SOME NEW ASPECTS OF SIMILARITY IN HYDRAULICS *

BY
E. O. MACAGNO **

In spite of the efforts of many hydrodynamicists over more than two centuries, a workable theory for fluid flow is still very much in the making, and many problems of fluid mechanics must be investigated in a purely experimental manner. The fundamental equations of this science were established during a period of great confidence in the powers of analytical methods; we can surmise that Euler and Saint-Venant, for example, had great hopes that their equations for fluid flow would be successfully employed in the solution of important problems of hydraulics that preoccupied their generations of scientists and engineers. The helping hand extended by Saint-Venant to Boussinesq [1] was probably not only a gesture of human kindness but a recognition of his accomplishments in the fields of laminar and turbulent motion; Saint-Venant, although aware of the difficulties of the subject, must have believed that Boussinesq's initial progress showed the way to be open. But one can already find in the works of Reynolds, a contemporary of Boussinesq, clear indication of a change in attitude stemming from realization of the tremendous obstacles actually in the way of theoretical progress. Reynolds considered necessary a synthesis of the turbulence characteristics by means of statistical properties upon which a theory could be developed; unfortunately, what was a rather easy task for the molecular "turbulence" proved to be extremely difficult for the macroscopic turbulence of fluid flow, and we still do not have a purely theoretical method for even the simplest turbulence phenomena. This may be due, in part at least, to our scanty knowledge of laminar flow which prevents us from having much insight into the basic mechanisms of turbulent flow, which in turn can be imagined as made up of very complicated laminar flows of random type. The ever-increasing refinement of observational techniques has shown us much more about turbulence characteristics than was known to all the previous generations of students of fluid motion, but we do not know if those characteristics are the ones that reveal the inner mechanisms and their interactions.

On the other hand, one should not be misled by the rather impressive display of theoretical calculations that can be found in the literature on laminar flow. In spite of continuing progress in this field, a critical examination soon shows that we still know little about the general properties of the Navier-Stokes equations, and practically nothing about truly three-dimensional laminar flows. It is true that important advances have been made within the boundary-layer approximation as well as in the linearized study of the onset of instability, but we know very little about the generation and evolution of eddies and secondary flows, and still less about transitional regimes. To picture the present situation, one could not find a better example than the classical problem of form resistance, which remains all-important in hydraulics. We still do not know how to calculate resistance coefficients for laminar flow around most of the simple obstacles, or for laminar flow through the least complicated of non-uniform pipes. Therefore, even for laminar regimes, not to mention transitional regimes, in view of the impotence of classical hydrodynamics we must still resort to experimentation as the hydraulicians did in the past.

Progress in the fields of direct experimentation with fluids or with analogs of fluid motion has been due mainly to improved knowledge of the basic criteria for dynamical similarity and for physical analogy. The concept of similarity was first given

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a scientific connotation by the Greek geometers, and it may appear a simple task to fulfill the requirements of geometrical similarity; this is not so, however, when such boundaries as hydraulically rough surfaces must be considerably scaled down for an experimental study. The concepts of kinematical and dynamical similarity were not systematized until the times of Galileo and Newton; the latter, in fact, was the one who actually laid the analytical foundations for a rational approach to physical similarity.

Stemming from Newton's general principles, special laws or criteria for similarity of fluid flows have been established during the past hundred years. Reynolds and Froude are two well-known names associated with the criteria for similarity when flows are controlled either by viscous and inertial forces or by gravitational and inertial forces. For more complicated situations, as exemplified by the resistance of nonuniform flow in open channels, the similarity requirements consist of two or more criteria to be applied simultaneously. If, for example, Reynolds' and Froude's criteria must be applied together, it will be found either that different fluids must be used in model and prototype, or that different gravitational forces must be applied to them. A study of the physical properties of available fluids shows that in our times the second alternative may be more feasible than the first [2].

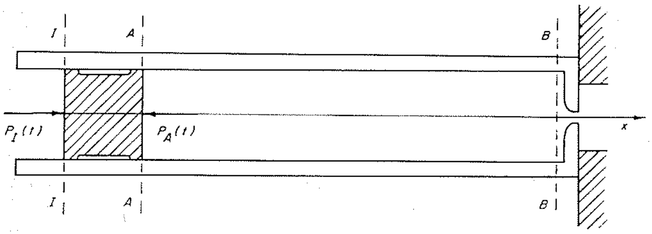
The ingenuity of research engineers since the times of Reynolds and Froude has provided the art of modeling with many clever devices: from the distortion of the model geometry to the simulation of roughness, and from the use of different fluids to procedures of transference of results from model to prototype that are more elaborate in form than simple multiplication by a scaling factor (*). Because no analog to turbulent flow has yet been found, experimentation for this type of flow must be done with real fluids, with the only possibilities of scaling down or up and of using different fluids. Unfortunately, the spectrum of physical properties is discrete and very limited, quite aside from economic, health-, and fire-hazard restrictions. If a research engineer, in order to have strict similarity of two flows, finds that he needs a fluid with one-thousandth the viscosity of water and one-tenth its specific weight, he knows very well that such a fluid does not exist. Or does it? If the research engineer can use a mathematical model, it surely does, for in such a model there are no limitations on the values than can be attributed to the physical properties. As more and more computational techniques become applicable in faster and faster electronic computers, the possibilities of simulating fluid flow in these machines become better and better.

There has been a tendency among users of hydraulic models to regard the models as fluid-flow computers; however, it may require a much greater stretch of the imagination to say now that digital computers may also serve as flow simulators. One could mention, though, that physicists have long since coined the expression « mathematical model » to indicate any set of mathematical relationships that describe and predict physical phenomena. But the more convincing effect springs from the daily use of the computer to study complicated cases of fluid flow. Adoption of the viewpoint of flow simulation in the computer becomes a temptation that is impossible to resist after seeing recent films that show calculated fluid flows emerging from plotting machines coupled to digital computers [3]. To illustrate such a point of view, several problems under investigation at the Iowa Institute of Hydraulic Research will be briefly discussed, with emphasis on their computational aspects.

The computer models have the advantage over the traditional models of a much higher freedom of variation in the parameters involved, and of changes in boundary and initial conditions. This can be illustrated by investigations of a simplified model of a hydraulic recoil mechanism (Fig. 1), for which it was desired to find in what manner the fluid discharge should be varied at the control nozzle to obtain a constant force on the supporting structure during most of the operation time. This requirement of ensuring a specific variation of pressure, or force, was new, as was the circumstance of having a variable length to the fluid column. Bergeron's [4] graphical method for pressure-wave analysis was applied at the beginning, but it proved too clumsy and inaccurate for the problem in question. The method was then converted into a numerical one and programmed for the digital computer, thereby gaining in accuracy and flexibility [5]. In attempting to obtain a rather smooth operation, the investigator initially imposed a certain condition on the second derivative of the discharge; if this condition were violated, the computer would try again with a different combination of data. In this way, it was found "experimentally" that the terminal position of the recoiling mass could never be reached without violating the imposed condition. Relaxing the condition to one imposed on the first derivative yielded sufficiently satisfactory results (Fig. 2). The interesting conclusion in regard to the operation of the computational model was that a study of the behavior of the derivatives, two quantities that would have been rather difficult to observe in a physical model, was quite easy and reliable.

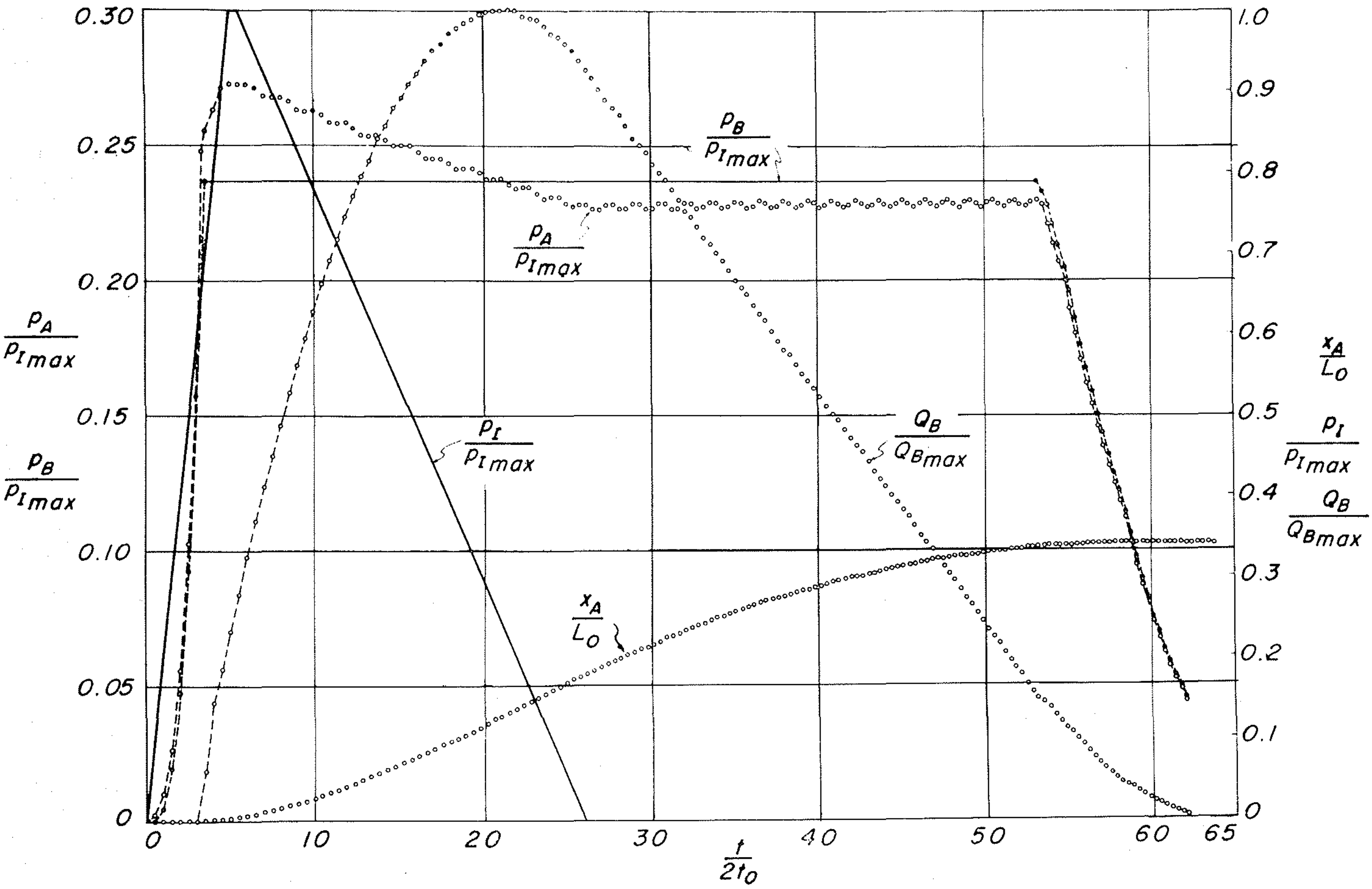
A second phase of the study of such a shock

(*) See the second paper by G. Murphy, in this same Engineering Institute.



1/ Definition sketch for simplified hydraulic recoil mechanism.

Schéma de définition d'un mécanisme de recul hydraulique simplifié.



Calculated values for the pressures p_A and p_B, the rate of flow Q_B, and the displacement x_A. Prescribed values are given by solid lines.

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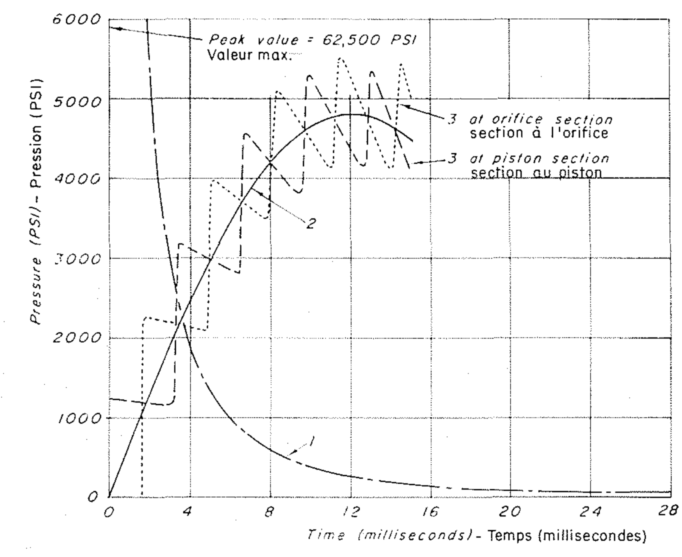
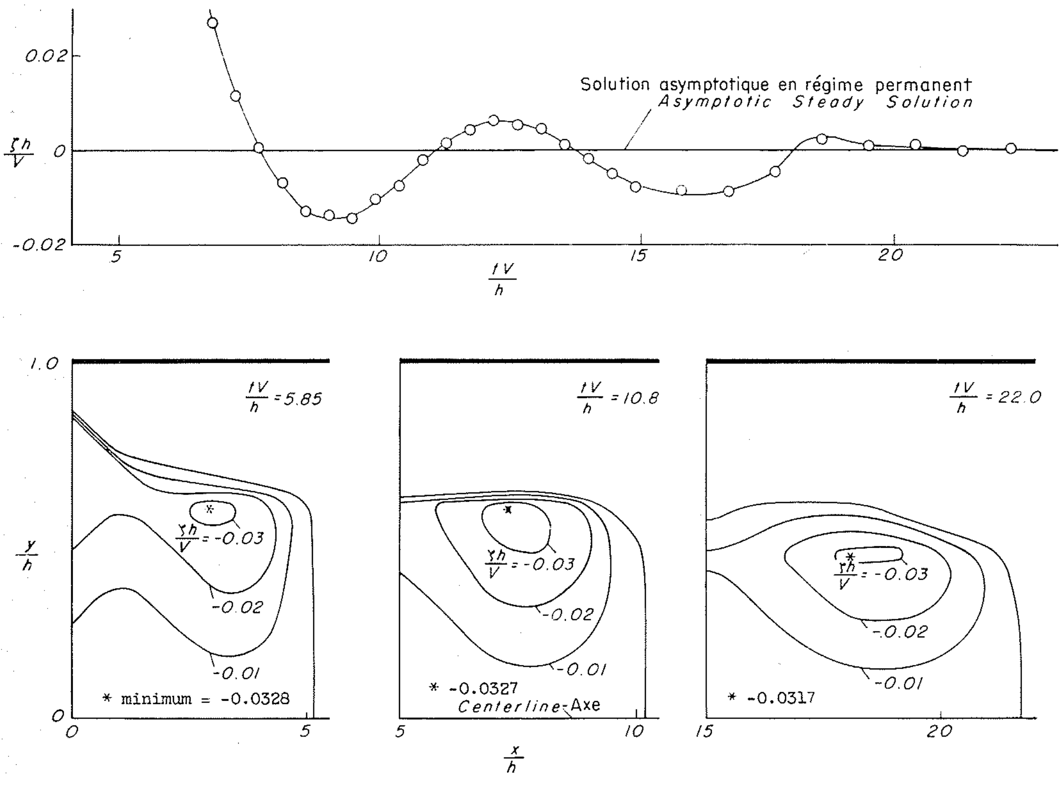
Valeurs calculées des pressions p_A et p_B, du débit Q_B et du déplacement x_A. Les valeurs prescrites sont données par les lignes continues.

absorber is also instructive in relation with the simulation of fluid flow in the digital computer. Due to the difficulties experienced in applying pressure-wave analysis to actual complex shock absorbers, an approximation was devised that consisted of discarding the wave-propagation effects due to elasticity of the fluid, while retaining the volumetric change due to pressure variation [6]. The idea was tested in a computational model in which the fluid could be given any bulk modulus of elasticity, including an infinite value, and in which any force diagram, including a Dirac function, could be used. The results for an impulsive force [7], calculated by Mr. A. Newsham, are shown in Figure 3; curves 3 represent the results with the complete pressure-wave analysis, and curve 2 corresponds to the approximation above described. Curve 1, for an incompressible fluid, shows that compressibility cannot be neglected without introducing serious errors. Curves 3 and 2 become almost coincident for times of application of the driving force greater than a few milliseconds. A feature of this model is worth special mention in this connection: the computational stability of the model in which waves were allowed was found to



3/ Pressure variation in shock absorber for impulsive driving force.

Variation de la pression à l'intérieur d'un amortisseur, sous l'effet d'une force motrice impulsive.



4/ Decay and convection of a disturbance in the flow-establishment region of a two-dimensional fixed conduit (Hinwood's Dissertation, 1965).

Dégradation et convection d'un phénomène de perturbation à l'intérieur de la région d'établissement de l'écoulement dans une conduite fixe à deux dimensions (Dissertation de Hinwood, 1965).

be much greater than that of the model without waves. No mathematical analysis of this computational stability was made; but it seems plausible, from a physical point of view, to conclude that the system in which propagation effects are permitted is more apt to spread disturbances that to intensify them.

One of the earlier uses of the digital computers was the calculation of neutral lines in the problem of flow instability. The investigation of the onset of instability by means of linearized Navier-Stokes equations leads to the solution of certain ordinary differential equations like the Orr-Sommerfeld equation, but then the fluid is so abstractly represented that the calculational facet becomes prevalent over the modeling facet [8]. The latter regains predominance, however, when the complete nonlinear equations are used to simulate the flow and to study the effect of induced finite disturbances upon it. In an investigation of the stability of flow with stratification of density, under way at the Iowa Institute of Hydraulic Research [9], Mr. J. B. Hinwood has recently represented the region of flow establishment in a two-dimensional conduit by means of a finite-difference form of the Navier-Stokes equations, including all nonlinear as well as unsteady terms. The equations are applied to points of a rectangular mesh covering the region of interest. From approximate considerations of computational stability, it was concluded that this would be higher if the unsteady terms were not eliminated from the model when determining the steady-state solution. The presence of the unsteady terms, on the other hand, has allowed the researcher to follow in a realistic way the evolution of the "flow" as it passes from the approximately assumed state to the final steady state. One aspect of this "observation" is given in Figure 4, in which the original disturbance is seen to die down at a fixed point and to be convected downstream.

A third example is provided by the investigation of two-dimensional and axisymmetric flows in abrupt conduit expansions. The purpose of this research project was to determine the characteristics of confined laminar eddies. For the case of a two-dimensional expansion, the Navier-Stokes equations in Cartesian coordinates are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial (p/\rho)}{\partial x} + \nu \nabla^2 u$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{\partial (p/\rho)}{\partial y} + \nu \nabla^2 v$$

in which u and v represent the velocity components in the x and y directions, p indicates the pressure, ρ the mass density, and ν the kinematic viscosity. If the stream function ψ is introduced, the velocity components can be expressed by means of:

$$u = \frac{\partial \psi}{\partial y} \qquad v = - \frac{\partial \psi}{\partial x}$$

In addition, the vorticity is given by:

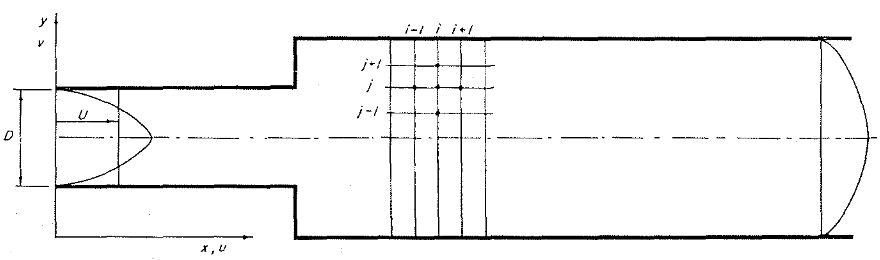
$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = - \nabla^2 \psi$$

By proper manipulation, the preceding equations can be reduced to the two following equations in finite-difference form:

$$\zeta_{I,J} = \frac{1}{4}(\zeta_{I-1,J} + \zeta_{I,J-1} + \zeta_{I+1,J} + \zeta_{I,J+1})$$
$$- \frac{R}{16}\,[(\psi_{I+1,J} - \psi_{I-1,J})(\zeta_{I,J+1} - \zeta_{I,J-1})$$
$$+ \quad (\psi_{I,J+1} - \psi_{I,J+1})(\zeta_{I+1,J} - \zeta_{I-1,J})]$$

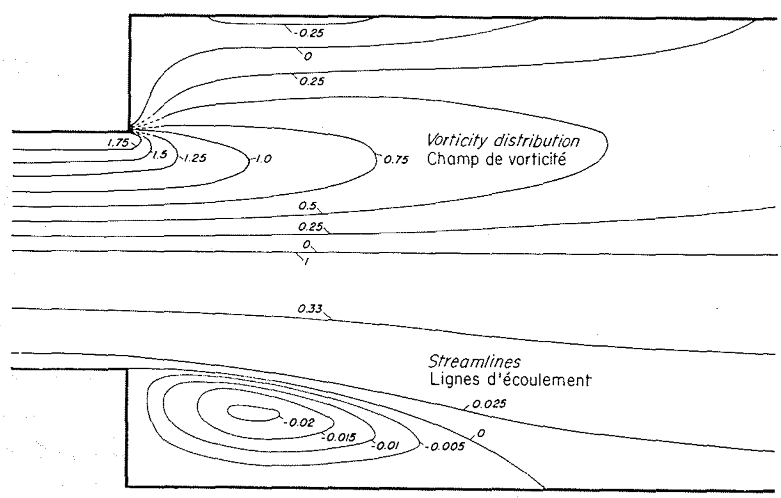
$$\psi_{I,J} = \frac{1}{4}(\psi_{I-1,J} + \psi_{I+1,J} + \psi_{I,J-1} + \psi_{I,J+1} - a^2\zeta_{I,J})$$

in which a indicates the size of the mesh. If one uses these equations over a lattice of squares drawn over the conduit expansions (Fig. 5), iterative procedures allow one to calculate, from an approximate initial distribution, the flow corresponding to a zero Reynolds number. The resulting flow pattern is then used for the calculation for an increased Reynolds number, say of 4, and so on. In Figure 6, the results of calculations of streamlines and of lines of constant vorticity are given for a Reynolds number of 48. For an axisymmetric expansion, the results of calculations are given in Figure 7, for a Reynolds number of 40. Photographic records of experimental flow in the axi-



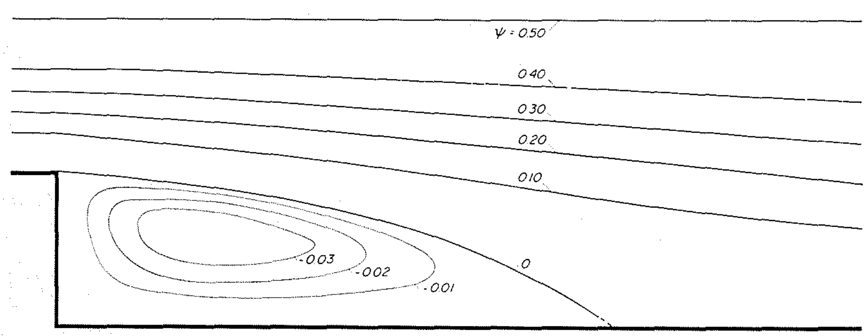
5/ Definition sketch for viscous flow in a conduit expansion.

Schéma définissant un écoulement visqueux par un élargissement dans une conduite.



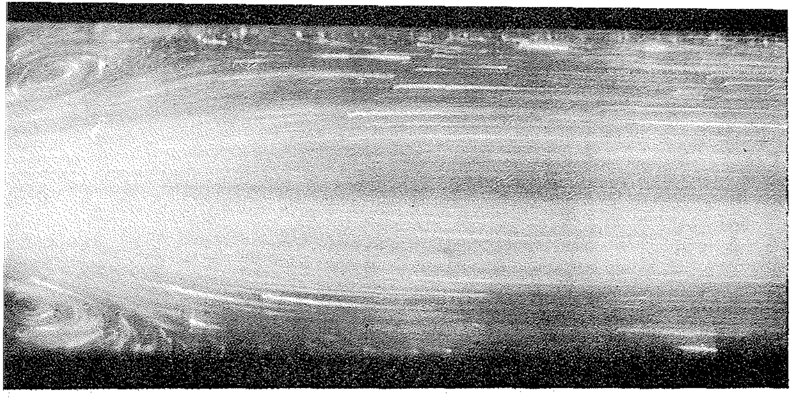
6/ Laminar flow in a two-dimensional expansion for a Reynolds number of 48 (values of stream function and vorticity are affected by an arbitrary factor).

Ecoulement laminaire par un élargissement plan, pour un nombre de Reynolds égal à 48 (les valeurs de la fonction de courant et de la vorticité sont soumises à l'influence d'un facteur arbitraire).

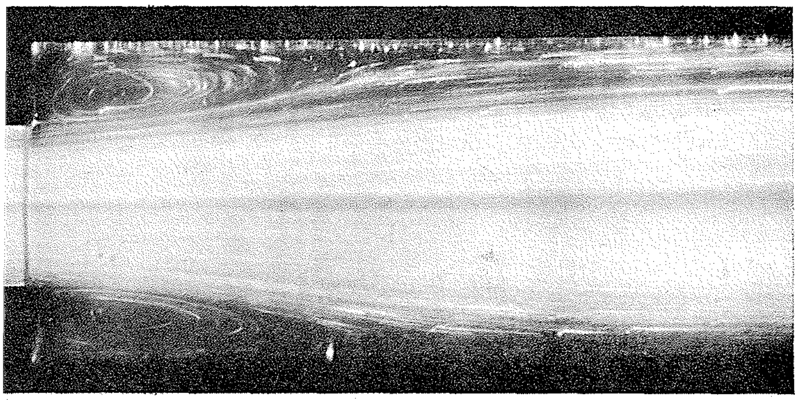


7/ Streamlines for steady laminar flow in an axisymmetric expansion for a Reynolds number of 40.

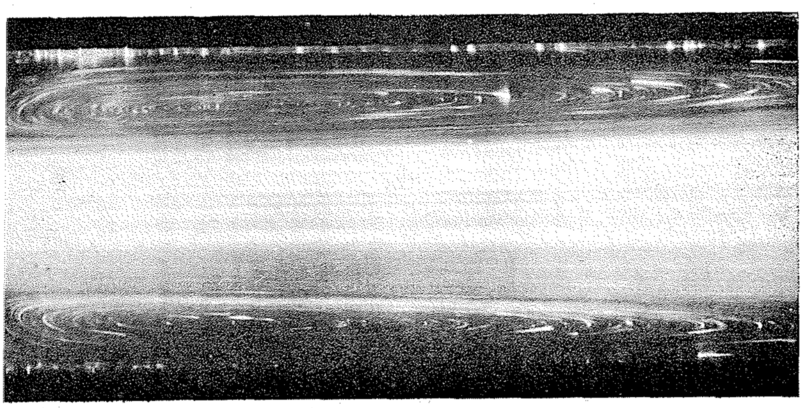
Lignes d'écoulement correspondant à un régime permanent laminaire dans un élargissement axisymétrique, pour un nombre de Reynolds égal à 40.



$R = 25$

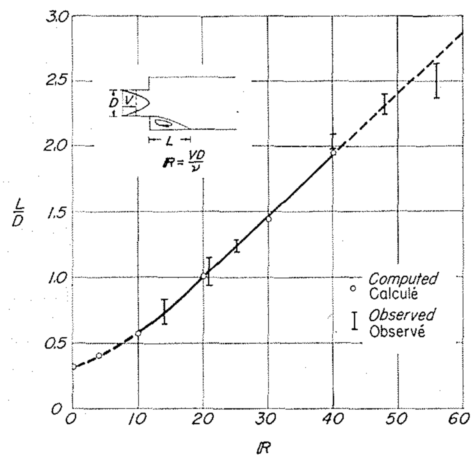


$R = 52$



$R = 230$

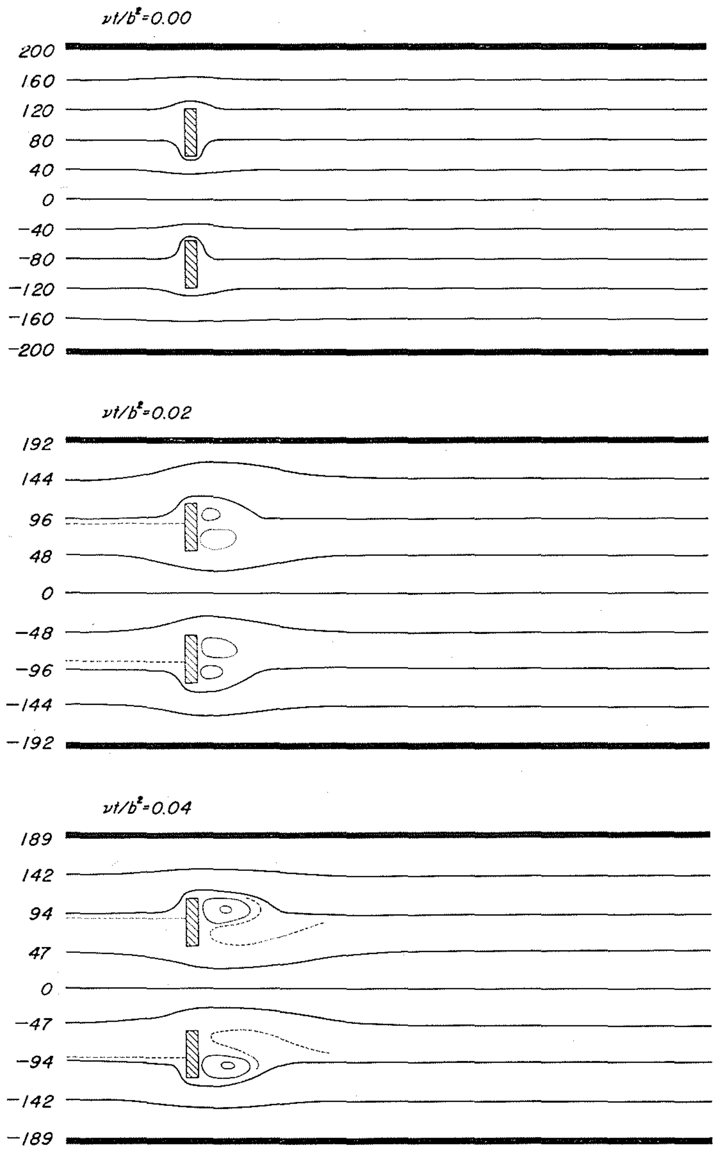
8/ Photographic record of flow in axisymmetric expansions.
Photographies d'écoulements par des élargissements axisymétriques.



9/ Eddy size for steady laminar flow in an axisymmetric expansion.
Dimensions du vortex dans un écoulement permanent laminaire par un élargissement axisymétrique.

symmetric expansion are shown in Figure 8. Experimental and calculated eddy lengths are given in Figure 9. In all cases, the Reynolds number is based on the velocity and the diameter of the narrow conduit.

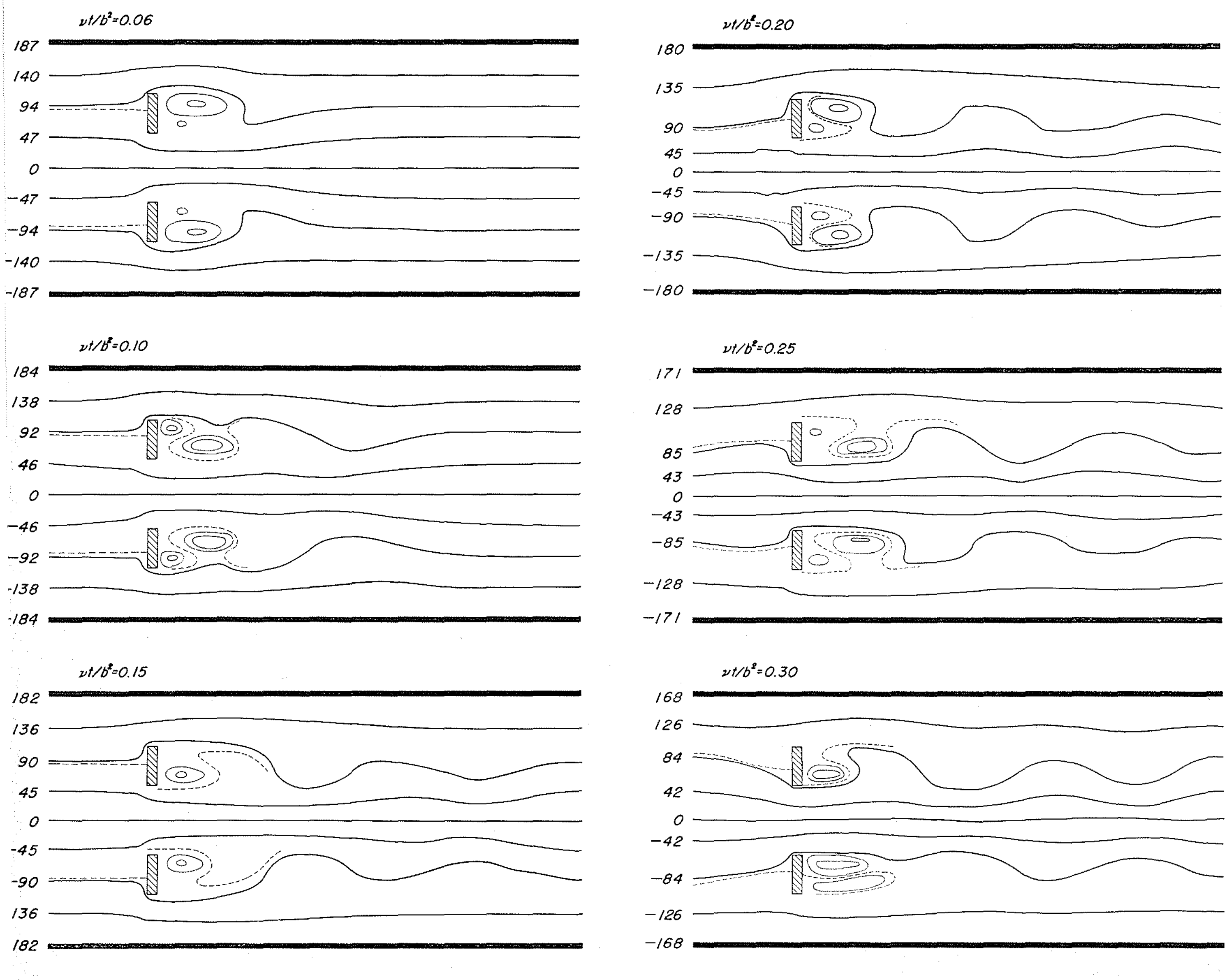
At the present time, wakes behind obstacles in a two-dimensional channel with moving walls are being studied using the method developed by J. Fromm [10] at Los Alamos Scientific Laboratory. This is done with the purpose of adapting this method afterwards to calculations in fixed conduits. Wakes for an infinite row of pairs of rectangular obstacles calculated by Mr. T. K. Hung are shown in Figures 10. In these figures, b indicates the dimension normal to the flow direction of the rectangular obstacles, and t, the time. The Reynolds



10/ Wake behind rectangular obstacles for a Reynolds number of 64.

number is based on the linear dimension, b, and on the initial velocity of the fluid. Next, this method will be adapted and extended to flow in fixed conduits for which it is expected to be very useful, because a large variety of flows will be presented by merely changing the temporal variation of the applied pressure gradient. There are, at the present time, indications that the use of the Navier-Stokes equations with unsteady terms results in a higher computational stability; experimentation with several cases in which steady-state solutions have already been determined, using now equations with unsteady terms, is expected to help in elucidating this conjecture.

In general, the simulation of fluid flow with the digital computers seems very promising, and good advantage should be taken of this new tool to investigate aspects of fluid mechanics that are difficult to study by other means. The "fluid" in the computer model can have any set of physical properties we find necessary; it can even have artificial properties, if this helps to simulate the desired conditions. In considering the problems that may arise when attacking three-dimensional problems we have already contemplated the possibility of introducing an artificial equation of state, better to handle the calculation of the pressure. The idea is not entirely a new one, artificial viscosity having been used to advantage in calculations of gas dynamics by von Neumann and Richtmeyer [11]. If artificial properties can be given to the Newtonian fluids, it is obvious that non-Newtonian fluids can be simulated with the digital computer, the only



llages à l'aval d'obstacles rectangulaires, pour un nombre de Reynolds égal à 64.

problem being that more and more parameters must be specified at each point of the lattice.

One of the difficulties in simulating flows with the computer lies in the fact that many points are needed with the present techniques. If geometry has to be represented, including the details of roughness of the solid surfaces, too large a number of points is necessary. This might appear as a minor problem if one's ambition were to simulate a turbulent flow; Corrsin [12] considered that the number of bits of information necessary to simulate details of decaying turbulence was an astronomical figure, ruling out the use of digital computers. Corrsin indicated that the analog computer could be considered, but then added this question: "in particular, how about an analog consisting of a tank of water?" In the present state of the art, it seems that efforts should be concentrated on investigating those aspects of laminar motion which are still unknown and constitute basic mechanisms for the turbulent flow. These mechanisms are only qualitatively recognized and not very well understood; but they can surely be described as related to the non-linear behavior of the Navier-Stokes equations. If truly turbulent situations cannot yet be reproduced, one should aim at modeling those that are only quasi-turbulent; one possibility in this direction, already at hand, consists in merely refining present calculations for unsteady two-dimensional flow in transitional regimes, so that the effect of disturbances can be studied when the flow becomes sufficiently disorganized to begin showing characteristics of turbulence.

Within the field of purely laminar flow, work should be started on the simulation of very simple but truly three-dimensional flows; their geometry could well remain unimportant, and the evolution of disturbances that start as two-dimensional and soon become three-dimensional could be one of the initial goals. Concentrated vortices have recently been discussed in an international congress [13] of fluid dynamicists, where it was shown that much knowledge is still to be gained in this field; it seems that computer simulation of vortices and the study of their interactions (as illustrated for annular vortices in a recent film [14]) could now be undertaken with good prospects of success.

As a final remark, it appears most opportune to note that the flows simulated with the computer are automatically measured in all their details. The essential question is now to model accurately enough. To realize the tremendous advantage that freedom from instrumentation can give us, one has only to think of the possibility of reading off the six components of the stress tensor at all points of a lattice in a three-dimensional flow as simulated by a digital computer. This can already be done for two-dimensional and axisymmetric flows, and it may be hoped that only a short time will elapse before it becomes possible for three-dimensional flows. Computers are still developing, and one could easily be kept busy in the near future by doing nothing more than investigating new problems of fluid flow in digital computers. If one judges by the results obtained in a few years, the accomplishments of the near future should be most rewarding.

Acknowledgments

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NOTE: When this paper was in the process of being typed, a paper on the same subject appeared in the March issue of our scientific magazine [15].

References

[1] ROUSE (H.) and INCE (S.). — *History of Hydraulics*, Dover, New York, 1963.

[2] MACAGNO (E. O.). — Semejanza mecánica, su aplicación a la hidráulica. *Ciencia y Técnica*, vol. 116, No 585, 1951.

[3] FROMM (J.). — Wake of an Obstacle. Film Y-181, Los Alamos Scientific Laboratory, 1963.

[4] BERGERON (L.). — Water Hammer in Hydraulics and Wave Surges in Electricity, *Wiley*, New York, 1961.

[5] MACAGNO (E. O.) and MACAGNO (M.). — Pressure-wave Analysis for Variable Length of a Fluid Column. *Proceedings IAHR Ninth Convention*, Paper III/40-1, Dubrovnik, 1961.

[6] MACAGNO (E. O.) and HO (H. W.). — Una aproximación en el movimiento de un liquido compressible. *Seminario de Hidráulica y Mecánica de Fluidos*, Paper III/13/1, Santiago, Chile, 1962.

[7] MACAGNO (E. O.) and NEWSHAM (A.). — Approximate calculation of the flow of compressible liquids (to be published in the near future).

[8] SANGSTER (W. M.). — The Stability of Stratified Flow on Nearly Vertical Slopes. Ph.D. Dissertation, University of Iowa, 1964.

[9] MACAGNO (E. O.) and HINWOOD (J. B.). — Instabilité dans la zone d'établissement d'un courant avec stratification de densité. *VIIIes Journées de l'Hydraulique*, Quest. I, Rapp. 10, Lille, 1964. (English version available upon request, Institute of Hydraulic Research, Iowa City, Iowa).

[10] FROMM (J.). — A Method for Computing Nonsteady, Incompressible, Viscous Fluid Flows. *Report LA*-2910, Los Alamos Scientific Laboratory, 1963.

[11] VON NEUMANN (J.) and RICHTMEYER (R. D.). — A Method for the Numerical Calculation of Hydrodynamic Shocks. *Journal of Applied Physics*, vol. 21, No. 3, 1950.

[12] CORRSIN (S.). — Turbulent Flow. *American Scientist*, Vol. 49, No. 3, 1961.

[13] KÜCHEMANN (D.). — Report on the I.U.T.A.M. Symposium on concentrated vortex motions in fluids. *Journal of Fluid Mechanics*, Vol. 21, Part. 1, 1965.

[14] ROUSE (H.) and O'LOUGHLIN (E.). — Characteristics of Laminar and Turbulent Flow. Film U-6159, University of Iowa, Iowa City, Iowa, 1964.

[15] HARLOW (F. H.) and FROMM (J. E.). — Computer Experiments in Fluid Dynamics. *Scientific American*, Vol. 212, No. 3, 1965.

[16] HARLOW (H. F.). — Numerical Fluid Dynamics. *The American Mathematical Monthly*, Vol. 72, No. 2, 1965. (This reference was added to copies of the paper printed after delivering the lecture.)

Résumé

Quelques aspects nouveaux de la similitude en hydraulique

par E. O. Macagno *

Malgré les efforts de beaucoup de chercheurs en hydrodynamique depuis plus de deux siècles, une théorie généralement applicable de l'écoulement des fluides reste toujours à dégager. Aussi, faut-il étudier beaucoup de problèmes selon la méthode purement expérimentale. Pour donner une idée de la situation actuelle, on ne peut mieux faire que de citer le problème classique de la résistance due aux formes, problème toujours d'importance capitale en hydraulique.

L'ingéniosité des chercheurs a fourni à l'art de l'expérimentation sur modèle plus d'un artifice astucieux. Si, pour assurer la similitude de deux écoulements dans des conditions complexes, le chercheur trouve qu'il a besoin d'un fluide n'ayant que le millième de la viscosité de l'eau et le dixième de son poids spécifique, il sait très bien qu'un tel fluide n'existe pas.

Et pourtant... peut-être bien que si. En effet, si notre ingénieur de recherches peut disposer d'un modèle mathématique, ce fluide existe certainement, car dans un tel modèle les valeurs que l'on peut attribuer aux propriétés physiques ne sont soumises à aucune limitation. A mesure que les techniques se multiplient au service d'ordinateurs électroniques toujours plus rapides, les possibilités de simulation des écoulements fluides par ces machines deviennent-elles plus intéressantes.

Nous présenterons dans cette communication une brève discussion de plusieurs problèmes actuellement en cours d'étude à l'Iowa Institute à Hydraulic Research. Les modèles sur ordinateurs ont cet avantage sur les modèles traditionnels, qu'ils permettent une plus grande liberté de variation des paramètres d'essai, et de modifications aux conditions de départ et aux limites. Cette thèse trouve son illustration dans l'étude d'un modèle simplifié d'amortisseur de chocs (fig. 1), pour lequel il importait de déterminer la manière de faire varier le débit fluide à la tuyère de régulation, de façon à obtenir une force constante sur l'ouvrage-support pendant la plupart du temps. Pour ce faire, nous avons eu recours à une conversion numérique de la méthode graphique de Bergeron. Les résultats obtenus sont consignés sur la figure 2.

Une deuxième phase de cette étude est également riche en enseignements concernant la simulation des écoulements au moyen d'un ordinateur numérique. En raison des difficultés rencontrées lors de l'application de l'analyse des ondes de pression à des cas réels d'amortisseurs complexes, nous avons imaginé une approximation qui consiste à faire abstraction des effets de propagation des ondes, tout en conservant les changements volumétriques dus aux variations de pression. Les résultats obtenus pour une force d'impulsion sont donnés par la figure 3; les courbes (3) représentent les résultats d'une analyse complète des ondes de pression; la courbe (2) correspond à l'approximation que l'on vient de décrire. Une étude expérimentale sur ordinateur montre que les courbes (2) et (3) deviennent quasiment coïncidentes pour des temps d'application de la force d'impulsion supérieurs à quelques millisecondes (ce qui ne correspond qu'à une petite fraction du temps total de recul).

Il est intéressant de noter que la stabilité de calcul fut plus grande dans le modèle où l'on a admis des ondes, que dans le modèle sans ondes.

Dans le cadre de sa thèse de doctorat, M. J. B. Hinwood a représenté la région d'établissement d'un écoulement dans un conduit à deux dimensions, au moyen de l'expression en différences finies des équations de Navier-Stokes, et compte tenu de tous les termes, tant non linéaires qu'instables, ainsi que d'une densité variable. La présence des termes variables fut nécessaire à l'étude des perturbations, mais elle se révéla également très utile pour suivre, de façon très « réelle », l'évolution de « l'écoulement » lors du passage de l'état approximatif admis au départ, à l'état final. Un aspect de cette observation se trouve dans la figure 4.

Un troisième exemple est fourni par l'étude d'un écoulement visqueux dans un conduit de section brutalement élargie. Pour cela, nous avons eu recours à une forme en différences finies des lois complètes de Navier-Stokes. Une partie des résultats obtenus est consignée dans les figures 6, 7, 9. Notons par ailleurs que ce programme de recherches comprenait également une étude préliminaire d'un écoulement accéléré autour de deux obstacles (cf. fig. 10).

L'une des difficultés de la simulation des écoulements visqueux à l'aide de cette technique se traduit par la nécessité d'un grand nombre de points. Si une représentation des caractéristiques géométriques s'impose, y compris celle du détail de la rugosité des surfaces des solides, le nombre de points nécessaires devient alors excessif. Cela pourrait paraître un problème d'importance mineure si notre ambition se bornait à la simulation des écoulements en régime turbulent. Dans l'état actuel de l'art, il semble que l'on devrait concentrer ses efforts sur l'étude des aspects encore inconnus du mouvement laminaire, qui constituent des mécanismes de base de l'écoulement turbulent.

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