SOME ROUGHNESS-CONCENTRATION EFFECTS ON BOUNDARY RESISTANCE

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Introduction

Prediction of the resistance which a given boundary presents to a steady, uniform flow raises problems which have been faced principally by resorting to experimentally determined resistance coefficients. It has generally been accepted that the most satisfactory form of relationship which expresses the resistance is the semi-logarithmic form, which, as shown by Millikan [1], may be derived by dimensional considerations alone. This type of relationship has been experimentally confirmed many times for both pipe and channel flows, for smooth surfaces and for many kinds of rough surfaces. Of the latter, perhaps the Nikuradse experiments are the most complete [2]. Although they were confined to pipe flow, the practice of expressing engineering types of roughness in terms of the size of the equivalent Nikuradse roughness has become widespread. Of course, the rough surfaces encountered bear little resemblance to Nikuradse's closely packed sand grains artificially fixed to the pipe walls, but the common basis of comparison in terms of a uniform and geometrically simple rough surface has proved valuable in the physical visualization of a certain roughness. Moreover, the use of one geometric variable results in convenient mathematical expression of the resistance relationship. For these reasons, the concept of an equivalent sand grain roughness will be utilized throughout this discussion.

Schlichting [3], in an early investigation of boundary resistance, measured the resistance of surfaces to which roughness elements had been attached. In his series of experiments, the roughness was applied to one boundary of an otherwise smooth rectangular conduit. The spacing or concentration of the elements, as well as their geometric form, was varied, and the roughness of each surface was expressed in terms of an equivalent uniform-sand roughness. Schlichting recognized that the usual types of roughness could not be geometrically described by a single dimension, such as a roughness-element height, but that a roughness density, such as "the number of individual roughness elements per unit of area," must also be specified. It would be expected that changes in the spacing of the roughness elements would cause changes in the effective roughness of a surface and that these changes would be of the same order of magnitude as changes caused by varying the roughness-element size. Schlichting observed the manner in which the equivalent sand roughness varied as the concentration of a given type of element was increased (Fig. 1). A large variety of roughness elements were used in his experiments, and since then several others have experimented with bars,

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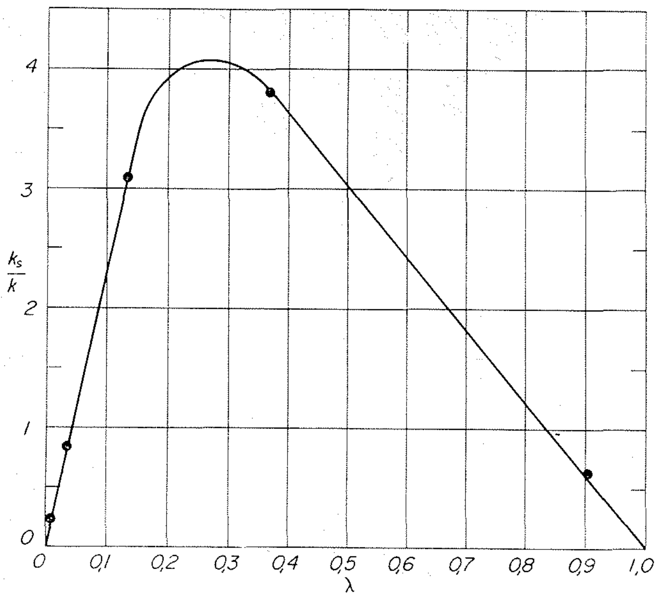


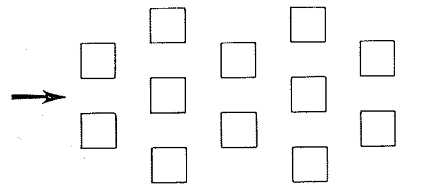
1/ Variation of the equivalent roughness size with concentration, after Schlichting [3].
Variation de la dimension de la rugosité équivalente en fonction de la concentration, d'après Schlichting [3].

short angles, square pipe threads, etc., varying their concentration and pattern while the boundary resistance was observed. Most of the results of these studies have been collected and compared by Koloseus and Davidian [4]. Extensive additional measurements were made by Koloseus and Davidian during an investigation primarily concerned with the effect of the presence of roll waves on resistance in open-channel flow. In this investigation, the concentration of cubical roughness elements was varied over a sixty-fourfold range, while the pattern was held constant. The present writers have since extended the work not only with cubes, but also with sand grains, for which the concentration was varied over an eighty-fold range, including as one extreme a roughness density which was similar to that of Nikuradse. The concentrations for the cube data, including those of Koloseus and Davidian [4], now extend over a three-hundred-sixty-fold range.

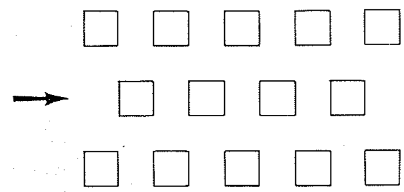
It must be mentioned here that some of the above investigations were made under conditions in which the roughness elements attached to the smooth bed could not be considered as a rough-textured surface, but rather as a series of boundary irregularities which were of the same order of magnitude as the flow depth itself and which caused relatively large local disturbances to the flow. The measured resistance included the effects of these large nonuniformities, and whenever the elements were large enough to cause appreciable disturbance to the flow, as evidenced for example by the formation of standing surface waves over each roughness projection, then the measured resistance must be considered separately from the resistance which is due to the shear stress generated by an unconfined fluid flowing past a roughened boundary. The investigation which is described here was designed to provide more complete information on the variation of the resistance coefficient with roughness concentration, under conditions which were independent of the free-surface effect described above, and to examine the nature and the magnitude of this type of free-surface effect. Emphasis has been placed upon the resistance measurements at high roughness concentrations, and the results have been compared with those of Nikuradse [2]. The form of the transition between smooth and fully roughwall flow is investigated for several sand concentrations, and the dissimilarity from the uniform roughness transition is noted. The series of experiments have, in addition, enabled observations to be made regarding the reference bed elevation and the value of the Kármán coefficient in open-channel flow.

Roughness characteristics

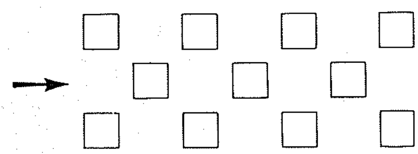
As mentioned above, the variation of the resistance coefficient with roughness concentration was studied experimentally by systematically varying the spacing of cubes and sand grains fixed to a smooth surface. The cubes were arranged in a fixed pattern, although the pattern used by the present writers was slightly different from that used by Koloseus and Davidian (see Fig. 2). The sand-grain distribution was somewhat random, as described later.



Pattern I - Disposition I



Pattern II - Disposition II



Pattern III (Koloseus and Davidian)
Disposition III (Koloseus et Davidian)

2/ Cube patterns.
Dispositions des éléments cubiques.

The uniform-cube roughness was obtained by having 6-inch-square plates cast of aluminium, an integral part of which was formed by the desired arrangement of 1/2-inch cubes. Care was taken to ensure that the top edges of the cubes were sharp and that the cubes joined the plates cleanly. Sets of plates were prepared having roughness concentrations of 0.11, 0.25, 0.44, and 0.70, the roughness concentration λ being expressed in each case according to Schlichting's definition, i.e., the ratio of the sum of the upstream projected areas of the roughness elements to the floor area,

$$\lambda = \frac{na}{A} \tag{1}$$

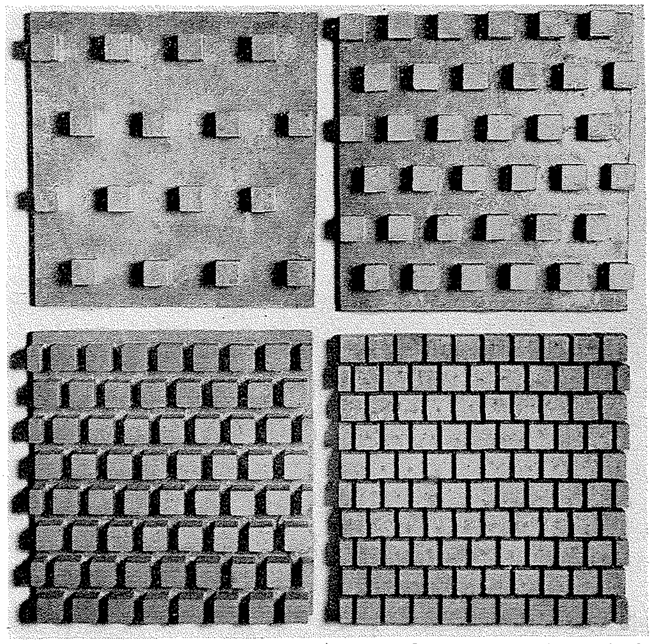
where n is the number of elements on the floor area A and a is the upstream projected area of each element. When applied to cubes, this measure of concentration varies from zero to one as the spacing of the cubes is reduced from infinity to zero. At a concentration of one, the top surfaces of the cubes form a continuous smooth surface.

The placement of the cubes on the plates was arranged to provide a uniform pattern when the plates were installed in the flume. By rotating each plate 90 degrees, a second pattern of roughness elements could be obtained. This afforded a ready means for observing the effect of this pattern change on the resistance coefficient for these particular concentrations. Four of these aluminum plates, each having a different concentration of cubical elements, are shown in Figure 3.

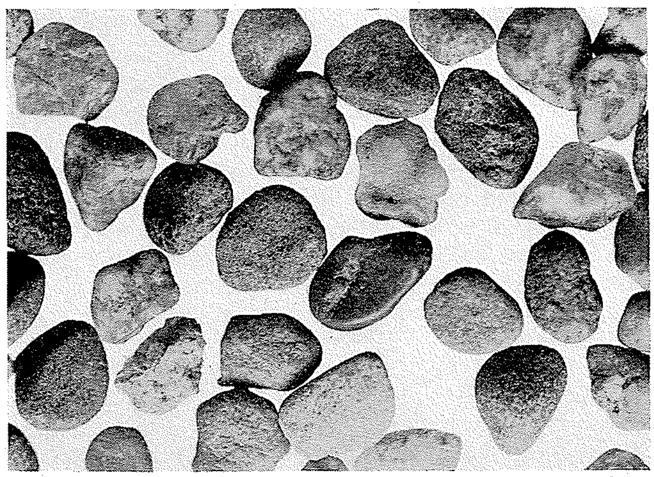
The uniform-sand roughness was produced using "Muscatine" sand passing a U. S. Standard Sieve No. 6 but retained on Sieve No. 8. The grains of this particular type of sand are quite angular (see Figure 4) and appear to be composed of a wide variety of minerals. The equivalent mean diameter of these sand grains was found by actual volumetric measurement of a counted number of grains to be 0.0096 foot. This value was used for the mean sand-grain height k.

Sand roughness concentrations of 0.0077, 0.09, 0.27, 0.45, and 0.64 were studied. All but the last of these concentrations were produced by first covering the glass floor of the 2-1/2 $\times$ 85-foot tilting flume with one coat of shellac and allowing this to dry, then covering the shellac with a coat of varnish, and allowing this to dry; the flume was next marked off into equal rectangular areas, a second coat of varnish was applied, and, while it was still wet, equal weights of sand were hand-sprinkled onto each area; finally, after the varnish has hardened, the entire surface was sprayed lightly with very thin varnish. Care was taken while sprinkling to obtain a fairly uniform density without influencing the essentially random spacing between the grains. Silhouette photographs of the five sand roughnesses are shown in Figure 5.

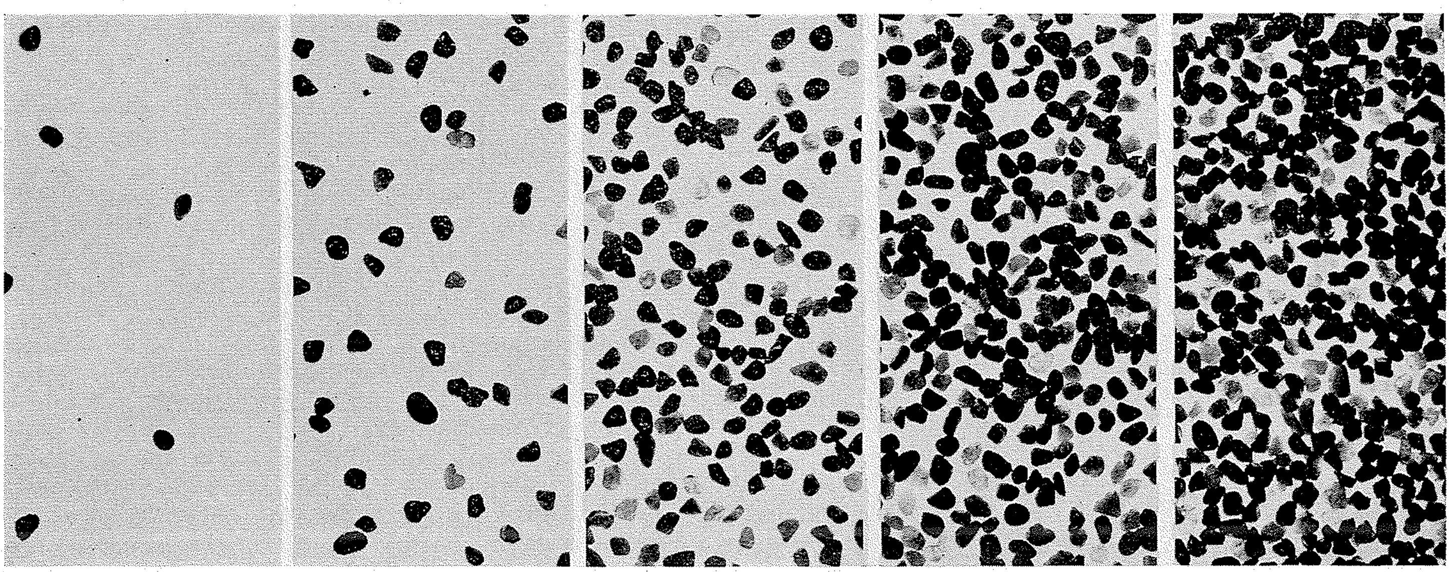
As mentioned above, the 0.64 sand-grain concen-



3/ Four concentrations of cube roughness, cast in 6 inch aluminum plates.
Quatre concentrations de rugosité, cubes moulés intégralement dans des plaques d'aluminium de 6 pouces.



4/ Photomicrograph of Muscatine sand, equivalent sphere diameter 0.0096-foot.
Photomicrographie de sable « Muscatine ». Diamètre de la sphère équivalente : 0,0096 pied.



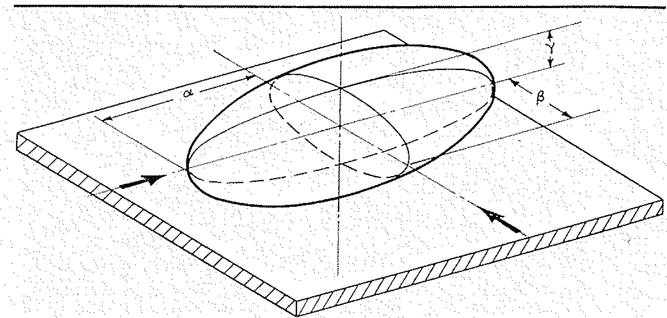
5/ Silhouette photographs of the five concentrations of sand roughness
Photographies, en silhouette, des cinq concentrations de rugosité constituées par du sable.

tration represented an attempt to reproduce in an open channel the pipe roughness used by Nikuradse. Glass plates 1/8 inch thick by 2 feet square were given the same preparatory treatment as described above. The plates were then set into shallow box frames and, with the varnish still wet, a known weight of sand was poured onto each plate to a depth of approximately 1/2 inch. The glass was tapped lightly from beneath to prevent any local arching of the sand above the glass. After allowing several days for the varnish to harden, the plates were tipped over and the excess sand poured off and weighed. Remaining on each plate was a layer of sand, one grain in thickness, adhering to the varnish. The roughened plates were sprayed as before with very thin varnish to ensure a good bond between the sand and the glass. A sufficient number of plates were produced in this way to cover the floor of the 2×30-foot flume for its entire length.

The maximum concentration of these irregularly shaped sand grains cannot be specified exactly, because it was computed using for a the projected area of a spherical grain of the same volume as the average grain, and the relationship between the actual projected areas of the grains and the projected area of the equivalent spherical grain is not known. If the grains can be represented as ellipsoids with semi-axes α, β, and γ, and if one assumes that the grain comes to rest in its most stable position, the grain on the smooth surface will appear as in Figure 6, and it will be oriented so that the direction of flow can be considered to be between the two extremes shown. Thus it may be seen that the ratio between the actual projected area of the grain and the projected area of the equivalent spherical grain will vary between the two extremes:

$$\sqrt{\frac{\alpha\gamma}{\beta^2}} \quad \text{and} \quad \sqrt{\frac{\beta\gamma}{\alpha^2}}$$

Microscope measurements were made on 100 representative grains of two of these axes, and the third was computed from the known mean volume of the grains. From these measurements, the two possible extremes of the above ratio were found to have the values 1.11 and 0.82. Thus it seems reasonable to assume that the concentration computed on the basis of spherical grains would be close to the concentrations actually achieved in the flume. At this point, it is interesting to note for comparison that hexagonally packed spheres correspond to a concentration of 0.905.



6/ Sand grain idealized as ellipsoid resting on smooth surface.
Représentation idéalisée d'un grain de sable, comme ellipsoïde reposant sur une surface lisse.

Experimental apparatus and procedure

Two flumes were used for making the resistance measurements. All the cube roughnesses except the one used to measure the free-surface effects were tested in the Institute's 2×30-foot glass-walled tilting flume, the slope of which is adjustable between zero and 2 degrees. Water is supplied from a constant-level tank up to a maximum capacity of 2.7 cubic feet per second. The stainless-clad floor of this flume was covered with the 6-inch aluminum plates over all but the upstream 5 feet of its length. Depth measurements were made along a section 12 feet in length at the downstream end of the flume. The sand roughness was applied to a section of a larger $2\text{-}1/2 \times 85$-foot glasslined tilting flume. The length of the roughened section was 45 feet, the section over which measurements were taken being 20 feet in length. This flume has a slope variation of 3 degrees. Water is supplied from a constant-level tank up to maximum capacity of 3.85 cubic feet per second.

The depth of flow was measured by means of an 1/8-inch-diameter static tube suspended from a carriage and fixed on the flume center line. This tube was connected to a micromanometer also connected to the carriage. A fixed cross-hair on the adjustable manometer leg permitted the water-surface elevation, and hence the depth of flow, to be read directly in thousandths of an inch. Minor corrections were applied where necessary to take into account small deviations in the rails and floor of the flume, even though these amounted to only a few thousandths of a foot. After the flow was established, the flume head—or tail—water gate was manipulated to obtain as uniform a depth of flow as possible. Depth measurements were taken every foot along the test section for each run, and plotted. The normal depth for each run was determined by inspection after sketching in the appropriate surface profile.

In tilting-flume experiments such as these, it may be shown that errors in the depth measurement will be introduced unless the manometer, which is leveled for each flume slope, is pivoted about a point which lies in the plane of the flume bed, and this point must be directly beneath the side piezometers of the static tube. With discrete roughness elements placed on a smooth bed, the bed plane is not uniquely defined. In every case, however, it was taken at the level which the bed would assume if all the roughness elements were melted down. The selection of this reference-bed elevation will be discussed more fully in a later section.

The cube roughness which was used to illustrate the free surface-effects consisted of 1.325-inch wooden cubes glued to the floor of the 85-foot tilting flume, at a concentration of 0.125. The cubes were finished with sharp edges, and were thoroughly sealed with epoxy paint to prevent swelling. In planning this phase of the investigation, it was realized that the local nonuniformities in flow caused by the cubes would introduce errors in depth measurement if static tubes were introduced. These errors were minimized by keeping the magnitude of the velocity head low in comparison with the depth of flow for all runs.

Experimental results

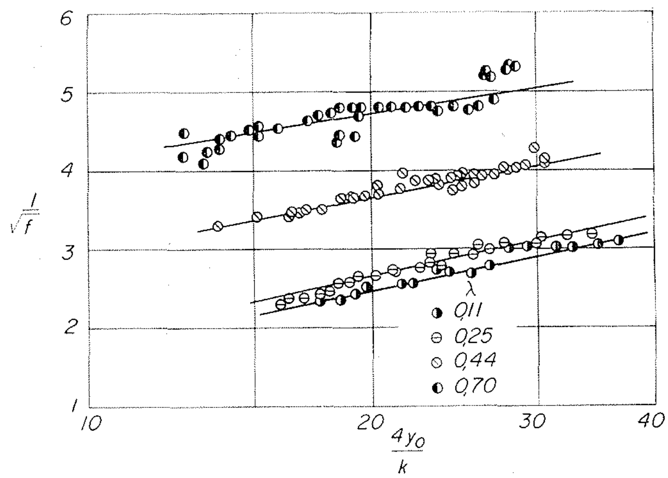
EFFECT OF CONCENTRATION :

The variation of the resistance coefficient $1/\sqrt{f}$ with relative roughness $4\,y_0/k$ is shown in Figures 7 and 8 for the cube and sand roughnesses, respectively. The factor y_0 is the normal depth of flow. In each of these plots a separate curve has been drawn for each value of the concentration λ. To each curve may be fitted an equation of the form:

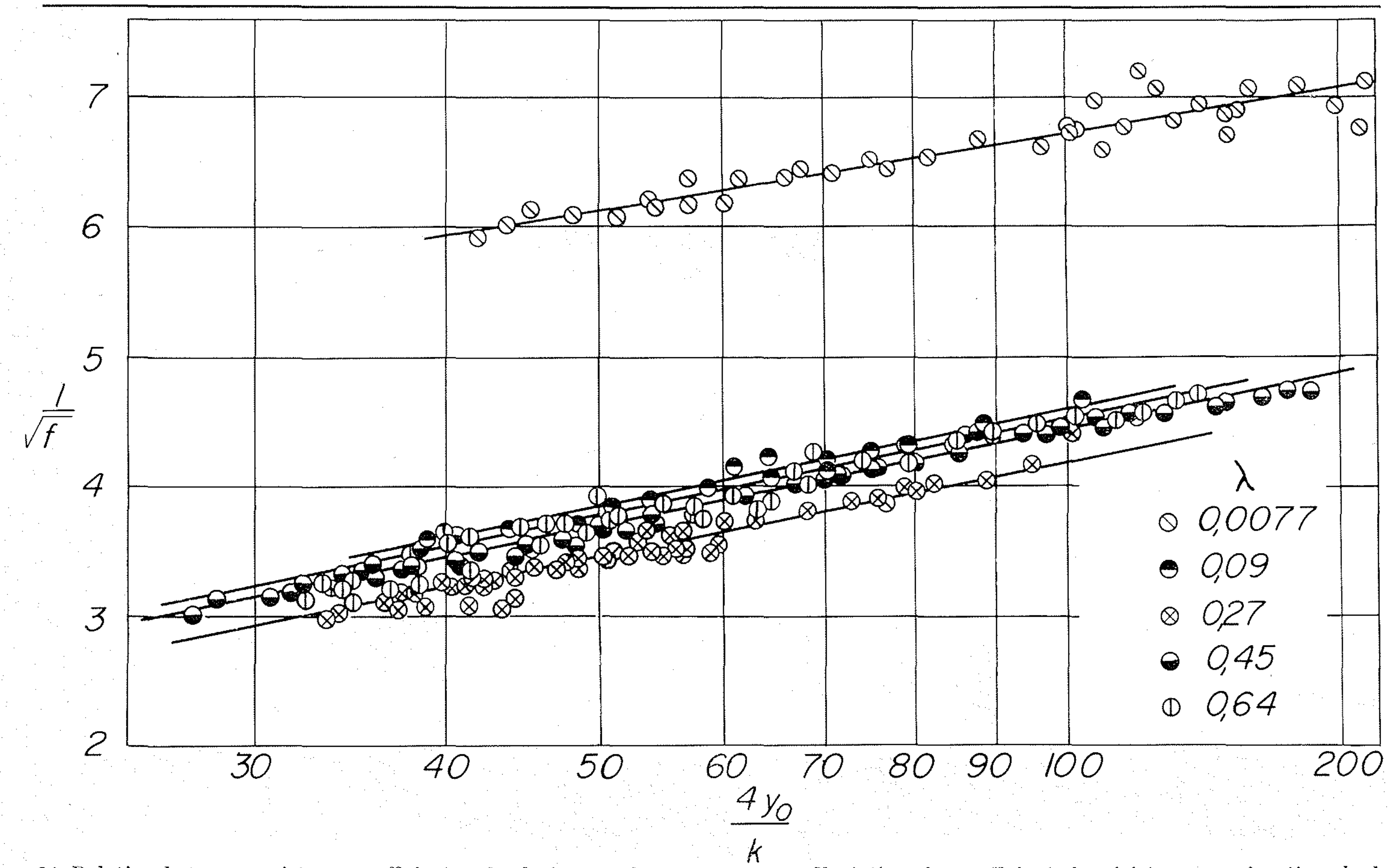
$$\frac{1}{\sqrt{f}} = \frac{A}{K} \log_{10}\left(\frac{Cy_0}{k}\right) \qquad (2)$$

where K is the Kármán coefficient, A is a constant, and C depends upon the roughness concentration.

Taking the slope of each curve for the moment to be constant, i.e., K is the same for every concentration, then the equivalent sand roughness k_s for



7/ Relation between resistance coefficient and relative roughness for four concentrations of cubical roughness elements.
Variation du coefficient de résistance en fonction de la rugosité relative, compte tenu de quatre concentrations d'éléments de rugosité cubiques.



8/ Relation between resistance coefficient and relative roughness for five concentrations of sand roughness elements.
Variation du coefficient de résistance en fonction de la rugosité relative, compte tenu de cinq concentrations d'éléments de rugosité en grains de sable.

any of the experimental roughnesses may be computed if it is assumed that a resistance equation can be written for flat surfaces roughened with the Nikuradse type of uniform sand, and if it is also assumed that this resistance equation is obtained by integrating the empirical velocity distribution observed by Nikuradse in pipes:

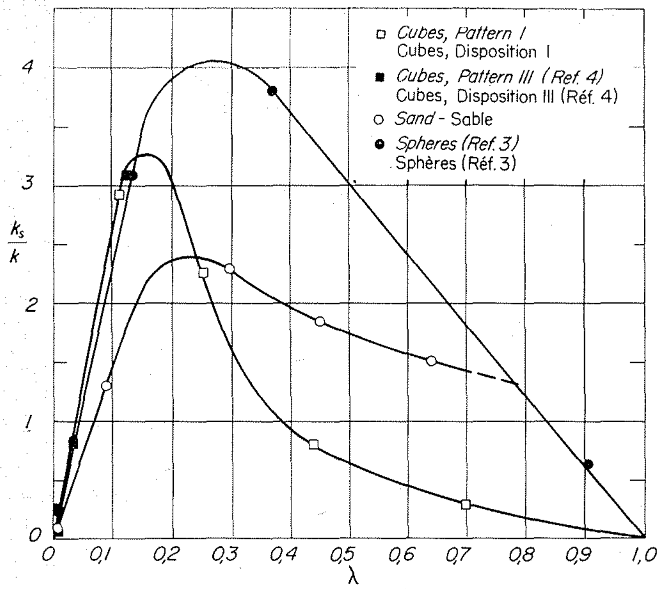
$$\frac{u}{u_*} = \frac{2.30}{K_N} \log_{10}\left(\frac{30\,y}{k_s}\right) \qquad (3)$$

where u is the velocity at a distance y from the boundary and u_* is the shear velocity.

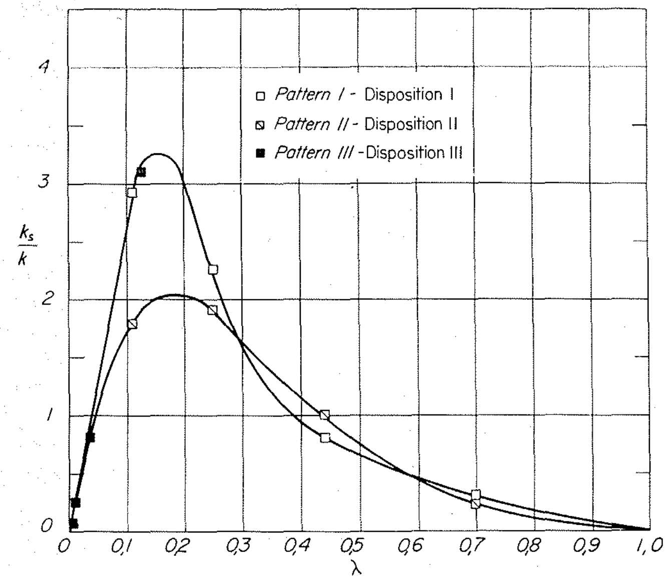
In Equation (3) K_N represents the value of the Kármán constant as determined by Nikuradse. Integration of this equation over the flow depth yields:

$$\frac{1}{\sqrt{f}} = \frac{0.81}{K_N} \log_{10}\left(2.75\,\frac{4\,y_0}{k_s}\right) \qquad (4)$$

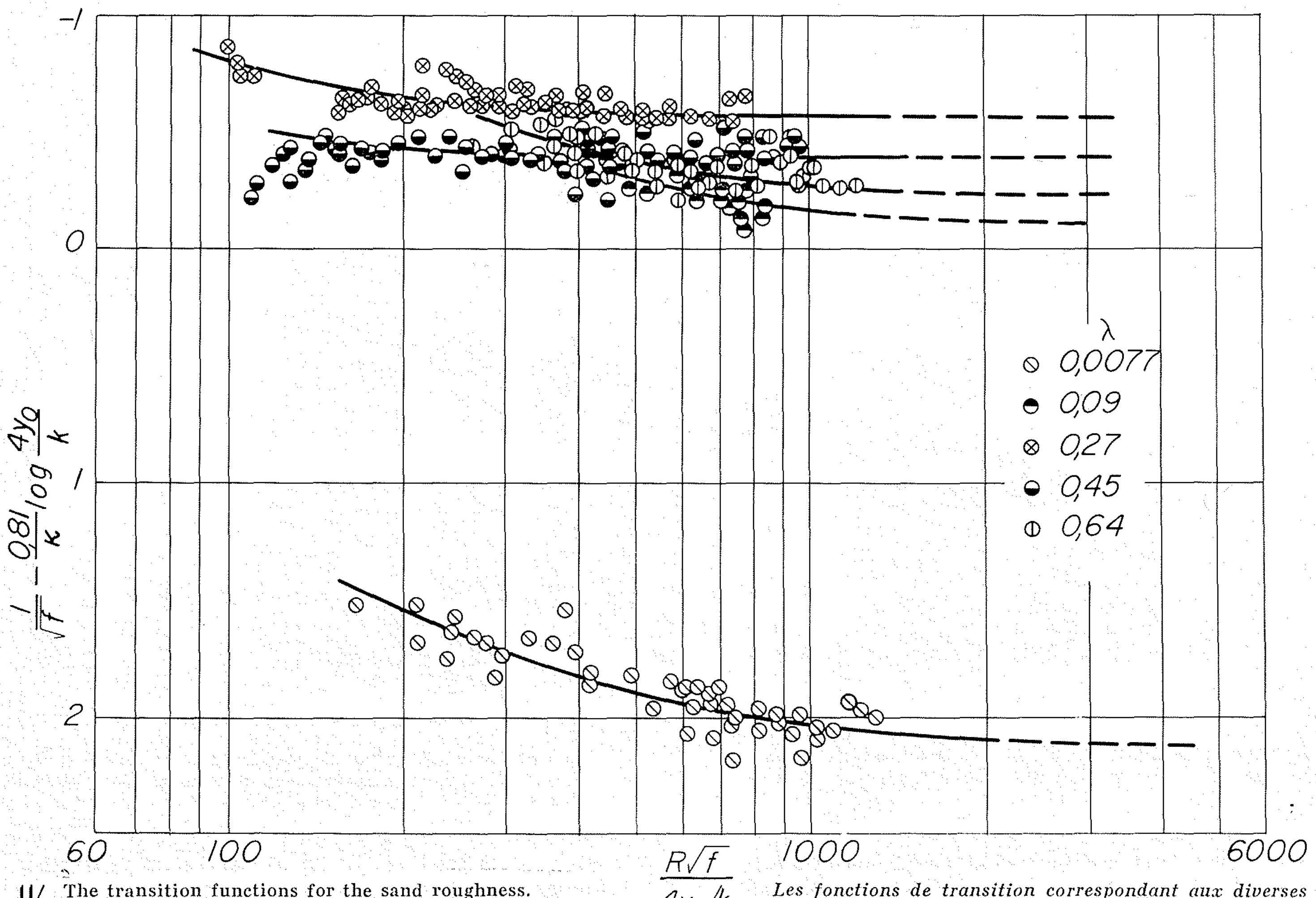
and it may be seen by comparing Equations (2) and



9/ Variation of the equivalent roughness size with concentration for cubes, spheres and sand grains.
Variation de la dimension de la rugosité relative en fonction des concentrations de cubes, de sphères et de grains de sable.



10/ Variation of the equivalent roughness size with concentration for different patterns of cubical elements.
Variation de la dimension de la rugosité relative en fonction de la concentration, compte tenu de différentes dispositions d'éléments cubiques.



11/ The transition functions for the sand roughness.
Les fonctions de transition correspondant aux diverses rugosités en grains de sable.

(4) that the equivalent sand roughness for the same value of the resistance coefficient is given by:

$$\frac{k_s}{k} = \frac{11}{C\,(A_{K_N}/0.81\,K)}\left(\frac{y_0}{k}\right)^{1-\frac{A\,K_N}{0.81K}} \qquad (5)$$

If the slope A/K of the curves in Figures 7 and 8, equals $0.81/K_N$, then the above expression simplifies to:

$$\frac{k_s}{k} = \frac{11}{C} \qquad (6)$$

If the Kármán "constant" changes from one concentration to another, then these changes will appear as differences between the slopes of the curves in Figures 7 and 8 for varying concentrations. Also, if the value of the Kármán coefficient for any concentration is different from Nikuradse's, then Equation (5) indicates that the equivalent sand-roughness depends upon the relative roughness. If this is to be the case, then the usefulness of the equivalent-roughness concept is greatly diminished. Fortunately, the exponent of the relative roughness in Equation (5) is almost zero for the data of Figures 7 and 8, so that the relative roughness has a negligibly small influence on the equivalent sand roughness computed using Equation (5). The results of this computation for the cube roughnesses are shown in Figure 9, with the results of Koloseus and Davidian, as well as Schlichting's sphere data. The sand equivalent roughnesses shown in Figure 9 were computed from Equation (5) after first determining the values of C in Equation (2) from the horizontal asymptotes of the transition curves in Figure 11.

It is immediately obvious from Figure 9 that the variation in the effective resistance of a surface caused by changes in concentration must be analyzed in terms of low and high concentrations. At a concentration of zero, the equivalent sand roughnes is zero—i.e., the surface is smooth. At low concentrations, there is almost exactly a one-to-one correspondence between the equivalent sand roughness and the concentration, no matter what the shape of the roughness element. At an intermediate concentration, the wake behind each element is not diffused before the flow reaches the next element. Morris [5] has described this wake interference type of flow in some detail. Further increases in the concentration simply increase the amount of mutual interference between the roughness elements, and each element becomes less effective in creating boundary resistance through the sheltering effect of its neighbors. The size of the equivalent sand roughness therefore decreases when the concentration is increased beyond that point at which wake-interference effects become pronounced.

It is easier to visualize the mutual interference between the roughness elements in the case of the cube roughness. Behind each cube is a separation pocket filled with relatively still water. If the spacing of the cubes is reduced to the extent that the cubes overlap with the nearby seperation pockets, then it is possible that the spaces between the cubes are filled completely with fluid which has no significant net motion in the direction of the outside flow. Morris has described this as skimming flow, and the resistance would certainly be expected to be not very different from that of a continuous surface formed by the tops of closely packed cubes. The cube concentration of 0.7 provided a good example of skimming flow, and the very low boundary resistance accompanying this type of flow could well explain the strongly concave shape of the cube curve in Figure 9. This physical behavior also suggests that the curve should become asymptotic to the horizontal axis as shown.

The somewhat different behavior shown in Figure 9 by the curves for the sand and Schlichting's spheres follows naturally from a consideration of the extent to which skimming flow may develop. In the case of the cubes, the surfaces of separation originating at the leading edges are all at the same elevation, and at high concentration these surfaces may be considered to form a continuous plane just above the tops of the cubes. Although this continuous surface is not smooth, it must be noted that no roughness elements protrude through it. On the other hand, the comparative irregularity of the sand grains results in the formation of a series of discrete separation pockets with the peaks of the grains protruding. Thus, even though the grains may be close enough to each other to cause appreciable interference between them and their separation pockets, the jagged outlines and peaks of the grains and the hap-hazard points of separation prevent the formation of an ideal skimming flow, and, as a result, the surface resistance is not reduced to the same degree as for surfaces over which skimming flow develops more fully. The falling limb of the curve for the sand therefore does not show the marked concave dip shown by the cubes. Schlichting's sphere-concentration data display the same trend as that described above for the sand. The more regular configuration of the spherical elements and their associated separation zones when compared with the sand suggest that the skimming flow phenomenon develops to an extent which is intermediate between that for the sand and the cubes. Thus, in the range of concentrations between 0.25 and 0.7, it is seen that the rate of decrease of the size of the equivalent uniform-sand roughness is a maximum for the cubes, a minimum for the sand, and an intermediate value for the spheres. This physical description of the onset of skimming flow and its effect on the resistance of a surface appears to provide a satisfactory qualitative explanation of the difference between the curves for the three types of roughness elements.

Effect of Pattern:

As mentioned previously, the cubes were fabricated so that a change in pattern could readily be made. Resistance-coefficient measurements were made on the second pattern and the value of the equivalent uniform-sand size was computed for each concentration. The results for this pattern are compared in Figure 10 with those for the previous cube pattern. Again, while such a pattern change has altered the shape of the curve, the general form of the function has not been changed. The maximum effective roughness again occurs at a concentration of about 0.2, and, at high concentrations, the closely packed roughness behaves like a smooth surface. The flatter slope of the falling limb may be explained in terms of the development of the skimming flow over the cubes. This second pattern appears to the oncoming flow as a number of columns parallel to the mean flow. The space between any two columns represents a discontinuity in the otherwise continuous surface formed by the coalescing zones of separation. The skimming flow and its accompanying decrease in surface resistance develops at a lower rate as the concentration is increased. At concentrations approaching unity, the width between the columns of elements becomes so small that the effect of the change in pattern vanishes. The difference in the peak values of the equivalent-uniform-sand size has not been examined in any detail, but is undoubtedly associated with the change in the flow pattern around each element caused by the change in the cube pattern.

COMPARISON WITH NIKURADSE'S EXPERIMENTS:

The maximum sand concentration shown in Figure 9 permits a comparison to be made between the channel-resistance measurements described here and the rough-pipe measurements of Nikuradse. There was no record made by Nikuradse of the roughness density which he achieved in his pipes, but the detailed description of the method of sand application indicates that the density would have been close to the maximum possible. The method of application of the sand grains in the channel experiments described here was designed to duplicate Nikuradse's procedure as far possible, but whatever differences were introduced by factors such as the lacquer and varnish characteristics, the amount of forced vibration applied to the boundaries, and the difference between applying the sand to a horizontal plate and a vertical pipe cannot be evaluated. The microphotograph of a sample of Nikuradse's sand grains also indicates that they were considerably more uniform in grain size and shape than the Muscatine sand. Nevertheless, the concentration of 0.64 achieved for the Muscatine sand should closely duplicate the density of the rough surfaces used by Nikuradse.

As shown in Figure 8, the equivalent roughness size for the 0.64 concentration was found to be 1.51 k. Probably the most influential factor causing a departure from the expected value of unity for k_s/k was the difference between the Nikuradse and Muscatine sands with respect to grain uniformity and sphericity. The addition of only a small number of large grains to a surface covered uniformly with small grains has been shown by Colebrook and White [6] to increase the equivalent roughness size by more than 50 %. In some earlier work, Schlichting [3] obtained in a single test for k_s/k a value of 1.64 for a closely packed sand roughness in a rectangular conduit. Thus it appears that, as well as the roughness height and the roughness concentration, at least one additional parameter expressing the degree of uniformity of the roughness elements is required in order to specify the resistance coefficient of a roughened surface. A parameter such as this has not been investigated in this series of experiments.

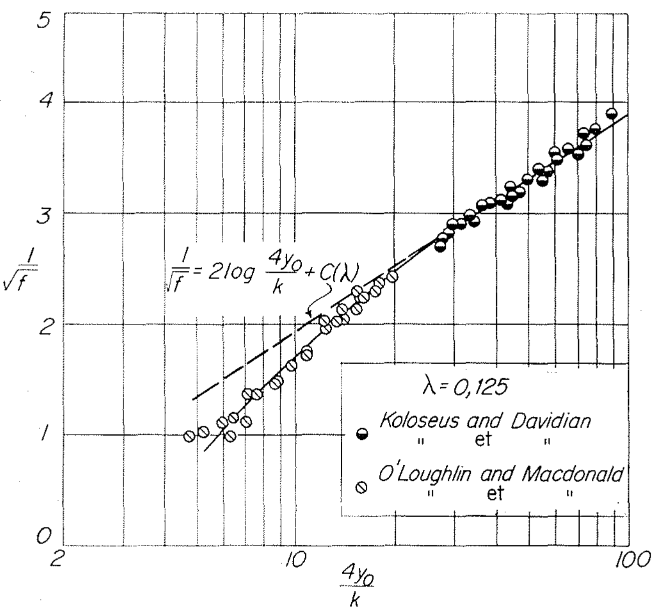
A further basis of comparison with the Nikuradse results is available in the transition function:

$$\frac{1}{\sqrt{f}} - \frac{0.81}{\mathrm{K}} \log_{10} \frac{4\,y_0}{k} = \Phi\left(\frac{\mathrm{R}\sqrt{f}}{4\,y_0/k}\right) \qquad (7)$$

which defines the relative influences of viscous and inertial effects on the resistance coefficient. In Figure 11 are shown the transition curves for each concentration of sand. The Reynolds number $\mathcal{R}$ is here defined as:

$$\mathcal{R} = \frac{4\,\mathrm{V}y_0}{\nu}$$

and the roughness height k is the equivalent sphere diameter of the sand grain, 0.0096-foot. It is evident from Figure 11 that the resistance coefficients measured for every sand concentration are not free from viscous effects—that is, each of the curves deviates from the horizontal as the roughness Reynolds number is changed. Of course, the nature of the rough boundary, with a large portion of the surface being smooth, would lead one to expect a variations of resistance coefficient with Reynolds number, but the magnitude of this variation would also be expected to be relatively minor, especially at the higher concentrations.



12/ Relation between resistance coefficient and relative roughness at small values of relative roughness.
Variation du coefficient de résistance, en fonction de la rugosité relative, aux faibles valeurs de ce dernier paramètre.

It is clearly seen from Figure 11 that the shape of the transition curve resembles the transition function for commercial-pipe materials rather than that obtained by Nikuradse for uniform-sand roughness. Whether this is the result of the nonuniformity of the sand grains of the presence of the smooth portions of the boundary cannot yet be stated with certainty. Roberson [7], in a digital-computer solution for the resistance of a boundary covered with cubes of varying concentrations, predicted transition curves of the commercial-roughness type, but the experimental measurements made to date with the cube roughness have not disclosed any dependence of the resistance coefficient on viscous effects.

THE REFERENCE BED ELEVATION:

Both the resistance coefficient and the Kármán coefficient for flow over a rough boundary at a given free-surface elevation depend upon the position of the bed plane to which depth measurements are referred. This plane is unambiguously defined if the geometric mean level suggested by Schlichting [3] is used. Obvious shortcomings are apparent when the roughness elements contain negligible volume, such as the short angles studied in detail by Sayre and Albertson [8]. Koloseus [9] has suggested that it would be more realistic in these circumstances to include an estimate of the volume of the separation pocket behind each element with the roughness-element volume. To be consistent with this practice, the volume of the separation pocket would need to be included no matter what the shape of the roughness element. For rounded elements, the additional volume would not be large enough to affect significantly either the Kármán "constant" or the resistance coefficient. For cubical elements, the same applies at low concentrations, but at the concentrations for which skimming flow is well developed it is physically naive to include in the flow passage a significant amount of the volume between the cubes. This behavior suggests the possibility of a link between the best bed elevation and the shape of the curves of Figure 9, which has already been discussed in terms of skimming-flow development. From velocity measurements made with the maximum sand concentration, it was observed that the best semi-logarithmic variation of velocity with depth was obtained when the data were plotted with respect to a plane 0.007 foot above the base of the grains, while the geometric-mean level was computed to be 0.004 foot above the base of the grains. The equivalent-sphere diameter of these grains was 0.0096 foot. The allocation of numerical coefficients to make any adjustment to the reference bed elevation computed with the Schlichting definition does not seem warranted yet, but it is worth noting qualitatively the interdependence between the bed-elevation selection, the roughness concentration, and the development of skimming flow.

Notwithstanding the uncertainty regarding the reference bed elevation, the Kármán coefficient was computed for all the data except the pattern II cubes, using for the datum elevation the geometric-mean level of all the roughness elements. These are shown in Table I. With the exception of the maximum cube concentration and the minimum sand concentration, all the values are closely grouped, with an average value of 0.33 (neglecting the two high values mentioned above). The authors are of the opinion that one is justified in claiming that this average value of K represents its true value in rough-channel flow rather than the value of 0.40 which is generally used, and the authors see no reason that this aevrage value should not have general applicability. No explanation is presently available for the departure of the two high values observed. Both high values were associated with relatively smooth boundaries.

Table I

Cubes		Sand	
Concentration	K	Concentration	K
0.11	.33	0.0077	.40
0.25	.32	0.09	.33
0.44	.36	0.27	.33
0.70	.46	0.45	.33
		0.64	.34

Effect of the Free-Surface Proximity:

Resistance coefficients were also measured over the range of depths at which the roughness elements were of the same order of magnitude as the depth. The aim of this series was to observe the manner in which the resistance coefficient variation with relative roughness departed from the semi-logarithmic law when the depth was small enough to allow the formation of surface disturbances over each roughness element. All the data obtained in this series have been plotted in Figure 12, together with some earlier resistance data obtained by Koloseus and Davidian [4] for 3/16-inch cubes at the same concentration but arranged in a slightly different pattern. A marked departure from the usual logarithmic resistance function is present for values of the relative roughness $4\,y_0/k$ less than about 12. For smaller relative roughnesses, the resistance coefficient $1/\sqrt{f}$ apparently decreases quite rapidly, indicating that the boundary made up of cubical obstacles upon a smooth plane presents a greater resistance to the oncoming flow than if the cubes are considered simply as a roughness in the hydraulic sense. In computing the parameters of Figure 12, the depth of flow was calculated with respect to a bed elevation arbitrarily chosen as the geometric mean level of the cubes and including the estimated volume of the still-water zone behind each cube. Some other reference elevation may be adopted, in which case the positions of the plotted points will be changed, in any case the resistance coefficient is seen to depart from the semi-logarithmic relationship in a manner which is as yet unpredictable.

Summary and conclusions.

Detailed observations have been made on the overall resistance in channel flow of surfaces which have been roughened by the addition of varying concentrations of regular cubical elements and natural sand grains. Emphasis has been placed on the concentrations of roughness for which wake interference becomes appreciable, up to the maximum concentrations which could be conveniently fabricated. By describing each surface in terms of an equivalent uniform-sand roughness, based on the rough-pipe experiments of Nikuradse, the effective resistance has been quantitatively related to the roughness concentration. The form of the relationship for each type of element is dependent upon the shape of the element, and an explanation for the difference between the relationships has been given in terms of the uniformity of the roughness elements and the degree to which any nonuniformity in shape permits the disruption of the flow phenomenon known as skimming. The extreme regularity of a cube-covered surface allows the development of skimming flow over the smooth upper surfaces of the cubes if the spacing of the cubes is small enough, resulting in a sharp drop in effective resistance. A change in the pattern of the cubes on the channel bed can be made to have the same effect as an alteration to the regularity or shape of the element. In each case, the development of a pure type of skimming flow is prevented by protruding roughness peaks, and the effective surface resistance as expressed by the equivalent sand roughness remains high, the rate of decrease depending upon the uniformity of the elements. At the maximum sand concentration, the nonuniformity effects is still evident, resulting in a relatively rougher surface than Nikuradse's. The necessary inference to be drawn is that yet a third parameter, besides roughness size and concentration, is required to describe the resistance characteristics of a roughened surface. This parameter must embody the combined effects of nonuniformities from one particle to another as well as the effect of the shape or angularity of the elements. Such parameters must, of course, be measured statistically when dealing with engineering surfaces.

The type of transition function characterizing the several sand concentrations appears to match that determined for commercial-pipe materials rather than the Nikuradse uniform-sand transition. This evidence of a viscous influence on the sand transition even to quite high values of the roughness Reynolds number may be due either to the effect of the smooth portions of the boundary or to the viscous influence on the flow around the roughness elements. The latter effect is not significant in the case of the sharp-edged cubes, and no viscous effect has been observed by Koloseus and Davidian even for low cube concentrations.

All of the data are dependent to varying degrees upon the level of the reference bed elevation. At low concentrations or at large relative roughness the problem diminishes in importance, but a high concentrations it appears that a rational and consistent result can be obtained if Schlichting's geometric-mean level is adopted, after being modified as suggested by Koloseus to include an estimated volume of the separation pockets in the roughness-element volume.

At extremely small values of the relative roughness, supplementary measurements demonstrated the nature of the resistance relationship, particulary with respect to its deviation from the semi-logarithmic expression. The magnitude of the local flow nonuniformities around each obstacle in the flow modifies the free-stream-velocity distribution to such an extent that the resistance coefficient obtained by integrating the logarithmic-velocity distribution is totally inadequate. It appears that the measured resistance is much higher than that predicted by the semi-logarithmic law, but the results depend to a large extent upon the interpretation of the measured quantities in computing the values of the depth and the velocity.

In closing, the authors wish to point out that the use of the Nikuradse roughened surfaces as a series of roughness standards is not entirely satisfactory, since these standard surfaces cannot be duplicated with any degree of certainty. Either spherical or cubical elements fixed to a smooth surface at a known concentration could provide a reproducible roughness standard for future needs.

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Résumé

Des effets de la concentration d'éléments rugueux sur la résistance

par

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La rugosité des parois ne peut pas être définie par la hauteur des aspérités seule; un paramètre tout aussi important est l'espacement des éléments rugueux. Schlichting [3] a fait une série d'expériences dans lesquelles il faisait varier l'espacement de plusieurs sortes d'éléments rugueux et il a exprimé la perte de charge mesurée en fonction des dimensions d'uniformes grains de sables utilisés par Nikuradse [2]. Plusieurs auteurs ont contribué aux résultats, qui ont été rassemblés par Koloseus et Davidian [4]. Des mesures additionnelles de pertes de charge ont été faites par Koloseus et Daviadian pour des valeurs de la concentration allant jusqu'à soixante-quatre fois la valeur minimale pour des éléments cubiques et cette gamme a maintenant été considérablement étendue par les auteurs de cette étude. Les modèles d'arrangement des cubes sur une surface lisse sont présentés à la figure 2. La perte de charge introduite par une surface lisse recouverte de grains de sable naturel pour des valeurs de la concentration couvrant une gamme de une à quatre-vingts fois la valeur minimale a été considérée dans cette étude. Des observations supplémentaires ont été faites sur la forme de la fonction de transition en comparaison avec la fonction de transition bien connue de Nikuradse; des observations ont été faites également sur le coefficient de Kármán pour des canaux découverts rugueux ainsi que sur la forme de la perte de charge pour de très faibles valeurs de la rugosité relative.

La concentration des éléments cubiques utilisés était 0,11, 0,25, 0,44 et 0,70, la concentration λ étant définie par l'équation (1). Ces éléments de rugosité ont été moulés en une seule pièce avec une base carrée d'aluminium de 6 pouces (fig. 3). Le sable avec un diamètre équivalent de 0,0096 pied a été collé sur le fond vitré du canal inclinable. Les caractéristiques des grains sont présentées à la figure 4 et les photographies des silhouettes des cinq granulations de sable à la figure 5. La concentration maximale de sable a été réalisée de façon à reproduire la rugosité étudiée par Nikuradse.

Les mesures ont été effectuées dans deux canaux inclinables différents de 30 à 85 pieds de longueur. La hauteur a été mesurée à l'aide d'un tube piézométrique et le coefficient de perte de charge a été calculé à partir de la hauteur normale évaluée. Le coefficient de résistance $1/\sqrt{f}$ est représenté aux figures 7 et 8 en fonction de la rugosité relative pour les éléments cubiques et le sable, une courbe particulière étant tracée pour chaque concentration. L'équation (2) peut être ajustée pour chaque courbe, et si la répartition des vitesses de Nikuradse (Eq. 3) est intégrée le long de la hauteur, donnant l'équation (4), alors le coefficient de rugosité de Nikuradse, k_s, peut être calculé par les équations (2) et (4) comme il a été fait pour l'équation (5). Le rapport k_s/k dépend de la rugosité relative si la valeur exprimentale du coefficient de Kármán est différente de la valeur du coefficient de Nikuradse K_n. S'ils sont égaux, l'équation (6) est alors applicable.

La variation de k_s/k avec λ, calculé à partir de l'équation (5), est présentée à la figure 9. Au-delà d'une concentration d'environ 0,2, la dimension de la rugosité équivalente décroît à cause de l'interaction des éléments rugueux décrite par Morris [5]. La variation de décroissance de la dimension de la rugosité équivalente uniforme avec l'augmentation de la concentration est trouvée être fonction de la variation de l'effleurement de l'écoulement, décrit aussi par Morris [5]. La forme et l'université des éléments et, à un moindre degré, l'arrangement influencent le degré auquel l'effleurement de l'écoulement se développe. La non-uniformité des grains de sable arrête l'effleurement des aspérités par l'écoulement, et la rugosité équivalente reste par conséquent élevée au maximum de la concentration. La non-uniformité relative des grains de sable résulte en un coefficient de résistance plus grand que celui de Nikuradse, comme il a été observé aussi par Colebrook et White [6].

Il apparaît que la fonction de transition équation (7), pour toutes les concentrations de sable, ressemble à la fonction de transition de la rugosité des tuyaux rugueux. Ces courbes montrent l'influence de la viscosité, mais il n'est pas apparent que ce soient les portions lisses des parois ou les changements possibles de l'écoulement autour des éléments qui causent un changement des pertes de charge en fonction du nombre de Reynolds. La forme générale des transitions est en accord avec celle prévue par Roberson [7].

Le choix d'une cote de référence pour le fond du canal est arbitraire; la moyenne géométrique suggérée par Schlichting [3] ne donne pas satisfaction pour des concentrations élevées. L'introduction de la poche de séparation comme paroi solide par Koloseus semble plus réaliste. La définition de la cote du fond de Schlichting a été utilisée pour cette étude. Les valeurs observées du coefficient de Kármán pour les cubes et pour le sable sont présentées dans le tableau I, la valeur moyenne étant 0,33.

Dans la figure 12, la différence avec la relation de la résistance semi-logarithmique est présentée pour de faibles valeurs de la rugosité relative. La présence de clapotis au-dessus de chaque élément indique que de grandes non-uniformités locales sont présentes dans l'écoulement, et il n'est pas justifié d'extrapoler l'équation (2) au-dessous de la limite $4\,y_0/k = 12$.

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