NOTULES HYDRAULIQUES

DIRECT SOLUTION FOR PROBLEMS IN PIPE FRICTION

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Pipe flow problems, in general, involve the following variables—the discharge of the fluid Q, the loss of head h_f (or pressure Δp) in the length of pipe L, the kinematic viscosity of the fluid ν, the pipe diameter D and the equivalent sand grain roughness of the material of the pipe k. These problems fall into three different categories. They are (1) To find the head loss (2) to determine the discharge (3) to find the pipe diameter, when the other variables are given.

The basic equation for the solution of these problems is the Darcy-Weisbach resistance equation, viz.

$$h_f = \frac{f L V^2}{2\, g D} \qquad (1)$$

where f: friction factor; and V: mean velocity of flow.

For the case of laminar flow, the friction factor is given [1] by the equation :

$$f = \frac{64}{\mathcal{R}} \qquad (2)$$

where $\mathcal{R} = (VD/\nu)$ and problems belonging to all the three categories can be solved directly by the use of equation (2). For turbulent flows, one has to make use of the diagrams prepared by Moody [2] or by Rouse [1]. These diagrams have been prepared with help of Colebrook-White equation (1) for the resistance of commercial pipes. Problems of the first category can be solved directly with the help of the above diagrams, while the form presented by Rouse enables a direct solution even for the second category of problems; however, using the above diagrams, one has to go through a process of trial and error for problems in the third category.

By forming a set of dimensionless parameters different from the ones in the above diagrams, the authors have shown that a direct solution can be obtained for all the three categories of problems.

Ist METHOD:

The resistance of a commercial pipe is expressed [1] by the Colebrook-White equation, namely

$$\frac{1}{\sqrt{f}} = 2 \operatorname{Log}_{10}\left(\frac{D}{2\,k}\right) + 1.74 - 2 \operatorname{Log}_{10}\left(1 + \frac{18.7\,(D/2\,k)}{\mathcal{R}\sqrt{f}}\right) \qquad (3)$$

Equation (3) has been found to be asymptotic both to the smooth and the rough-pipe equations and to follow closely the trend of experimental points in the transition region.

Equation (3) after necessary manipulations can be written as:

$$\frac{0.903\,B}{A^5} = 2 \operatorname{Log}_{10}(A^2 C) + 1.74 - 2 \operatorname{Log}_{10}\left(1 + \frac{6.60}{A}\right) \qquad (4)$$

where:

$$A = \frac{k\sqrt{gDS}}{\nu} \qquad (5)$$

$$B = \frac{QS^2 k^5 g^2}{\nu^5} \qquad (6)$$

$$C = \frac{\nu^2}{2\,k^3 g S} \qquad (7)$$

and:

$$S = \frac{h_f}{L} \qquad (8)$$

By substitution of various values of A and C in Equation (4), the parameter B has been found and

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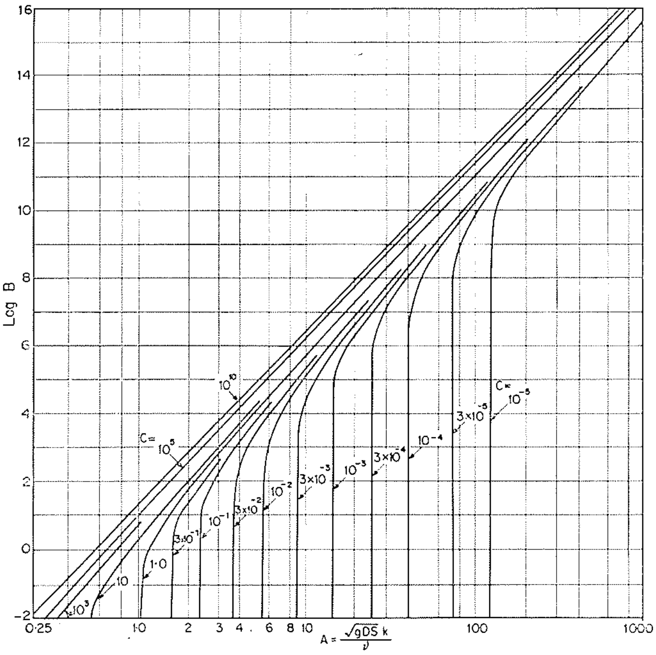
[1] ROUSE (H.): Elementary mechanics of fluids. *John Wiley and sons*, Inc. New York.

[2] MOODY (L. F.): Friction factors for pipe flow. *Trans. ASME*, November 1944.



1/ Diagram for solution of Colebrook's equation.
Graphique pour la solution de l'équation de Colebrook.

the relation between these parameters has been shown as Figure 1. The diagram covers the flow in the transition and the rough boundary region. Extension of the diagram to smooth boundary flows was felt to be unnecessary as the method suggested by Powell [3] yields direct solutions in such a case.

Figure 1 yields a direct and accurate solution for the pipe diameter, given Q, ν, S and k. Also, for known values of D, ν, S and k, Q can be found directly from the above diagram, but the diagram will have to be constructed to an exaggerated B scale for accurate solutions of problems in this category. Direct solution for S, however, cannot be obtained with the help of this diagram, if Q, ν, D and k are known; but this category of problems can be solved directly using the Moody's diagram.

IInd METHOD:

The functional relationship between the various parameters involved in pipe friction problems can be written in the form:

$$\frac{\Delta p}{L} = \varphi_1 (k, \rho, Q, D, \mu) \tag{9}$$

where $\Delta p = \gamma h_f$, γ being the specific weight of fluid;

ρ : Mass density of fluid;

μ : Dynamic viscosity of fluid.

By dimensional analysis, the above relation can be written as:

$$\frac{\Delta p \rho k^3}{L\mu^2} = \varphi_2 \left(\frac{Q}{\nu k}, \frac{k}{D}\right) \tag{10}$$

[3] POWELL (R. W.) : Resistance to flow in smooth pipes found directly. *Civil Engineering,* November 1954, p. 62.

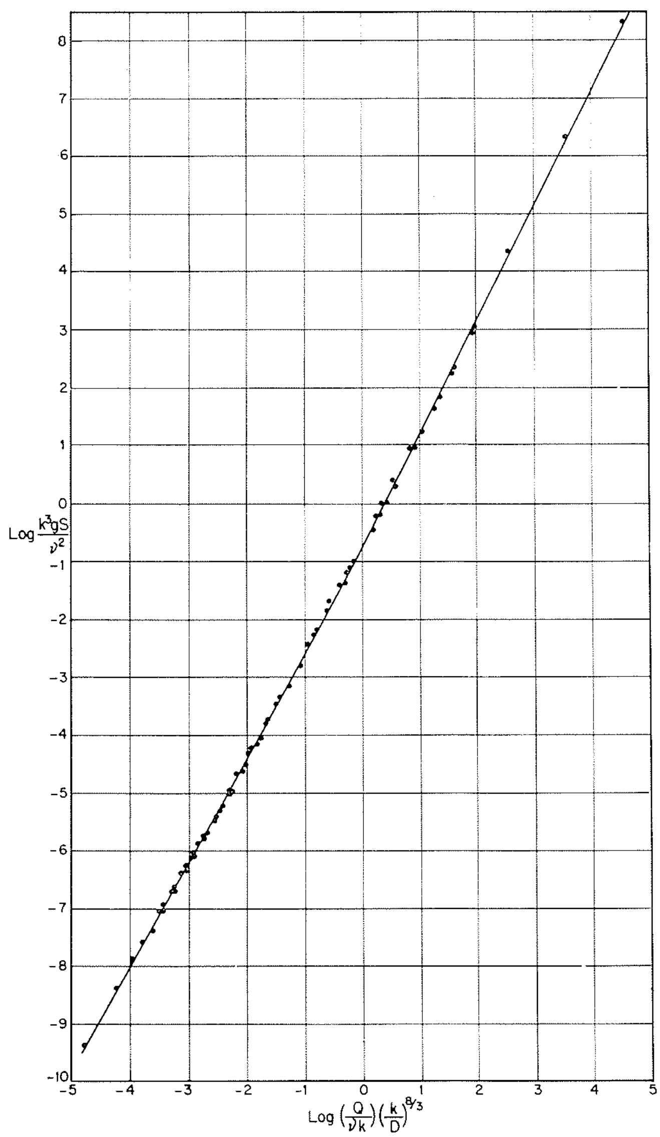
which can be put in the form:

$$\frac{k^3 g S}{\nu^2} = \varphi_3 \left(\frac{Q}{\nu k}, \frac{k}{D}\right) \tag{11}$$

The relationship between the above parameters can be obtained with the help of the Moody's diagram. For different values of $\mathcal{R}$ and k/D, f was obtained from the Moody's diagram and thus the parameters k^3gS/ν^2 and $Q/\nu k$ computed. A study of the plot between $Q/\nu k$ and k^3gS/ν^2 with k/D as the third variable over a range of smooth, transition and rough regions, indicated that with slight approximation, a unique relation can be obtained between the parameters $(Q/\nu k)\ (k/D)^{8/3}$ and k^3gS/ν^2. The relationship between these two parameters has been shown in Figure 2 and can be expressed by the following equations:

$$\left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} = 2.51 \left(\frac{k^3 g S}{\nu^2}\right)^{0.547}$$

$$\text{for } \frac{k^3 g S}{\nu^2} < 0.35 \quad \text{or} \left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} < 1.60 \tag{12}$$



2/ Relation between k^3gS/ν^2 and $(Q/\nu k)\ (k/D)^{8/3}$.
Relation entre k^3gS/ν^2 *et* $(Q/\nu k)\ (k/D)^{8/3}$.

$$\left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} = 2.40\left(\frac{k^3 g S}{\nu^2}\right)^{0.507}$$

$$\text{for } \frac{k^3 g S}{\nu^2} > 0.35 \quad \text{or } \left(\frac{Q}{\nu k}\right)\left(\frac{k}{D}\right)^{8/3} > 1.60 \qquad (13)$$

The data used in developing the above relationship cover a range of Reynold's number VD/ν from 4 000 to 7.10^7 and from the completely smooth to the completely rough region [up to $(k/D) = 4.10^{-2}$] Equations (12) and (13) yield direct solutions for all categories of problems concerning pipe friction. Despite the fact that the above equations are empirical ant not exact, they have the advantage over Figure 1 that in this method the inherent errors involved in interpolating and reading Figure 1 are absent. Further, equations (12) and (13) are valid for the smooth, transition and rough boundaries, which is not the case for Figure 1.

The errors in the diameter (for given Q, ν, k and S) obtained from equations (12) and (13) as compared to the value obtained using Moody's diagram are found to be very small (about $\pm$ 3 %). In view of the fact that a standard pipe diameter has to be used ultimately and also since the method of successive trials by using the Moody's diagram is seldom carried to the exact value, equations (12) and (13) seem to be suited for problems of this category. The errors in the value of Q obtained from the above equations seem to be of the order of $\pm$ 5 %. However, the errors in the values of S computed from these equations are of the order of $\pm$ 10 % and in some stray cases as high as $\pm$ 15 %.