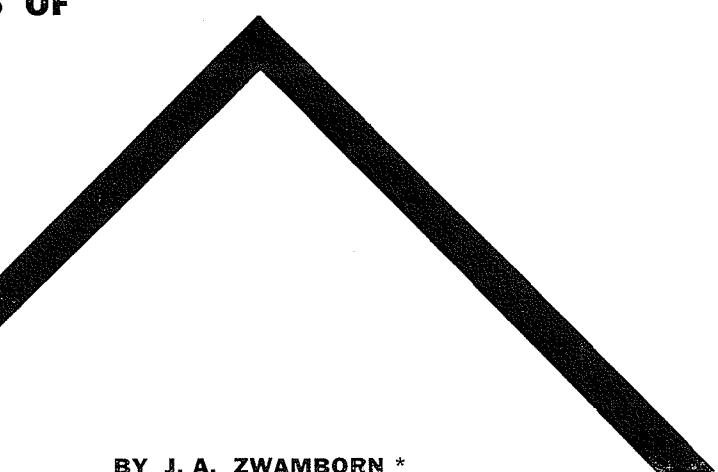


REPRODUCIBILITY IN HYDRAULIC MODELS OF PROTOTYPE RIVER MORPHOLOGY



BY J. A. ZWAMBORN *

Nomenclature

$C = V/(RS)^{1/2}$	= Chézy coefficient;
C'	= grain roughness coefficient;
d	= diameter of bed material;
d_m	= mean grain diameter;
d_{90}	= 90% smaller grain diameter;
f	= function;
$F = V^2/gh$	= Froude number;
g	= acceleration due to gravity;
h	= water depth;
L	= horizontal dimension;
q_s	= bed-load per unit width;
Q	= flowrate;
R	= hydraulic radius;
R_*	= V_*d/v = grain Reynolds number;
S	= slope;
t	= time;
T_s	= time (sediment);
v	= flow velocity;
V	= average velocity;
V_*	= \sqrt{ghS} = shear velocity;
W	= settling velocity;
Δ	= relative sediment density (submerged);
μ	= ripple factor.

The subscript r denotes prototype to model ratio.

Similarity criteria for river models

For uni-directional flow and neglecting Coriolis effects, the equation of motion is:

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + g \frac{\partial h}{\partial x} + \frac{g}{C^2} \frac{V|V|}{R} = 0$$

while, for continuity,

$$\frac{\partial h}{\partial t} + h \frac{\partial v}{\partial x} + v \frac{\partial h}{\partial x} = 0$$

Hydraulic similarity is ensured when all the terms in the two differential equations are equal for model and prototype. This condition is fulfilled when:

$$V_r = h_r^{1/2} \quad (\text{Froude law}),$$

$$t_r = L_r/V_r \quad (\text{hydraulic time scale}), \text{ and}$$

$$C_r = (L_r/h_r)^{1/2} = S_r^{-1/2} \quad (\text{Chézy roughness ratio}).$$

The value of g may be assumed the same in model and prototype and the river may be assumed sufficiently wide so that the hydraulic radius may be replaced by the water depth. The flow rate ratio $Q_r = L_r h_r^{3/2}$ satisfies the Froude law, an essential condition in reproducing meandering rivers [1].

The roughness of movable-bed models is inter-related with the type of bed material which, together with the flow characteristics, determine the bed shape. On the basis of work by Einstein, Meyer-Peter and Müller, and following the logarithmic velocity distribution law, Frýlink [2] defines a ripple factor as follows (turbulent flow):

$$\mu = \left(\frac{C}{5.75 \sqrt{g} \log 12 h/d_{90}} \right)^{3/2}$$

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With $C = 5.75 \sqrt{g} \log 12 h/d_{90}$ being the grain roughness coefficient, the overall channel roughness C may be found from:

$$C = \mu^{2/3} C'$$

Hence the condition for dynamic similarity becomes:

$$\mu_r^{2/3} C'_r = S_r^{-1/2}$$

Frylink [2] suggests that $\mu = f_1 (hS/\Delta d)$ only. Thus in case that $(hS/\Delta d)_r = 1$ then:

$$C_r = C'_r = S_r^{-1/2}$$

In a more recent study based on dimensional analyses Yalin [3] concludes that, mainly:

$$C = f_2 (\mathcal{R}_*, hS/\Delta d, h/d)$$

Apart from the grain Reynolds number, \mathcal{R}_* , this agrees with the above expression for C . It thus appears that, according to Yalin's results, μ is both a function of $hS/\Delta d$ and \mathcal{R}_* . Especially for sand of a size below 520 micron it is the author's opinion, however, that the dependence on \mathcal{R}_* as shown by Yalin is quite negligible. It thus follows that, provided $(hS/\Delta d)_r = 1$ and the model value of \mathcal{R}_* does not deviate too much from its prototype value, the friction criterion is satisfied when:

$$C'_r = S_r^{-1/2}$$

Various approaches to ensure sediment motion similitude have been attempted. It is invariably concluded, however, that the same two parameters, namely $hS/\Delta d$ and \mathcal{R}_* best describe the whole complex [4, 5, 6]. Unfortunately the similarity conditions $(hS/\Delta d)_r = 1$ and $\mathcal{R}_{*r} = 1$, together with the Froude and friction criteria generally lead to completely impractical model scales. More realistic results are obtained when the first ratio is replaced by $(V_*/W)_r$, and when a certain deviation is tolerated in applying the second condition. V_*/W and \mathcal{R}_* describe the sediment processes as well generally as $hS/\Delta d$ and \mathcal{R}_* and, in the case of saltation and suspended load, V_*/W is considered a superior parameter for scaling purposes. Moreover, Yalin [7] has shown that:

$$V_*/W = f_3 (hS/\Delta d, \mathcal{R}_*), \text{ thus } hS/\Delta d = f_4 (V_*/W, \mathcal{R}_*)$$

Hence both criteria are basically the same, and also in the friction criterion discussed earlier $hS/\Delta d$ may therefore be replaced by V_*/W .

It may thus be concluded that good similarity in river morphology can be expected between model and prototype when, for the model, Froude's law ($V_r = h_r^{1/2}$), the sediment motion criterion:

$$[(V_*/W)_r = 1],$$

and the friction criterion ($C' = S_r^{-1/2}$) are satisfied, and when \mathcal{R}_* is of the same order of magnitude in model and prototype. The permissible deviation from $(\mathcal{R}_*)_r = 1$ will depend on the type of problem and can only be judged satisfactorily from a direct correlation between model and prototype occurrence-

ces. The hydraulic time scale follows from $t_r = L_r/V_r$ and the sedimentological time scale for a particular flood from:

$$(T_s)_r = L_r h_r \Delta_r / (q_s)_r$$

in which q_s is the bed-load per unit width.

Umfolozi and Notchwan River models

A few movable-bed hydraulic models of South African rivers, which were designed in accordance with the above scale laws, are now discussed.

UMFOLOZI RIVER MODEL.

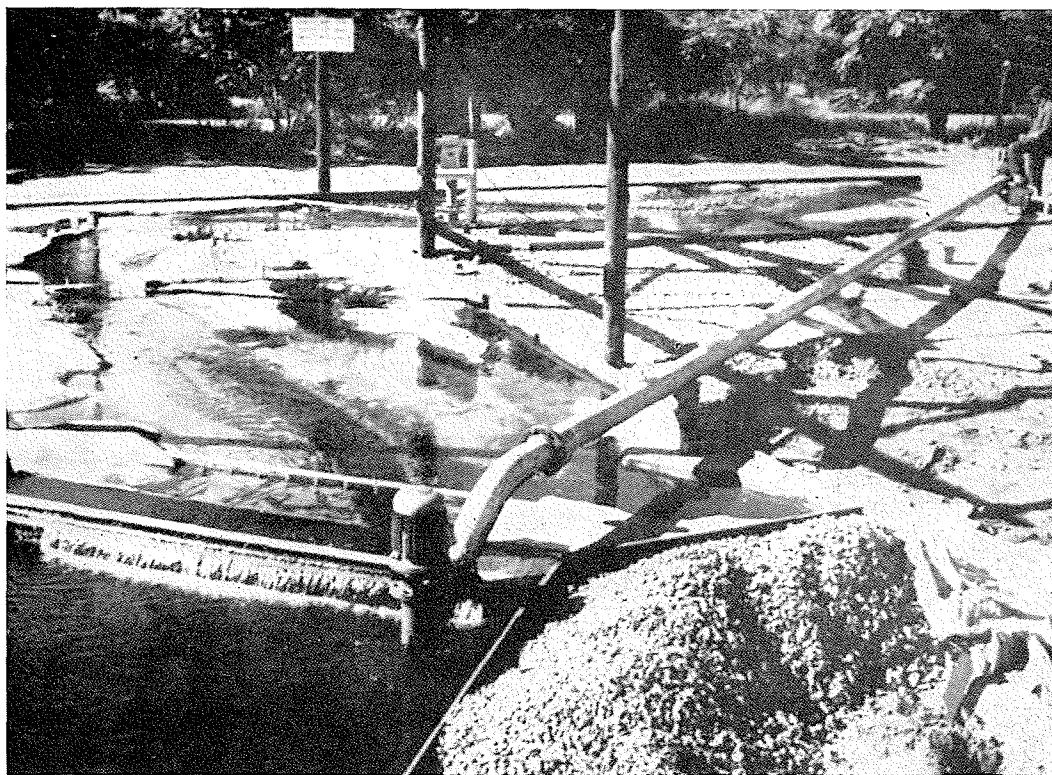
The object of this model study was to establish the best site for a national road bridge, about 1,000 ft long, across the Umfolozi River, and to study its influence on river morphology. Four miles of river were reproduced in the model to a horizontal scale ratio, $L_r = 250$ and a vertical scale ratio, $h_r = 100$ ($S_r = 1/2.5$) (see Figure 1). Floods up to 400,000 cusecs, scaled down in accordance with the Froude law ($Q_r = L_r h_r^{3/2} = 250,000$) were reproduced. Crushed anthracite, (S.G. 1.35) was used for the movable bed in those areas of the model where bed movement was found to occur. Model bed-load rates were calculated for different flow-rates using Kalinske's formula for the river cross-section about 1 mile upstream of the bridge site. In this cross-section the required sediment was added to the river flow in the correct proportion by means of a specially designed dry feeder apparatus (see top centre on Figure 1). The calculated bed-load rates were verified in the model for the various flow rates assuming stable bed conditions at the feeding point and were found to be correct within about 10%. The size distribution of the model sediment was found by applying the condition:

$$(V_*/W)_r = 1 \quad (\text{or } W_r = (V_*)_r = (hS)_r^{1/2} = 6.33)$$

to the whole size range of the prototype sand (see Figures 2 and 3).

The friction criterion was very nearly satisfied ($C_r = 1.52$ against $S_r^{-1/2} = 1.58$), which led to an easy reproduction of flood levels recorded in nature.

Based on mean grain size diameters, the values of \mathcal{R}_* were 4.3 and 36.6 for model and prototype respectively [$(\mathcal{R}_*)_r = 8.5$]. Plotted against $V_*/W_{\text{mean}} = 5.5$ in a graph similar to that by Albertson et al. [8] (see Figure 4), it becomes clear that the various regions with a particular bottom form, as determined from flume tests by numerous investigators, do not apply to prototype conditions [9]. The prototype point falls in the region where, for the same values of V_*/W and \mathcal{R}_* , antidunes would occur in a test flume. The prototype Froude number (\mathcal{F}) is only of the order of 0.5. This rules out the possibility of antidunes in nature. Under model conditions \mathcal{R}_* should therefore be between, say, 1 and 6 for a comparable Froude number and

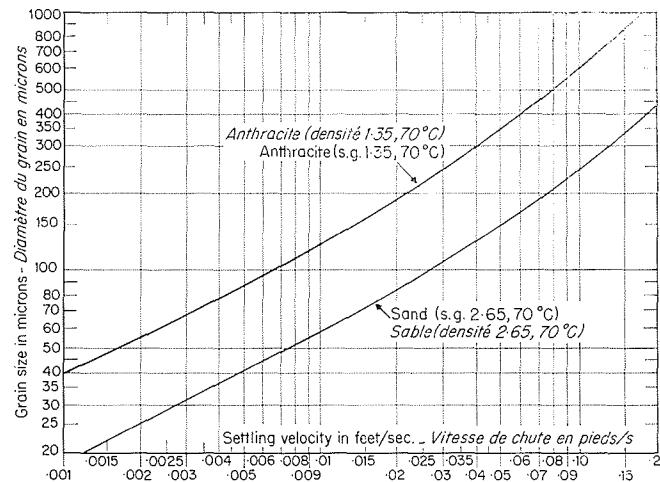


1/ General view of the model in operation.

Vue générale du modèle en fonctionnement.

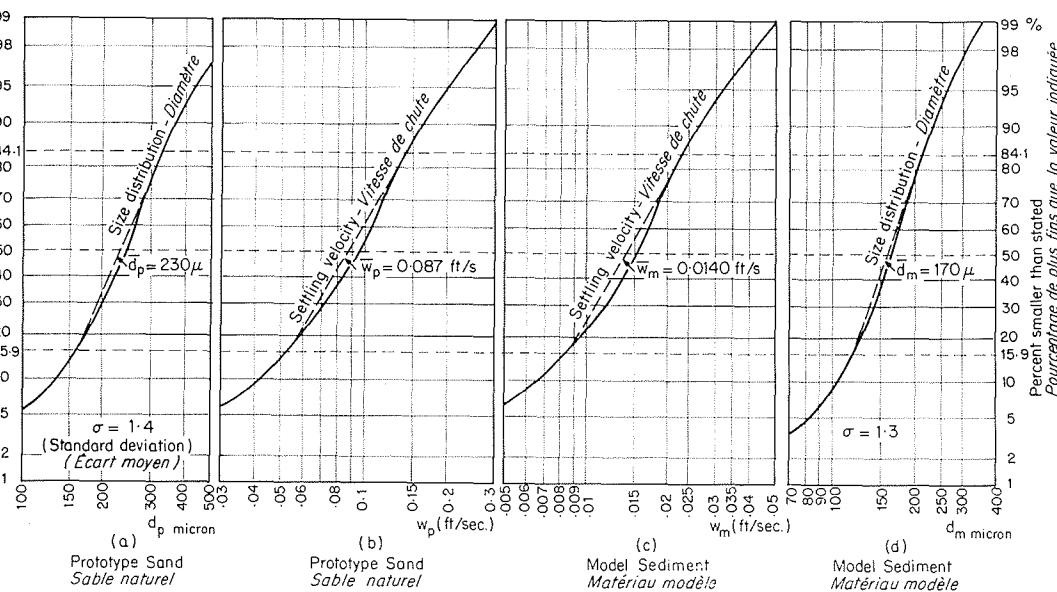
bottom configuration (the model value of $\mathcal{R}_* = 4.3$ complies with this condition). It thus follows that the relationship between V_*/W , \mathcal{R}_* and \mathcal{T} does not uniquely describe bottom processes for both model and prototype conditions and apart from the product hS the actual water depth (h) or in dimensionless form h/d will probably be of importance [10, 11]. This is contrary to a conclusion reached by Yalin [3].

From the Umfolozi study it was concluded that, apart from practical considerations (reasonable model scales), it appears necessary to deviate from the condition $(\mathcal{R}_*)_r = 1$ in order to arrive at comparable sediment behaviour in model and prototype. The model value of \mathcal{R}_* should preferably fall in



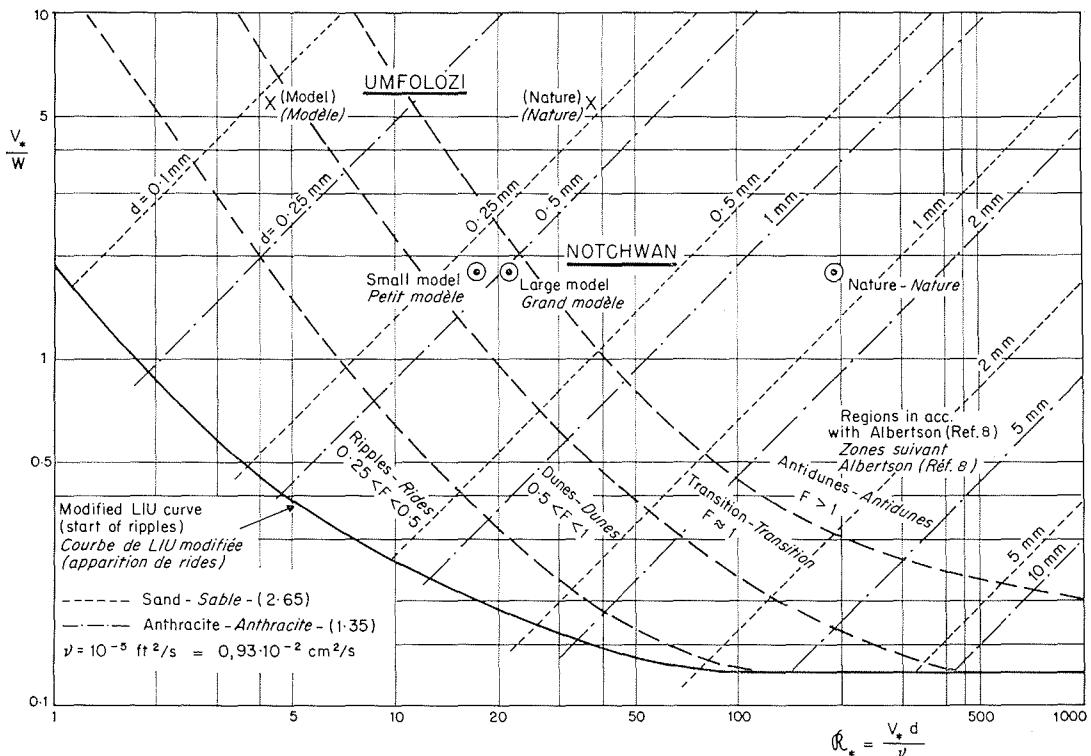
2/ Settling-velocity curves.

Vitesse de chute.



3/ Determination of size distribution for model coal.

Détermination granulométrique du charbon sur le modèle.



4/ Bed-shape criteria.

Critères concernant la forme du lit.



Large scale model of Notchwan River bridge

the region with a Froude number comparable to prototype conditions (Fig. 4).

Whereas the hydraulic time scale simply follows from $t_r = L_r/V_r = 25$, the bed-load transport ratio must be known in order to arrive at a sedimentological time scale. Due to the steep slope of the river (about 1 in 2,000), its tractive force falls completely outside the region covered by modern bed-load formula. For a flood of 200,000 cusecs and by using du Bois' formula for the prototype and Kalinske's formula for the model, it follows that $(q_s)_r = 560$ and thus $(T_s)_r = 200$.

It may be noted that after verifying a number of models against prototype occurrences the following very approximate relation was found:

$$(T_s)_r \approx 10 t_r.$$

NOTCHWAN RIVER MODELS.

The Notchwan River has an average slope of 1 in 200. The bed consists of coarse sand ($d_m = 780$ micron) and the river carries floods up to about 12,500 cusecs. The study was concerned with river training just upstream of a road bridge. A curved guide wall was constructed and possible scour depths were required for foundation protection purposes. Two models were built designed on the same similarity criteria as the Umfolozi model (see Figure 5). The duplication was considered necessary to study

possible scale effects regarding scour depths. The main particulars of each model are described below.

SMALL SCALE NOTCHWAN MODEL:

$$\begin{aligned} L_r &= 120 & V_r &= 6 \quad W_r = (V_*)_r = 3.3 \\ &&& \text{(or } 3.9 \text{ using hydraulic radii)} \\ h_r &= 36 & t_r &= 20 \quad C_r = 1.41 (S_r^{-1/2} = 1.82) \\ S_r &= 1/3.3 \quad (T_s)_r \approx 180 & \mathcal{R}_* &= 17.5 \text{ (model)} \\ &&& \text{and } 192 \text{ (nature)} \end{aligned}$$

The mean diameter of the required model sand (S.G. 2.65) was found in the same way as described for the Umfolozi model. For $W_r = 3.3$, 285 micron mean diameter sand and for $W_r = 3.9$, 260 micron sand is required. The available sand used in the study was of 280 micron mean diameter which was found to be satisfactory.

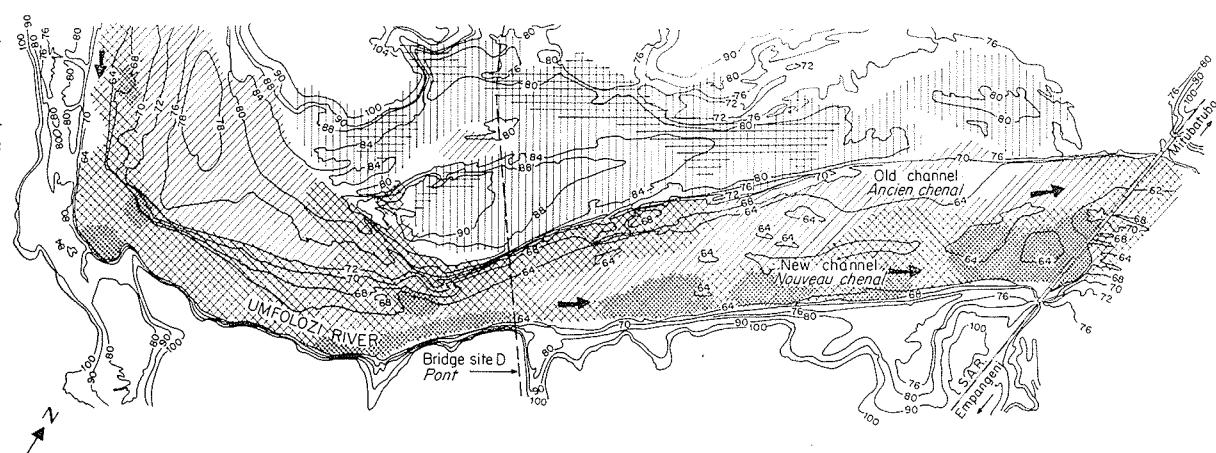
A deviation of the roughness criterion is apparent (compare the values of C_r and $S_r^{-1/2}$) which indicates a too great a distortion of the model. It was therefore necessary to use artificial roughness elements in the upstream portion of the river in order to reproduce flood levels correctly [12].

With a prototype value of $\mathcal{F} \approx 1$, it is again clear from Figure 4 that the value of $(\mathcal{R}_*)_r = 11$ is acceptable for reproducing correct bottom configuration.

6/

Topography just after the July flood.

Topographie aussitôt après la crue de juillet.



LARGE SCALE NOTCHWAN MODEL:

$$\begin{aligned} L_r &= 24 & V_r &= 4 & W_r = (V_*)_r &= 3.26 \\ h_r &= 16 & t_r &= 6 & C_r' &= 1.20 (S_r^{-1/2} = 1.22) \\ S_r &= 1/1.5 & (T_s)_r &\approx 60 & \mathcal{R}_* &= 21.1 \text{ (model)} \\ &&&&& \text{and } 192 \text{ (nature)} \end{aligned}$$

The mean diameter of the required model sand (S.G. 2.65) is 290 micron whereas 280 micron sand was used. $(\mathcal{R}_*)_r = 9.1$.

The friction criterion was fully satisfied and it was therefore no problem in correctly reproducing known flood levels.

Model-prototype correlation of river morphology

Subsequent to the completion of the model studies described in the above, a very severe flood occurred in the Umfolozi River. The effects on the regime of the river were recorded and, since the model was still available, this unique opportunity was used to verify the similarity conditions described earlier on. Moreover, tests with different sediments were made in both the Umfolozi and Notchwan models to establish the importance of some of the governing parameters.

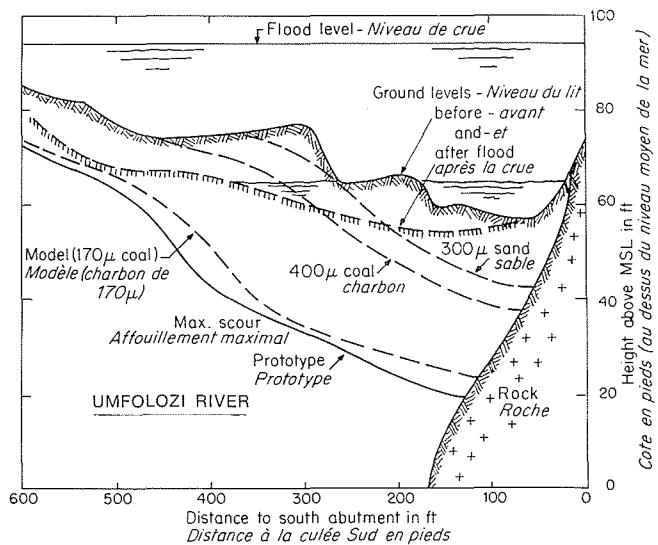
UMFOLOZI MODEL CORRELATION TESTS [13].

A flood with a peak flow of 300,000 cusecs (approximate return period 300 years) which occurred in July, 1963 caused major changes in the Umfolozi River bed. Immediately after the flood the new situation was accurately surveyed and is compared in Figure 6 with a similar survey made before the flood. The most striking differences between conditions before and after the flood are at the inside of the bend (extreme left Figure 6) where scour of some 5ft average occurred, and just downstream of the recommended bridge site D, where the river developed a new, more southerly course. At site D the actual river channel, which was originally 250ft wide, eroded to a total width of 600ft, while the flood plain north thereof generally silted up, in some places up to 12ft.

The flood was reproduced to scale in the model and the topography after the flood, as well as the areas where erosion or deposition occurred during the flood, are shown in Figure 7.

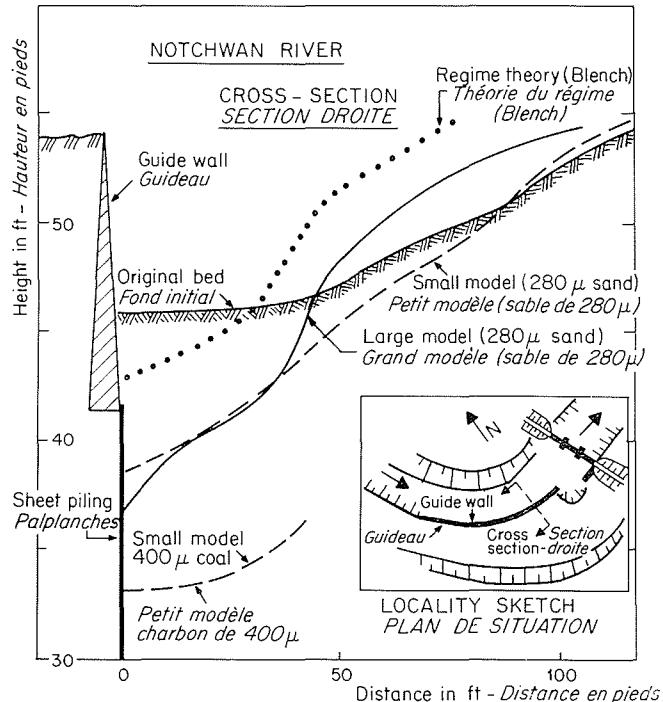
The general resemblance between the model and prototype results is apparent (compare Figures 6 and 7). This applies to both qualitative and quantitative changes [13].

The tail of a flood tends to fill in scour holes formed during peak flow. Special methods must therefore be used to measure maximum scour depths. At bridge site D soil profiles were available down to a depth of 100ft below ground level. From a comparison with washboring and gamma-ray density-logging results, obtained immediately after the flood, the maximum scour line at site D was established [13, 14] (Fig. 8). A similar line was recorded in the model by using closely spaced



8/ Scour depths at bridge site D.

Profondeurs d'affouillement à l'emplacement du pont D.



9/ Scour depths Notchwan River.

Profondeurs d'affouillement, Notchwan River.

fluorescent sand piles. The top of the undisturbed part of the sand pile yielded the maximum scour level which occurred during the test.

The very good agreement between model and prototype scour depths (within 10 % for the maximum value) is evident from Figure 8. Moreover, the model result was easily reproducible.

In Figure 8 are also shown maximum scour lines obtained from model tests with 400 micron anthracite ($V_*/W = 1.2$ against the required 5.5) and 300 micron sand ($V_*/W = 0.06$).

The much-too-small scour depths using this material emphasizes the importance of the parameter V_*/W .

NOTCHWAN MODELS CORRELATION TESTS.

In Figure 9 scour depths for the equivalent of a 12,500 cusecs flood in both the small scale and the large scale Notchwan models (see previous section) are compared. The agreement is very good indeed, apart from the inside of the bend (see right hand side of Figure 9). The little-distorted large model shows deposition here which is not the case for the more-distorted small model. In the latter case, however, deposition is not possible since this would require a cross slope steeper than the angle of repose of the sand. This also indicates that the distortion of the small model (3.3) was a bit too much (see previous section).

Further evidence of the importance of the parameter V_*/W was gained by using 400 micron anthracite ($V_*/W = 3.1$ against 1.8) in the small model which resulted in far greater scour (Fig. 9). For $(V_*/W)_r = 1$, the required size of anthracite is 620 micron. This was not available but some tests with 500 micron anthracite indicated that 620 micron would most probably yield the same scour as the 280 micron sand.

Finally the large model was remoulded in accordance with the Regime Theory similarity conditions:

$$(h_r = 10, L_r = 32,$$

$$d_m = \text{prototype sand} = 780 \text{ micron}) [15].$$

The recorded scour, however, was far too small (Fig. 9).

Conclusions

For rivers of a nature similar to the ones described above (V_*/W large or large tractive force) it may be concluded that:

- (i) provided the similarity criteria $V_r = h_r^{1/2}$, $(V_*/W)_r = 1$ and $C_r = S_r^{-1/2}$ are adhered to, river bed changes will be reproduced very nearly correct to scale;
- (ii) in order to reproduce river bed behaviour correctly in the model, it was found necessary to deviate from the criterion $(\mathcal{R}_*)_r = 1$;
- (iii) the value of \mathcal{R}_* for the model should fall in the region of similar bed form and of equal Froude number (see Figure 4); and

- (iv) values of $(\mathcal{R}_*)_r$ up to about 10 did not appear to affect similarity as a whole.

Further research is being undertaken to correlate also the river bed changes for conditions after the Umfolozi bridge is built. Sand piles with fluorescent sand for scour recording are employed for this purpose.

Bibliographie

- [1] BYKER (E. W.) et al. — Some scale effects in models with bed-load transportation. Proceedings I.A.H.R. Conference, Lisbon 1957.
- [2] FRYLINK (H. C.). — Discussion des formules de débit solide de Kalinske, d'Einstein et de Meyer-Peter et Müller, *II^{es} Journées Hydraulique*, Soc. Hydrotechn. de France, Grenoble (1952).
- [3] YALIN (S.). — On the average velocity of flow over a movable bed. *La Houille Blanche*, Grenoble, No. 1 (January 1964).
- [4] EINSTEIN (H. A.) et CHIEN (N.). — Similarity of distorted river models with movable beds. *Transactions A.S.C.E.*, vol. 121 (1956).
- [5] BOGARDI (J.). — Hydraulic similarity of river models with movable bed. *Acta Technica Academiae Scientiarum Hungaricae*, tome XXIV, fascicule 3-4, Budapest (1959).
- [6] YALIN (S.). — Ueber die dynamische Ähnlichkeit der Geschiebebewegung. *Die Wasserwirtschaft*, Heft 8 und 9, Stuttgart (1960).
- [7] YALIN (S.). — An expression for bed-load transportation. *J. Hyd. Div. A.S.C.E.*, HY3, (May 1963).
- [8] ALBERTSON (M. L.) et al. — Discussion on paper 1197 by Liu. *J. Hyd. Div., A.S.C.E.* Vol. 84, HY1 (February 1958).
- [9] ZWAMBORN (J. A.). — Discussion on paper 4195 by Bogárdi. *J. Hyd. Div., A.S.C.E.* Vol. 91, No. HY5 (September 1965).
- [10] SIMONS (D. B.) et RICHARDSON (E. V.). — Forms of bed roughness in alluvial channels. *J. Hyd. Div., A.S.C.E.* Vol. 87, No. HY3 (May 1961).
- [11] NORDIN (C. F.) et BEVERAGE (J. P.). — Discussion on paper 3525 by Yalin. *J. Hyd. Div., A.S.C.E.* Vol. 90, No. HY1 (January 1964).
- [12] International Course in Hydraulic Engineering. — The use of artificial roughness in movable-bed models. Waterloopkundig Laboratorium, Delft, report S80 (August 1962).
- [13] ZWAMBORN (J. A.). — Correlation between model and prototype morphology of rivers. *CSIR special report*, No. MEG 334, Pretoria, (January 1965).
- [14] KÜHN (S. H.) et WILLIAMS (A. A. B.). — Scour depth and soil profile determinations in river beds. *Proceedings 5th Int. Conf. on Soil Mech. and Found. Eng.*, Paris (July 1961).
- [15] BLEANCH (T.). — Regime behaviour of canals and rivers. *Butterworths Scientific Publications*, London (1957).

Résumé**Reproduction de la morphologie des fleuves sur modèle réduit**

par J. A. Zwamborn *

La similitude entre le modèle et la nature est généralement assurée lorsque :

$$V_r = h_r^{1/2} \quad (\text{loi de Froude});$$

$$t_r = L_r/V_r \quad (\text{échelle de temps hydraulique});$$

et $C_r = (L_r/h_r)^{1/2} = S_r^{-1/2}$ (coefficients de rugosité de Chézy).

La rugosité d'ensemble, C , d'un modèle à fond mobile est fonction, à la fois de la nature du matériau constituant le fond, et des caractéristiques de l'écoulement. Frýlink [2] définit un coefficient, μ , pour les rides du fond, par la relation :

$$\mu = \left(\frac{C}{5,75 \sqrt{g} \log 12 h/d_{90}} \right)^{3/2}$$

Soit, $C' = 5,75 \sqrt{g} \log 12 h/d_{90}$ représentant le coefficient de rugosité des grains, la rugosité d'ensemble peut se déterminer à partir de la relation : $C = \mu^{2/3} C'$

L'auteur conclut, sur la base des études de Frýlink [2] et de Yalin [3], que μ est fonction principalement de V_*/W , de sorte que, lorsque $(V_*/W)_r = 1$, le coefficient de rugosité est donné par : $C_r = C'_r = S_r^{-1/2}$

Puisque le mouvement des matériaux solides est également fonction principalement de V_*/W , et (bien que dans une moindre mesure) du nombre de Reynolds \mathcal{R}_* du grain, on peut s'attendre (à condition, toutefois, que les lois de similitude précipitées soient respectées) à ce que la similitude des caractéristiques morphologiques de la rivière soit correcte lorsque $(V_*/W)_r = 1$, et lorsque les valeurs de \mathcal{R}_* sont du même ordre de grandeur sur le modèle et dans la nature.

Un certain nombre de modèles hydrauliques à fond mobile ont été étudiés en fonction des critères de similitude précédents, et employés pour résoudre des problèmes d'aménagement fluvial en Afrique du Sud.

Le modèle du pont de la rivière Umfolozi a été réalisé avec une échelle horizontale $L_r = 250$, et une échelle verticale $h_r = 100$ (voir la figure 1). Le matériau du fond sur le modèle était de l'anthracite, dont la granulométrie répondait à la condition $(V_*/W)_r = 1$ (voir les figures 2 et 3). Le critère de frottement a pu être respecté à peu de choses près, avec $C'_r = 1,52$, au lieu de $S_r^{-1/2} = 1,58$, ce qui a permis la reproduction, sans difficulté, des cotes de crue connues, sur le modèle. Les valeurs de \mathcal{R}_* étaient 4,3 pour la nature, et 36,6 pour le modèle. Cet écart s'est montré nécessaire pour permettre la reproduction d'un profil du lit « modèle » correspondant à celui existant dans la nature, pour lequel le nombre de Froude est égal à $F = 0,5$ (voir la figure 4) [9]. Il apparaît ainsi que V_*/W , \mathcal{R}_* , et F ne définissent pas les processus d'évolution du fond de manière unique [10, 11].

Une forte crue s'est produite dans la rivière Umfolozi après l'achèvement de l'étude sur modèle réduit. L'influence de cette crue sur le régime de la rivière a été enregistrée, et puisque le modèle était encore disponible, il a été possible d'y reproduire le passage de la crue, et ensuite de confronter les résultats obtenus et les données « nature » (voir les figures 6 et 7). Les profondeurs d'affouillement à l'emplacement envisagé pour le pont ont été relevées tant sur le modèle que dans la nature [13, 14] (voir la figure 8). On voit que ces données s'accordent bien, tant en ce qui concerne la morphologie fluviale en général, que les profondeurs d'affouillement en particulier [13]. La figure 8 indique également les profondeurs d'affouillement maximales atteintes lors des essais sur le modèle, pour l'anthracite de granulométrie 400 μ ($V_*/W = 1,2$ au lieu de la valeur requise, soit 5,5), et pour du sable de granulométrie 300 μ ($V_*/W = 0,06$). Ces profondeurs d'affouillement sont toutefois bien trop faibles, ce qui met en évidence l'importance du paramètre V_*/W .

Un autre problème traité sur modèles était la détermination de la profondeur d'affouillement le long d'un mur de guidage au pont de la rivière Notchwan. Deux modèles ont été étudiés et réalisés suivant les mêmes critères de similitude que pour le modèle de l'Umfolozi (fig. 5). Les profondeurs d'affouillement relevées sur le petit modèle ($L_r = 120$; $h_r = 36$; sable de 280 μ) et sur le grand modèle ($L_r = 24$; $h_r = 16$; sable de 280 μ) sont comparés sur la figure 9. La concordance des résultats est considérée comme étant bonne, compte tenu de la grande différence entre les dimensions de ces deux modèles. En outre, le résultat obtenu avec du charbon de granulométrie 400 μ ($V_*/W = 3,1$ au lieu de 1,8) indique ici aussi qu'une déviation par rapport à $(V_*/W)_r = 1$ n'est guère admissible (fig. 9).

Enfin, le grand modèle a été reprofilé en fonction des conditions de similitude de la théorie du régime ($h_r = 10$; $L_r = 32$; $d_m = 780 \mu$) [15], mais on voit (fig. 9) que la profondeur d'affouillement relevée est beaucoup trop faible.

Ceci amène à formuler les conclusions suivantes, en ce qui concerne les rivières présentant des forces d'entrainement relativement élevées :

1. à condition de respecter les critères de similitude $V_r = h_r^{1/2}$, $(V_*/W)_r = 1$, et $C'_r = S_r^{-1/2}$, on peut s'attendre à ce que l'évolution du lit fluvial soit reproduite à peu de choses près correctement à l'échelle sur le modèle;
2. afin de pouvoir reproduire le comportement du lit correctement sur le modèle, il a été nécessaire de s'écartez du critère $(\mathcal{R}_*)_r = 1$;
3. la valeur de \mathcal{R}_* correspondant au modèle doit se situer, en principe, dans la région de similitude des profils du lit et d'égalité des nombres de Froude (voir la figure 4);
4. les valeurs de $(\mathcal{R}_*)_r$ jusqu'à l'ordre de 10 ne semblaient guère modifier la similitude dans l'ensemble.

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