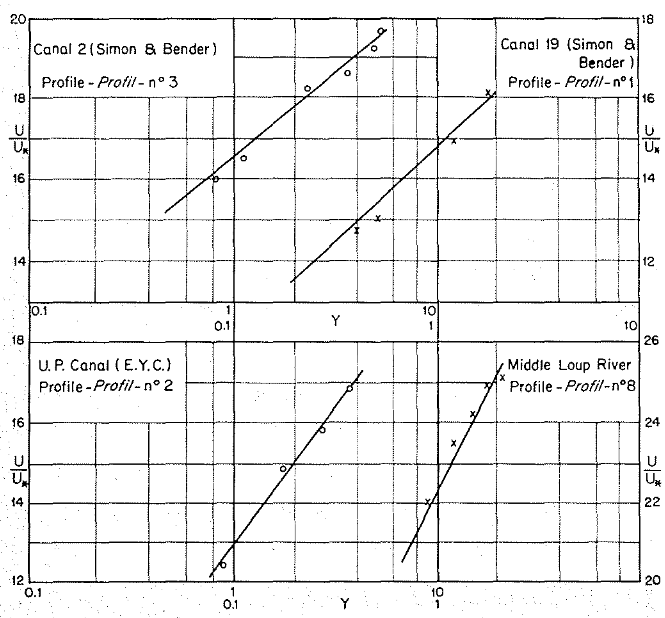
The collected field and flume data were analysed to study the variation of $\mathcal{K}$ and k'_s with change in flow, fluid and sediment characteristics.

Variation of Kármán constant $\mathcal{K}$

The coefficient $\mathcal{K}$ initially appeared in the literature as a universal constant of turbulence exchange in Von-Kármán's hypothesis of turbulence similitude. From Nikuradse's experiments of artificially roughened pipes, the value of $\mathcal{K}$ has been found to be 0.4, although Vanoni (quoted in 1) while reanalysing the data of Nikuradse, found $\mathcal{K}$ to vary from 0.32 to 0.42. The review of literature indicates that the accepted value of $\mathcal{K}$ is in the vicinity of 0.4 in case of pipes.

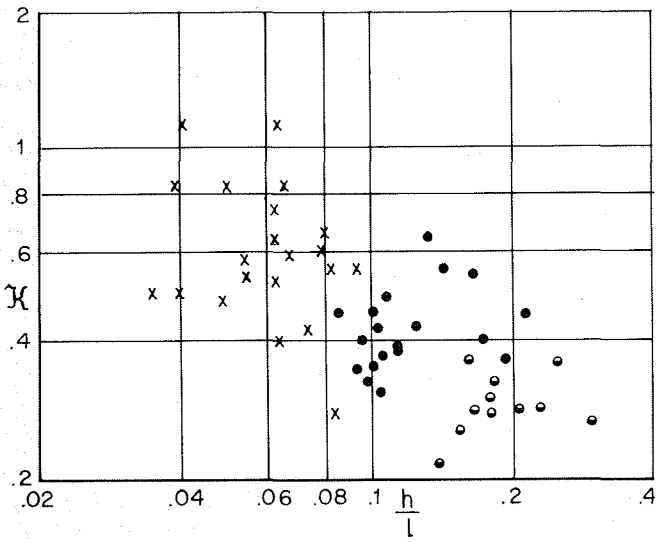
There is evidence to show that the value of $\mathcal{K}$ in rigid boundary open channels with artificial roughnesses is not constant but varies appreciably. It has been found by Rand [11] 1952, and Albertson and Sayre [1] 1961, that the value of $\mathcal{K}$ is different for different roughness patterns and it is constant for one type of roughness pattern.

On analysing the data for alluvial channels it has been found that the value of $\mathcal{K}$ varied appreciably and sometimes the values are even greater than unity. Figure 2 shows the variation of $\mathcal{K}$ with h/l for the data collected by U.S.G.S., Barton-Lin, and Laursen. Furthermore in majority of runs in which value of $\mathcal{K}$ are greater than 0.4, the value of h/D is of the order of 0.1 or so. Figure 3 shows that $\mathcal{K}$ increases wifh the increase of in h/D. On the other hand Rand [11] 1952, has found that $\mathcal{K}$ decreases with increase in h/D for artificial roughnesses in open channels. This apparent discrepancy may be explained on the basis of combined effect of relative roughness and sediment transport on the Kármán's constant.

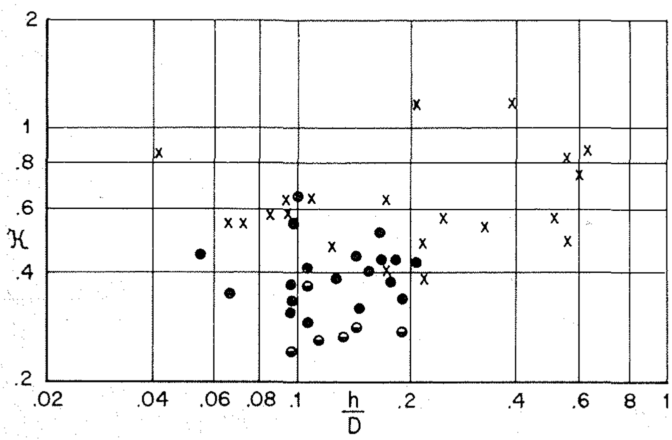


Variations de la loi logarithmique /1
Variation of logarithmic law

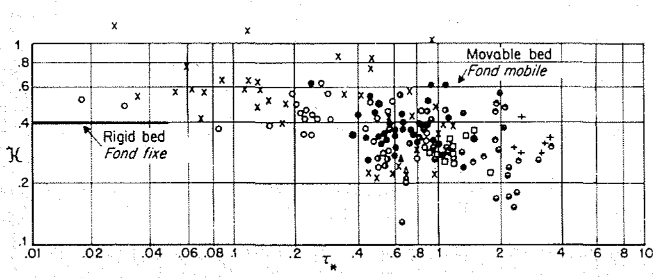
The concept of Kármán constant and its evalua-



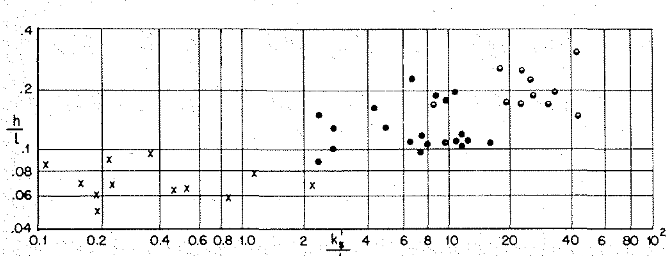
Variations de K en fonction de h/l /2
Variation of K *with* h/l



Variations de K en fonction de h/D /3
Variation of K *with* h/D



Variations de K en fonction de τ_* /4
Variation of K *with* τ_*



Variations de k'_s en fonction de h/l /5
Variation of k'_s *with* h/l

tion in earlier stages was exclusively based on either smooth pipes or pipes roughened with uniform sand grains. For such boundary characteristics, the flow was of wake interference type and the values were in the vicinity of 0.4. If the roughness heights as compared to the depth of flow can be varied (as in case of artificial roughnesses) it is expected that $\mathcal{K}$ values will be different than 0.4. The reason for larger values of k under these circumstances is that the local vortices are set up by the roughnesses and these increase the level of turbulence. In such cases there is not only the turbulent exchange in its customary sense but also an exchange of momentum due to "Convective currents" induced by these roughnesses.

In order to study the variation of $\mathcal{K}$, Figure 4 is drawn which shows the variation of $\mathcal{K}$ with τ_*. Preliminary analysis failed to systematise the scatter on Figure 4 by inclusion of Froude number. Hence Figure 4 shows the plot of $\mathcal{K}$ against τ_* only. It can be seen from this plot that with increase in τ_*, $\mathcal{K}$ decreases. With the increase in τ_*, the sediment load increases which damps the turbulence and causes the decrease in the value of $\mathcal{K}$. The size of ripples and dunes is also the function of τ_* and thus with the increase in τ_* the value of $\mathcal{K}$ will also change. These effects are accounted by single parameter τ_*.

Tentatively a line is drawn on this figure to show that up to $\tau_* = 0.05$ the value of k is 0.4. For τ_* values < 0.05 the sediment will not move and therefore, the values of $\mathcal{K}$ in such case, would be identical to those in case of rigid boundary open channel. Contrary to common belief, it can be seen that $\mathcal{K}$ in alluvial channels can assume any value from 1 to as low as 0.15. In spite of appreciable scatter there is a definite tendency for $\mathcal{K}$ to decrease with τ_*. This figure can be used for predicting the value of $\mathcal{K}$.

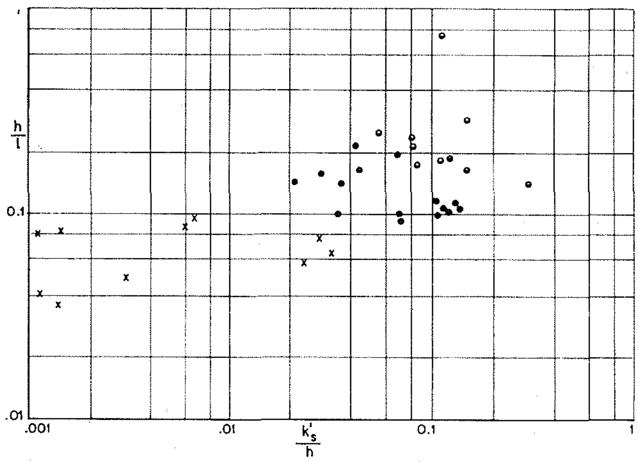
Variation of lenght parameters k'_s

As discussed earlier, the length parameter varied with change in flow, fluid and sediment characteristics. The length parameter k'_s is dependent on resistance to flow. The size of ripples and dunes govern the resistance to flow. Figure 5 shows that with the increase in h/l, the values of k'_s/h also increase. Figure 6 indicates that k'_s/d increases with the increase of h/l. These figures clearly show that values of k'_s are dependent on sizes of ripples and dunes.

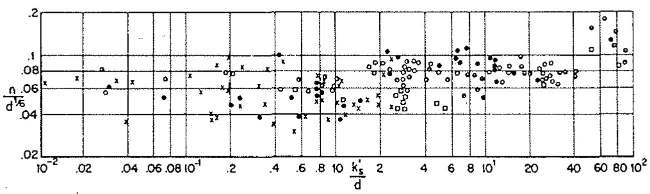
a) VARIATION OF k'_s/d, WITH τ_* & $\mathcal{F}$.

The length parameter k'_s is governed by size of ripples and dunes. The size of ripples and dunes depends upon τ_* and $\mathcal{F}$. Therefore k'_s/d is a function of τ_* and $\mathcal{F}$, as discussed earlier.

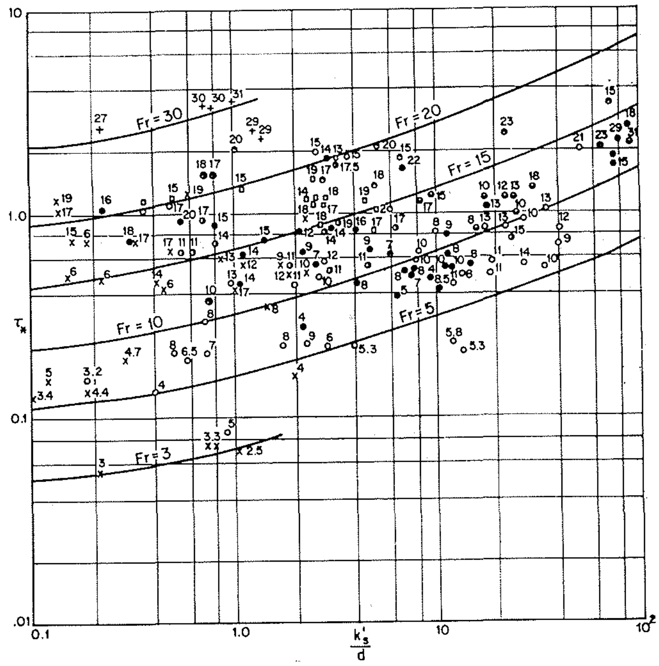
The figure 7 shows the plot k'_s/d versus τ_* with Froude number as third variable. In spite of scatter, there is a definite tendency for k'_s/d to vary with $\mathcal{F}$ also. There is a deviation in case of Laursen's data for 0.04 mm size., it does not follow the trend. In case of fine sediment load there is a zone of heavy sediment concentration near the bed. The mixture of fine sediment and water behaves like a fluid and many times there is no clear demarcation between the fluid and boundary. But disregarding Laursen's data of 0.04 mm size the lines



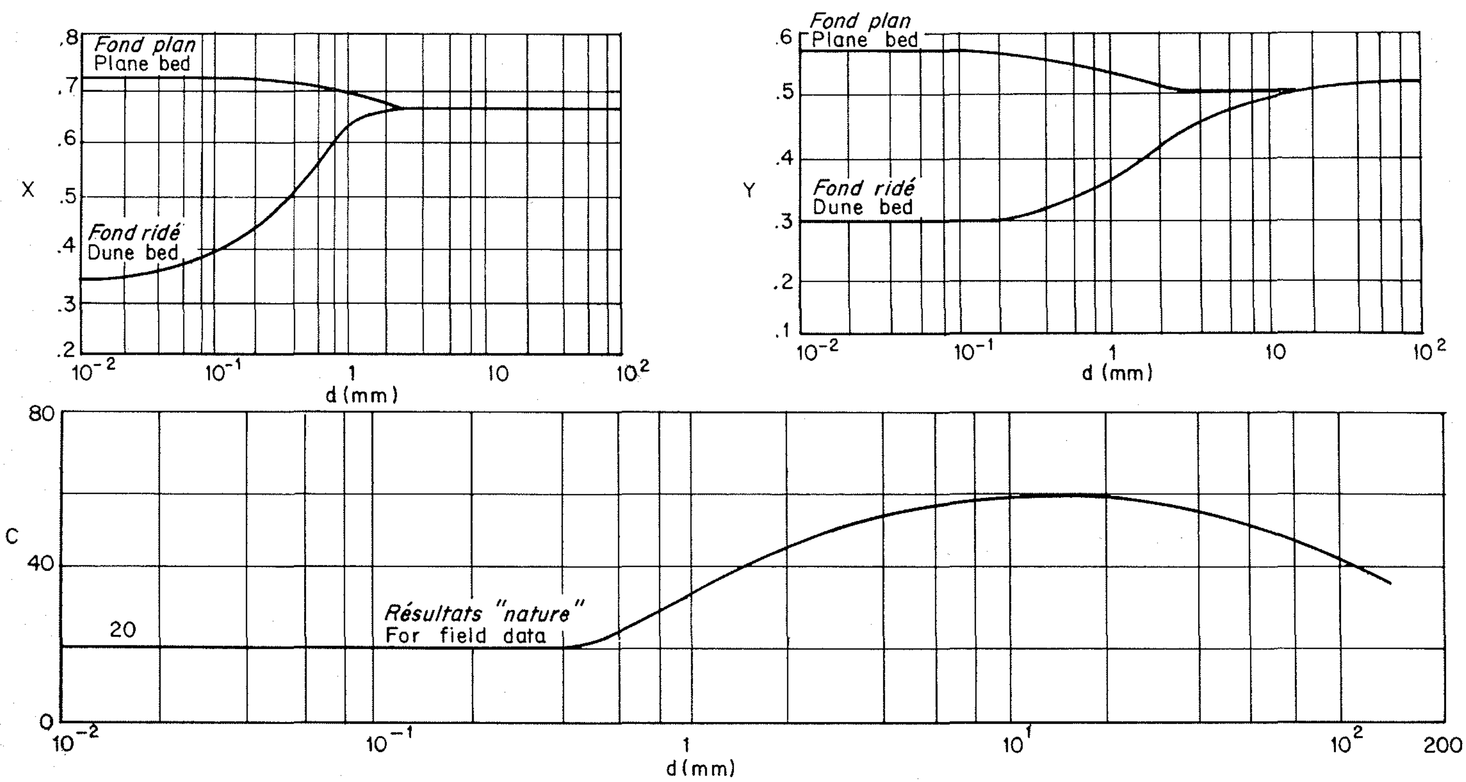
Variations de k'_s/h en fonction de h/l **/6**
Variation of k'_s/h *with* h/l



Variations de k'_s/d en fonction de τ_* et de $F = U/\sqrt{(\Delta\gamma_s/\rho_f) . d}$ **/7**
Variation of k'_s/d *with* τ_* and $F = U/\sqrt{(\Delta\gamma_s/\rho_f) . d}$



Variations de k'_s/d en fonction de h/d **/8**
Variation of k'_s/d *with* h/d



Variations de x, y et c en fonction de d en mm (en dimensions F.P.S.) **/9/** *Variation of* x, y *and* c *with* d *mm (in F.P.S. system)*

of constant $\mathcal{F}$ can be drawn. Thus k'_s/d is related with τ_* and $\mathcal{F}$. For known values of τ_* and $\mathcal{F}$, k'_s/d can be predicted from Figure 7. Usually R, S, d and $\Delta\gamma_s/\rho_f$ are known in alluvial channels; later a method has been suggested to predict the approximate mean velocity of flow.

b) VARIATION OF k'_s/d WITH MANNING COEFFICIENT n:

The value of Manning's n has been computed from the following equation:

$$n = \frac{1.486}{U} R^{2/3} S^{1/2}$$

Figure 8 shows the plot of k'_s/d versus $n/d^{1/6}$. There is a trend for increase in k'_s/d with increase in $n/d^{1/6}$ but the scatter seems to be very great and no definite law can be formulated.

Prediction of mean velocity of flow U

In order to known k'_s, one must know the mean velocity of flow U. For computing U, the slightly modified version of Liu-Hwang [19] 1961, formula is used. It is as follows:

$$U = CR^x S^y$$

where C, x and $y = f(d)$. The value of x and y can be obtained from author's graph (see Fig. 9). The c-d graph of author was modified in 1961 [5]. Knowing the values of R, S and d, mean velocity of flow can be predicted.

Velocity distribution in vertical

After studying the variation of $\mathcal{K}$ and k'_s the following procedure is recommended for computing the velocity distribution of D, S, d, ρ_s and ρ_f are known.

Steps:

1. Knowing the bed material size, find the values of C, x and y (Fig. 9) and compute U.
2. Compute τ_* and $\mathcal{F}$.
3. Knowing τ_*, read the value of $\mathcal{K}$ from Figure 4.
4. Knowing τ_* and $\mathcal{F}$, read the value of k'_s from Figure 7.
5. Knowing $\mathcal{K}$ and k'_s, velocity distribution can be predicted from equation (4).

Conclusion

As a result of the present investigation, the significance of various factors influencing the flow in alluvial channels has been brought to light. The parameters of primary importance are τ_* and Froude number. The Froude number of this form

$$\frac{U}{[(\Delta\gamma_s/\rho_f) \cdot d]^{1/2}}$$

has been found to be more adequate.

The equation of velocity distribution in alluvial channel has been found to be of following form:

$$\frac{u}{u_*} = \frac{2.3}{\mathcal{K}} \log_{10} \frac{y}{k'_s}$$

in which $\mathcal{K}$ and k'_s depend on flow, fluid and sediment characteristics.

TABLE 1

Summary of the field data used in the analysis

No.	Stream or canal	d (in mm)	Range of D (in ft)	Range of Q (in cfs)	Symbol
1	U.P. Canals (India) [10].	0.189-0.365	2.96-13.07	376-9703	o
2	American Canals [12].	0.029-0.805	2.61- 8.50	43-1031	○
3	Niobrara River [2] (Nebr. USA). . . .	0.28	1.40- 3.60	215-916	◓
4	Middle Loup River [6] (USA).	0.30	1.10- 2.40	393-479	□

TABLE 2

Summary of the flume data used in the analysis

No.	Investigation	d (in mm)	Flume	Symbol
1	Laursen [4].	0.04 & 0.10	105' × 3' & 1.5' deep	◒
2	Pien [4].	0.18	70' × 0.94' & 0.82 deep	⊕
3	Barton & Lin [4].	0.18	65' × 4' & 2.0' deep.	●
4	Einstein & Chien [4].	0.27	40' × 1.0' 1.0 ft deep.	+
5	U.S.G.S. [4].	0.45	150 ft × 8 ft & 2.0 ft deep.	x
6	Vanoni & Brooks [18].	0.091	40' × 10.5 in. & 10 in deep.	△

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Résumé

Distribution des vitesses dans les canaux alluvionnaires

par R. J. Garde * et A. S. Paintal **

Il est nécessaire de connaître la distribution verticale des vitesses dans les canaux alluvionnaires pour traiter de nombreux problèmes concernant le débit solide en suspension, la stabilité du profil...

Dans le cas d'un écoulement turbulent sur paroi rigide, les théories de longueur de mélange de Prandtl et de similitude de Karman conduisent à la loi de distribution logarithmique des vitesses (1), vérifiée par Nikuradse pour les conduites artificiellement rugueuses (équations 2 et 3). Keulegan a montré d'autre part que les lois de Nikuradse étaient transportables à des écoulements à surface libre.

Mais, pour les canaux alluvionnaires, la constante de Karman $\mathcal{K}$ et le paramètre de longueur k_s dépendent des caractéristiques de l'écoulement, du fluide et des sédiments. Les auteurs proposent l'équation (4) dérivée de l'équation (3) de Nikuradse pour les parois hydrauliquement rugueuses. Utilisant les résultats publiés par de nombreux chercheurs, ils ont déterminé les lois de variation des paramètres $\mathcal{K}$ et k'_s.

* Reader in Civil Engineering, University of Roorkee (India).

** Lecturer in Civil Engineering, University of Roorkee (India).

Variation de la constante de Karman $\mathcal{K}$:

Ce paramètre dépend de la rugosité des ondulations du lit et est influencé par la présence de sédiments en suspension. L'analyse dimensionnelle et l'expérience montrent que $\mathcal{K}$ ne dépend que de l'indice de mobilité du lit :

$$\tau_* = \gamma_f RS/g\,(\rho_s - \rho_f)\,d$$

et du nombre de Froude :

$$\mathcal{F} = \frac{U}{[(\gamma_s - \gamma_f)/\rho_f . d]^{1/2}}$$

Voisine de 0,4 pour des écoulements sur fond fixe, la constante de Karman varie dans de larges proportions dans le cas d'écoulements sur fonds alluvionnaires (fig. 4). En effet, si τ_* augmente, le débit solide croît et la turbulence, caractérisée par $\mathcal{K}$, diminue. L'influence du paramètre $\mathcal{F}$ n'a pas été examinée.

Variation de k'_s :

Le paramètre k'_s/d ne dépend également que des paramètres τ_* et $\mathcal{F}$, dans le cas d'un écoulement sur fond alluvionnaire comportant des rides et des dunes. Les résultats expérimentaux ont permis de définir la loi de variation de k'_s/d (fig. 7).

Les auteurs ont pensé qu'il serait possible de définir une relation (équation 9) entre le paramètre k'_s et le coefficient de rugosité de Manning n, à l'image de l'équation (8) correspondant au paramètre k_s d'Einstein-Chien (équation 7); mais la dispersion expérimentale ne permet pas de préciser une telle relation (voir fig. 8).

Remarque :

Les figures 4 et 7 donnent les paramètres de l'équation de distribution verticale des vitesses, connaissant τ_* et $\mathcal{F}$. Les auteurs proposent pour le calcul de la vitesse moyenne U figurant dans $\mathcal{F}$ la relation $U = CR^x S^y$ de Liu-Hwang (fig. 9).