CONFIGURATION OF THE FREE SURFACE ABOVE A VERTICAL JET

BY
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AND
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Introduction

Flow within a submerged jet has been the subject of numerous studies conducted since 1947 at the Iowa Institute of Hydraulic Research. Analytical approximations of flow conditions within a submerged jet discharging into a semi-infinite fluid were made by Albertson, Dai, Jensen, and Rouse in their paper "Diffusion of Submerged Jets" [1] in 1950. This paper was based, for the most part, upon previous experimental findings made by Baines [2] and Dai [3]. Velocity distributions and turbulence characteristics of the jet were investigated, and expressions for the flow rate, energy losses, and momentum transfer were evaluated.

The effect of removing one of the idealized conditions imposed upon these initial studies, namely, that of an infinite fluid, was investigated by Murray, Nelson, and Steven [4] in 1956 and by Mross [5] in 1960. Murray, Nelson, and Steven investigated the streamline pattern and pressure distribution produced within a submerged jet when deflected normally by a wall, while Mross studied the velocity distribution of a horizontal jet bounded above by a free surface.

One of the remaining phases of this study, and the subject of this particular paper, is the effect of a free surface immediately above a vertical jet. It will be assumed that the results of previous work on submerged jets are approximately applicable in the region beneath the free surface. Thus, attention will be focused upon the carry and shape of such a jet above the free surface as the efflux velocity and nozzle submergence depth below the free surface are varied.

The configuration of a free surface above a jet may be described by the following seven variables, which are defined in Figure 1 :

$$f_1 (V_0, \Delta z, z_0, d_0, r, \rho, \gamma) = 0 \qquad (1)$$

Since three different dimensional categories are represented by these variables, the π-theorem states that four groups of dimensionless variables may be

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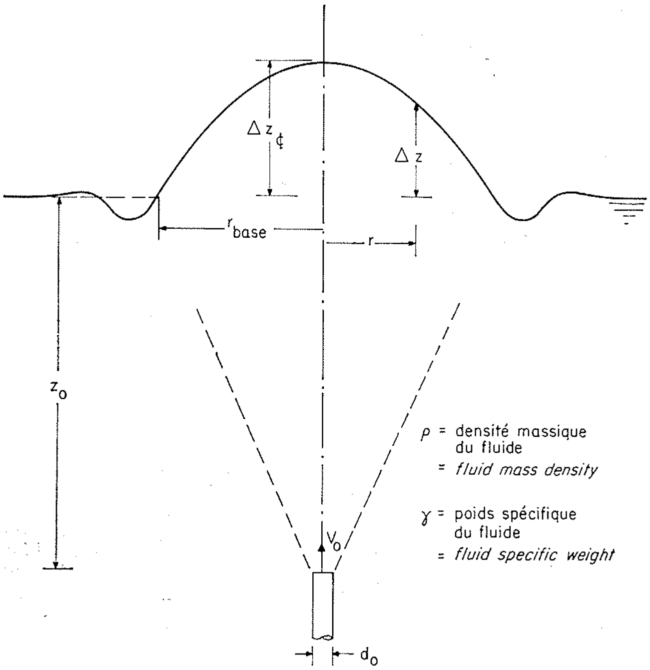
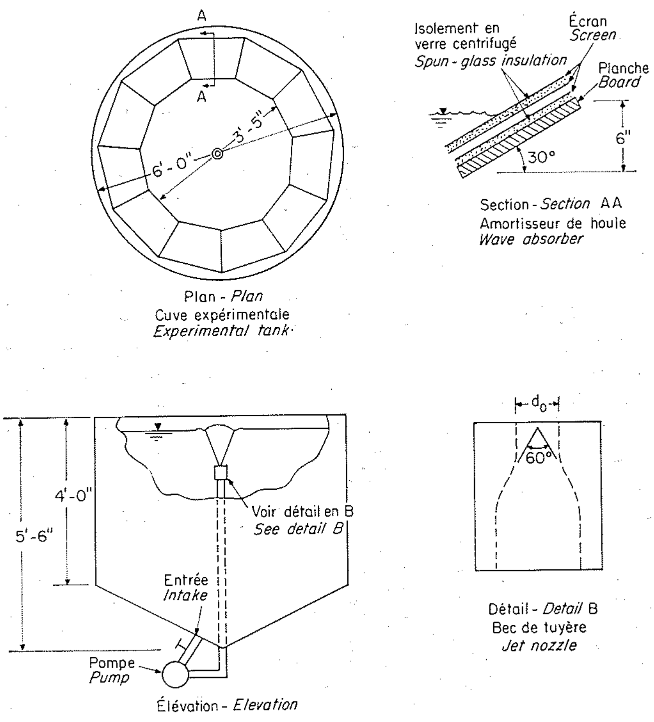


1/ Variables describing a vertical jet beneath a free surface.
Variables pour décrire un jet vertical sous une surface libre.

formed. Therefore, the relative surface configuration will be found to depend upon three dimensionless terms as shown below:

$$\frac{\Delta z}{z_0} = F_1\left(\frac{z_0}{d_0}, \frac{r}{d_0}, \frac{V_0}{\sqrt{gz_0}}\right) \qquad (2)$$



2/ The experimental apparatus.
L'appareillage expérimental.

The last term, $V_0/\sqrt{gz_0}$, will be recognized as a Froude number.

At this point it should be noted that the dynamic viscosity might have been included as an eighth variable in the dimensional analysis. This would have introduced the Reynolds number as a fifth term. Since it is reasonable to assume that the rise of a jet above a free surface will depend in part upon the velocities in the submerged portion of the jet, and since these velocities were shown by Baines to be partially dependent upon the Reynolds number (2), the inclusion of the Reynolds number in this analysis could be justified. However, since the Reynolds number plays a comparatively minor role in the process of jet diffusion, the Reynolds number effect will be neglected in this study.

The foregoing analysis may be simplified further when free surface measurements are taken at large values of z_0/d_0. Then, assuming that the nozzle acts as a point source of momentum, V_0 and d_0 can be absorbed in the single term :

$$\frac{M}{\rho} = \int v^2 dA = \left(\frac{\pi d_0^2}{4}\right) V_0^2$$

Thus,

$$f_2\left(\frac{M}{\rho}, \Delta z, z_0, r, \rho, \gamma\right) = 0 \qquad (3)$$

and the three resultant dimensionless terms then become:

$$\frac{\Delta z}{z_0} = F_2\left(\frac{M/\rho}{gz_0^3}, \frac{r}{z_0}\right) \qquad (4)$$

Hence, the free-surface profiles can now be shown with one three-dimensional plot instead of a series of plots as required by the original analysis.

Another important fact becomes apparent if the displacement of the free surface is considered only along the jet centerline. Then the variable r is excluded and

$$\frac{\Delta z_{CL}}{z_0} = F_3\left(\frac{M/\rho}{gz_0^3}\right) \qquad (5)$$

Therefore, provided that the nozzle is submerged deeply enough to be considered a point source of momentum, a single curve should suffice to show the free-surface displacement at the centerline of any submerged vertical jet.

Apparatus

A schematic of the experimental apparatus is shown in Figure 2. Experiments were conducted in a 6-foot-diameter steel tank that had been used for a previous study at the Iowa Institute of Hydraulic Research. Two-inch copper pipe was used to feed water from the bottom of the tank to a 1/6-horsepower centrifugal pump located beneath the tank. Water was then discharged through another 2-inch copper pipe that entered at the bottom and extended to several feet below the top of the tank. A nozzle with a convergence angle of 60° was used at the end of this discharge pipe to assure minimum boundary-layer development at the entrance of the jet into the surrounding fluid. Four different nozzles ($d_0 = 1/2$ inch, 1 inch, 1-1/2 inches, and 2 inches) were used for the experiments. This type of recirculatory system was chosen because of its relative simplicity and its inexpensive construction.

It was necessary to install a wave-absorbing beach, shown in Figure 2, to minimize disturbances generated by the jet at the free surface. The wave absorber was constructed of eleven 1-foot-wide boards hinged together at an angle of 30° to the horizontal. Two layers of spun-glass insulation were held in place on the face of the beach by three layers of screen mesh, as shown in Figure 2. Without this device, waves would have been reflected by the steel sides of the tank, causing a gradual increase in amplitude of the waves until the sides of the tank were overtopped.

Surface-profile measurements were made with a point gage that traversed the free surface in two directions at right angles to each other. Flow rates were determined by reading the difference in pressure between two piezometer taps located within the nozzle and using a corresponding calibration curve. All measurements were linear and were read to a thousandth of a foot.

Results

The centerline displacement of the free surface above a vertical jet is shown in Figure 3. Measurements were made with a point gage at nozzle-submergences of 1, 4, 9, 16, and 25 nozzle diameters. This family of curves shows that the relative free-surface displacement $\Delta z_{CL}/z_0$ decreases as the relative submergence depth z_0/d_0 decreases and the momentum-flux term $(M/\rho)/gz_0{}^3$ is held constant.

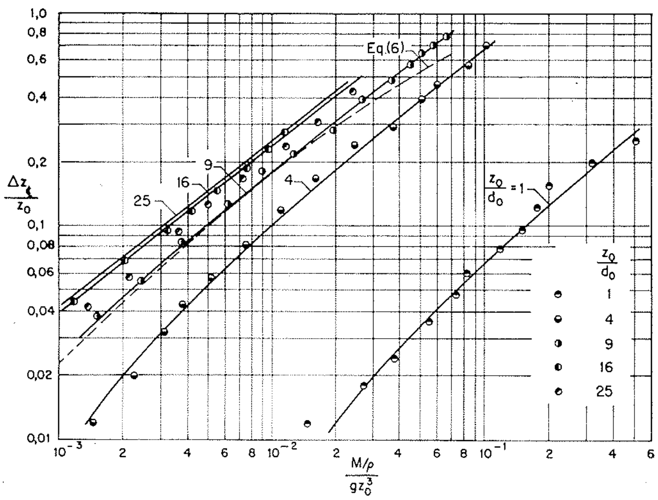
This may be verified intuitively by examining the expression for the momentum flux,

$$\frac{M}{\rho} = \int v^2 dA$$

Now, if both the submergence depth z_0 and the momentum flux M/ρ are held constant, and if the nozzle diameter is increased (decreasing z_0/d_0), then the velocity of the jet must decrease to keep M/ρ constant. Since any displacement of the free surface is proportional to velocities within the jet, the relative surface displacement must then decrease as z_0/d_0 is decreased.

It may also be pointed out that as the parameter z_0/d_0 is increased, the curves describing the relative centerline displacement come closer and closer together. This supports the conclusion drawn from the dimensional analysis that a single curve can be used to show the relationship between $\Delta z_{CL}/z_0$ and $(M/\rho)/gz_0{}^3$ for any value of z_0/d_0 if z_0/d_0 is large enough for the nozzle to approach the condition of becoming a point source of momentum.

Careful examination of Figure 3 will show that the experimental points for $z_0/d_0 = 25$ fall between the curves for $z_0/d_0 = 16$ and $z_0/d_0 = 9$. A

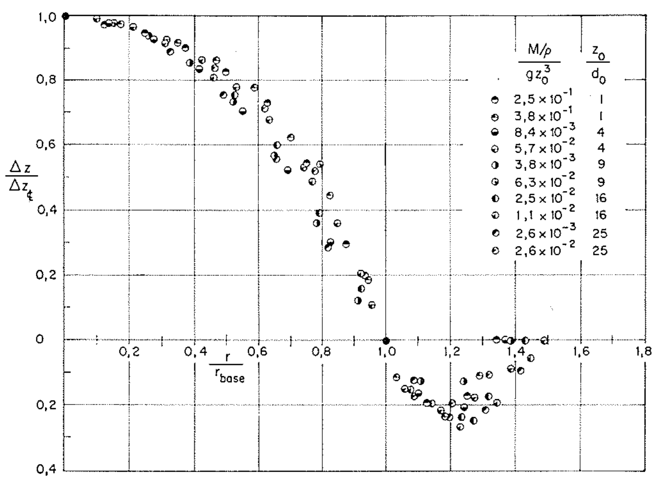


3/ Relative centerline displacement of the free surface above a vertical jet.

Déplacement relatif de l'axe de la surface libre au-dessus d'un jet vertical.

previous investigation by one of the writers [6] indicated that measurements for submergence depths of 33, 40, and 50 nozzle diameters coincided with the ones taken for $z_0/d_0 = 25$, despite the fact that these points all lay between curves for the higher and lesser relative submergences of $z_0/d_0 = 16$ and $z_0/d_0 = 9$ respectively. However, the same experimental equipment was used for both investigations, and it was decided that the wave-absorbing beach might be influencing flow conditions within the jet for these larger submergences, since the spread of the jet at the free surface varies directly with the submergence depth of the nozzle. Thus, the results for submergence depths of 33, 40, and 50 nozzle diameters have been omitted, although the measurements taken at a submergence of 25 nozzle diameters have been retained. It is the writers' considered opinion that this curve for 25 nozzle diameters submergence should lie just above the curve for 16 nozzle diameters as shown in Figure 3.

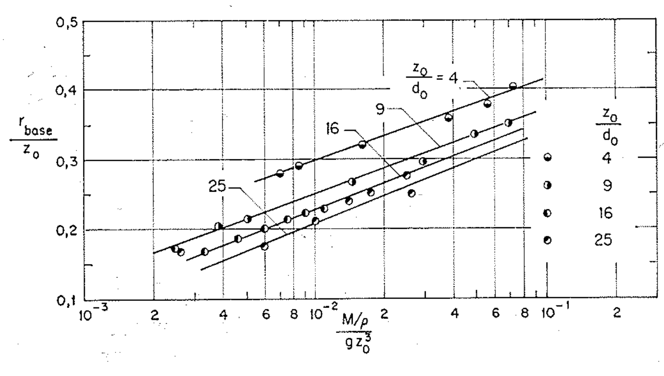
Complete surface profiles were measured at each one of the various submergence depths. The dimensional analysis indicated that these profiles could be shown in one three-dimensional plot for large relative submergences, i.e., a point source of momentum. However, the surface profiles at lesser submergences depend upon both the relative submergence depth and the momentum-flux term and would require a separate family of curves for each relative submergence depth. Because this would have involved more time and expense than the project warranted, it was decided to plot all the surface profiles in the alternative way shown in Figure 4. In this figure the relative surface displacement for one half of the symmetrical profile is plotted against the radius divided by a new variable, r_{base}, which is defined in Figure 1. The curves for each relative submergence depth seem to coincide closely enough to justify the use of this method for showing the free-surface profiles.



4/ Generalized profiles of the free surface above a vertical jet.
Profil généralisé de la surface libre au-dessus d'un jet vertical.

It is interesting to observe the dip shown in the profile of Figure 4 for values of r/r_{base} between 1.0 and 1.4. This is caused by water in the crown of the profile meeting the free surface at that point. The extreme unsteadiness of this wave helps to account for the experimental scatter of points in this region.

Introduction of the variable r_{base} in Figure 4 makes it necessary to express this variable as a function of the momentum flux. This is done in Figure 5, which is a plot that is analogous to the centerline-displacement plot of Figure 3. Again it may be observed that the ratio r_{base}/z_0 is dependent upon both the momentum flux and the submergence depth for smaller submergences and that the influence of z_0/d_0 becomes smaller and smaller as the relative submergence becomes greater. However, Figure 5 shows that for constant values of the momentum-flux term, the variable r_{base}/z_0 decreases for increasing values of relative submergence. Thus, increasing the relative submergence depth while keeping the momentum-flux term constant has the effect of both increasing $\Delta z_{\text{CL}}/z_0$ and decreasing r_{base}/z_0, which may be seen by comparing Figure 3 with Figure 5. It is regrettable that more experimental points were not obtained for Figure 5, but the extreme difficulty of measuring small surface profiles for $z_0/d_0 = 1$, as well as the very practical considerations of time and funds available for the project, limited the work that could be done in this direction.



5/ The relative radius at the base of the free surface profile of a vertical jet beneath a free surface.
Le rayon relatif à la base du profil en surface libre d'un jet vertical sous une surface libre.

In concluding this section of the paper, the writers would like to point out some of the difficulties involved in obtaining measurements of the free surface above such a jet. Turbulence within the jet causes the surface profile to fluctuate continuously for the larger submergence depths. In addition, air entrainment occurs at higher efflux velocities. Therefore, attempts were made to obtain average measurements of displacement with the point gage. These measurements were repeated with a surprising degree of consistency, but the ever-present possibility of experimental error is considerably increased because of these difficulties.

An analytical approximation

The centerline displacement of the free surface may be roughly approximated by the velocity head of the center streamline in an infinitely submerged jet at a distance of $z_0 + \Delta z_{\text{CL}}$ from the nozzle. This approximation is based upon the assumption that the variation of total head along the center streamline of a vertical jet beneath a free surface is identical to that for a completely submerged jet. To be more specific, the centerline displacement is approximated as:

$$\Delta z_{\text{CL}} \approx \frac{V^2}{2g} \tag{6}$$

where V is computed from the following equation given in reference (1) for an infinitely submerged jet:

$$\left(\frac{V}{V_0}\right)(z_0 + \Delta z_{\text{CL}}) = 6.2\, d_0 \tag{7}$$

Equation (7), like the curve it was used to approximate, is valid only for distances z/d_0 large enough that the nozzle acts essentially as a point source of momentum. Each value was converged upon by a method of successive approximation, and the dashed line plotted in Figure 3 shows the result of this approximation.

Conclusions

The average free-surface characteristics for any vertical jet beneath a free surface have been presented in this paper, and the following conclusions can be drawn from the experimental results:

(1) The relative centerline displacement of the free surface depends primarily upon the momentum flux for values of z_0/d_0 great enough that the nozzle acts as a point source of momentum. For lesser submergences, the relative centerline displacement varies not only with the momentum flux but also with the relative submergence depth, z_0/d_0.

(2) The dimensionless surface profiles tend to be geometrically similar over the range of relative submergence depths investigated in these experiments.

(3) It should be possible to extrapolate values of free-surface displacements from the results of these experiments for submergence depths greater than 25 nozzle diameters. This is because the nozzle tends to act as a point source of momentum for larger submergence depths, and thus, for submergences greater than 16 nozzle diameters, the free-surface characteristics are approximately independent of the relative submergence depth.

(4) The centerline displacement of the free surface above a vertical jet may be roughly approximated by the velocity head of the center streamline in an infinitely submerged jet at a distance $z_0 + \Delta z_{CL}$ from the nozzle.

Acknowledgments

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Appendix-Notations

The following symbols have been adopted for use in this paper :

d_0 = nozzle diameter;
g = acceleration of gravity;
M = momentum flux of the jet;
r = radial distance from the jet centerline;
r_{base} = radius of the profile at the elevation of the undisturbed free surface;
V = centerline velocity at a distance $z_0 + \Delta z_{CL}$ from the nozzle of a completely submerged jet;
V_0 = efflux velocity at the nozzle of the jet;
Δz = displacement of the free surface above the undisturbed free surface;
Δz_{CL} = centerline displacement of the free surface above the undisturbed free surface;
z_0 = vertical distance from the jet nozzle to the undisturbed free surface;
γ = fluid specific weight; and
ρ = fluid mass density.

Résumé en français à la page suivante.

Résumé

La configuration de la surface libre au-dessus d'un jet vertical

par

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et

Sheng-Tien Hsu **

On a fait l'étude expérimentale d'un jet aximétrique vertical immergé au-dessous d'une surface libre. Les essais ont été conduits par analyse dimensionnelle; les résultats ont été présentés sous forme de graphiques adimensionnels des profils de la surface libre (fig. 4), et du déplacement relatif de l'axe, en fonction de la profondeur relative de submersion de la buse et d'un terme adimensionnel tenant compte du flux de quantité de mouvement (fig. 3).

Les déplacements de la surface libre ont été mesurés pour des profondeurs de submersion de la buse égales à 1, 4, 9, 16 et 25 diamètres de buse. La figure 3 montre que le déplacement relatif de l'axe $\Delta z_{CL}/z_0$ devient progressivement de plus en plus indépendant de la profondeur relative de submersion de la buse, lorsque la valeur de z_0/d_0 augmente; lorsque la profondeur de submersion dépasse $z_0/d_0 = 16$, la buse se comporte essentiellement comme une source ponctuelle de flux de quantité de mouvement. Sur cette même figure 3, la courbe correspondant à $z_0/d_0 = 25$ n'a pas été tracée sur les points expérimentaux, mais a été alignée sur les autres courbes, car les auteurs estimaient que la position de ces points se trouvait influencée par la plage amortisseuse de houle représentée sur la figure 2. Le rôle de cette plage était de limiter autant que possible les perturbations provoquées par le jet à la surface libre.

On a constaté que les profils adimensionnels de la surface libre tendaient à être géométriquement semblables sur l'ensemble de la gamme des profondeurs de submersion relatives considérées dans les expériences. Ceci signifiait que tous ces profils pouvaient se ramener à peu près à une courbe unique, celle de la figure 4. La dépression localisée de la surface libre, correspondant à des valeurs de r/r_{base} comprises entre 1,0 et 1,4, s'explique par le fait qu'en ce point, l'eau contenue dans la protubérance du profil rencontre la surface libre.

L'introduction d'une nouvelle valeur variable de r_{base} dans la figure 4, a conduit à la nécessité d'exprimer r_{base}/z_0 en fonction du flux de quantité de mouvement, et de la profondeur de submersion relative. Ceci a fourni le graphique de la figure 5, qui est exactement analogue au graphique représentant le déplacement de l'axe en figure 3.

On a également constaté que le déplacement de l'axe de la surface libre était donné, approximativement, par la hauteur cinétique du filet liquide médian d'un jet infini, et submergé, à une distance égale à $z_0 + \Delta z_{CL}$ de la buse. Ceci permet de substituer dans l'équation (6), la vitesse correspondant à l'axe, tirée de l'équation (7), pour obtenir Δz_{CL}. La ligne en traits discontinus de la figure 3 montre le résultat de cette approximation.

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