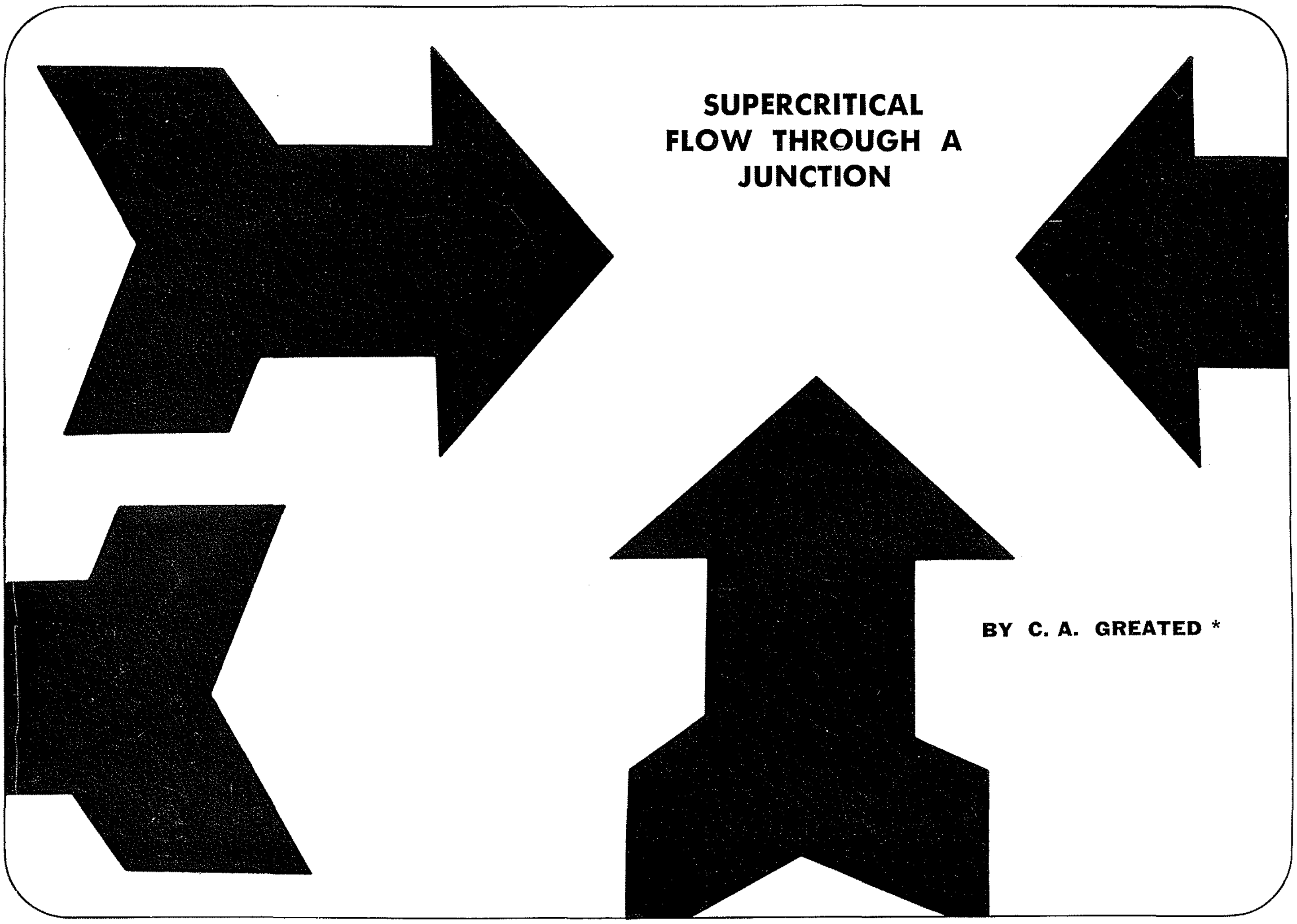
SUPERCRITICAL FLOW THROUGH A JUNCTION



BY C. A. GREATED *

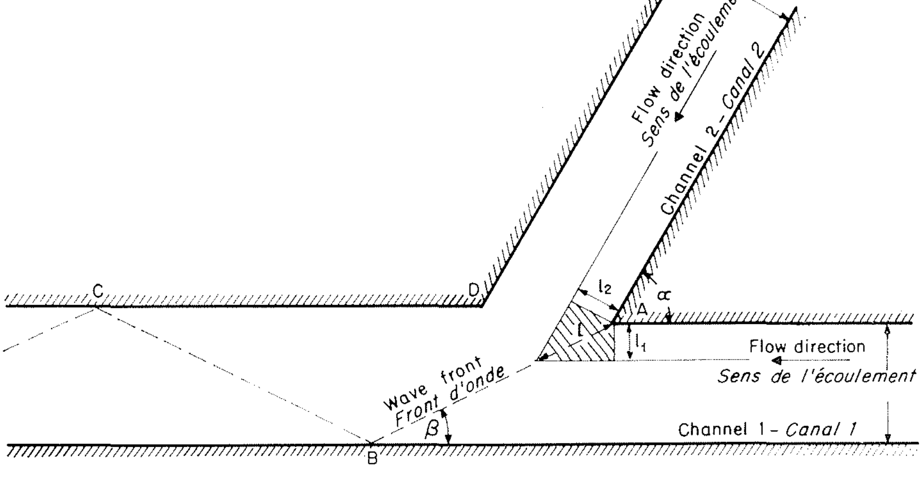
When flows pass through a junction at supercritical velocities, oblique standing waves are formed downstream, which oscillate back and forth across the channel as seen in the photographs (Fig. 2). In order to study these a number of tests were carried out in the laboratory of Southampton University using 5 in. wide rectangular channels intersecting at 60 degrees. It was found that accurate predictions of wave positions could be obtained by a direct application of the momentum principle.

Consideration has only been given to junctions of horizontal rectangular channels taking the simple form shown in Figure 1 with the velocities in all three channels supercritical. Fluid elements from the two incoming streams meet along the line AB and then move towards B. This produces a standing wave which reaches a maximum height at B, as the water meets the boundary, and is then deflected back, as shown, at approximately the same angle. The next highest standing wave will occur on the opposite wall at C and so forth down the channel until eventually the waves are completely damped out by friction.

When applying the momentum equations, friction forces are ignored and the waves are assumed to extend an infinite distance downstream. In order to find the position of B and of successive wave crests, it is necessary to calculate the angle β of the wave with the main channel.

Referring to Figure 1; the momentum in a direction perpendicular to AB, of the fluid entering the shaded area is equal to the net force, giving:

$$\frac{\omega d_2^2 l}{2} - \frac{\omega d_1^2 l}{2} = \frac{\omega d_1 l_1 V_1^2}{g} \sin \beta - \frac{\omega d_2 l_2 V_2^2}{g} \sin (\alpha - \beta)$$



1/ Form of junction.
Plan du confluent.

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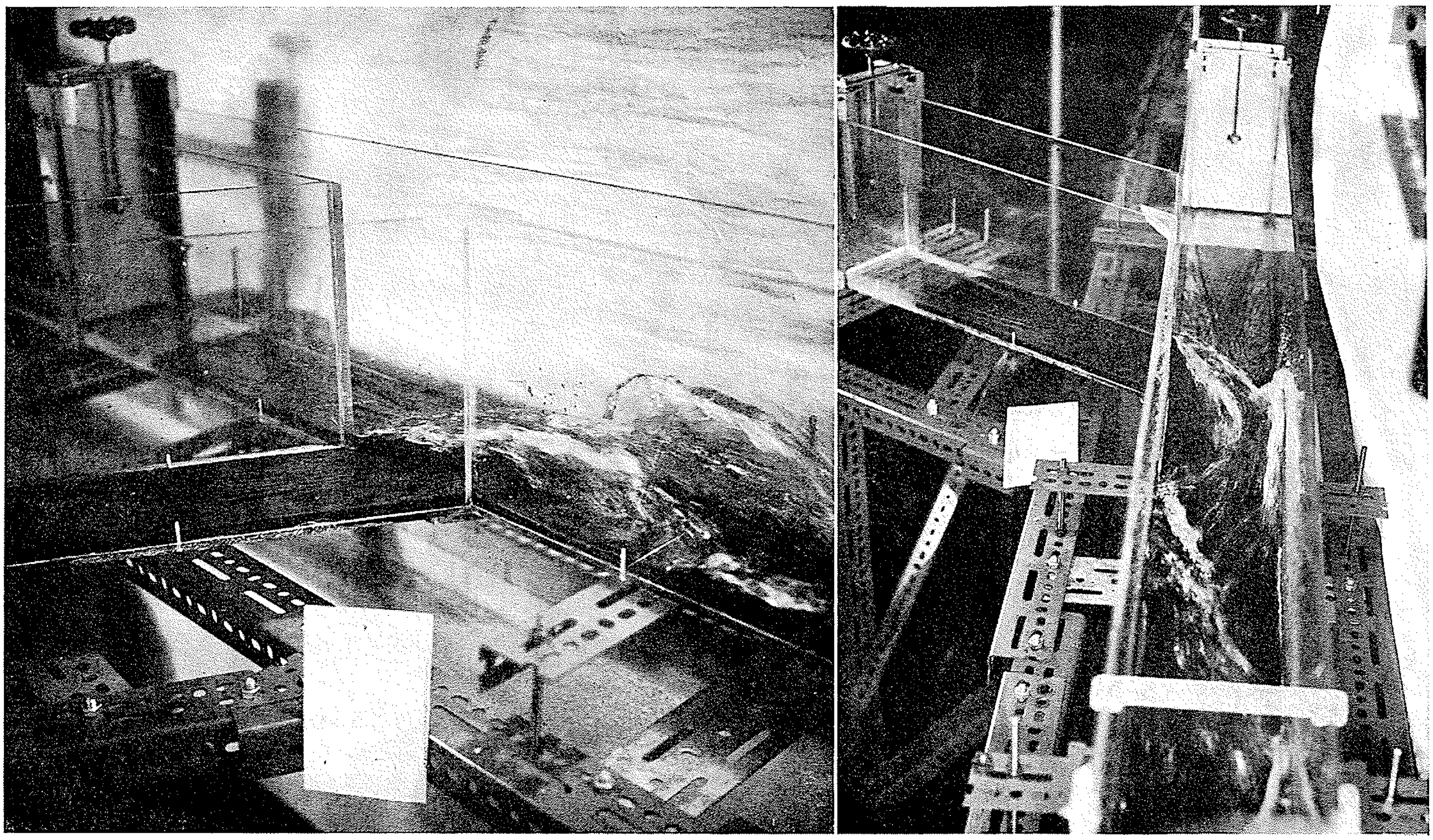
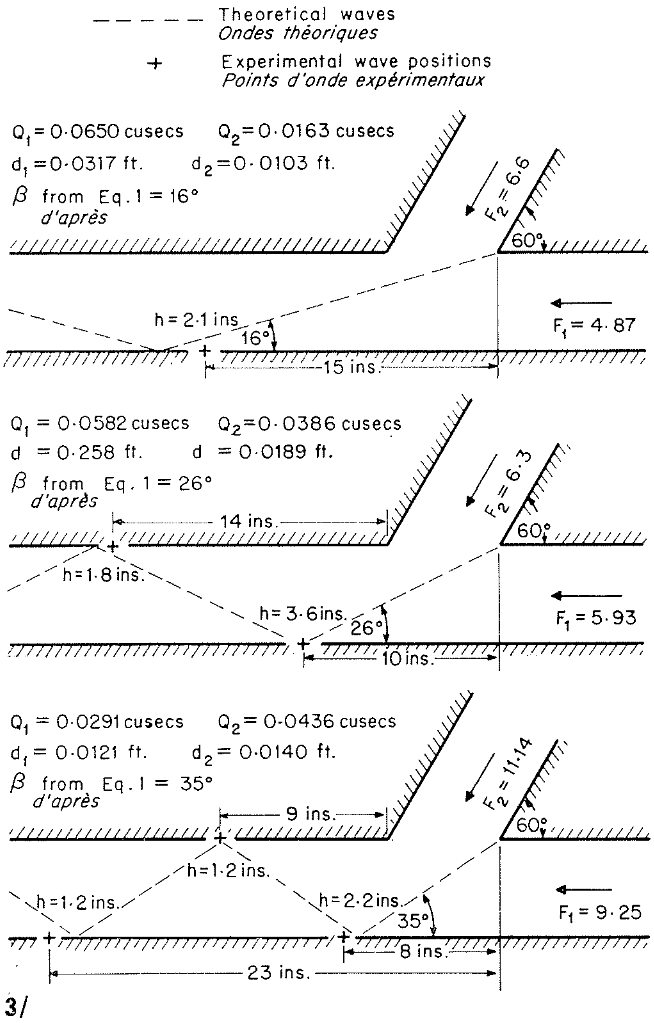


2/ *a*) Side view of junction showing build-up of water on far wall.

Vue latérale du confluent des veines liquides. On voit bien l'accumulation de l'eau contre la paroi arrière.

b) General view of crosswave formation, looking upstream.

Vue d'ensemble de la création de l'onde transversale, en regardant vers l'amont.



3/

3/ Comparison of measured and predicted wave positions.

Confrontation des positions d'onde mesurées et théoriques.

where the lengths l, l_1 and l_2 are as shown in the figure. Writing $l_1 = l \sin \beta$, $l_2 = l \sin (\alpha - \beta)$ and regrouping the terms in dimensionless form gives.

$$1 - D^2 = 2 D^2 \mathcal{F}_1^2 \sin^2 \beta - 2 \mathcal{F}_2^2 \sin^2 (\alpha - \beta) \quad (1)$$

For given values of D, $\mathcal{F}_1$ and $\mathcal{F}_2$ this can be solved by simple iteration to give the required values of β.

In the laboratory tests the positions of the maximum water levels on the walls were recorded, together with the corresponding heights h measured above the channel bed. The positions at which the centre of the waves met the wall did not always coincide exactly with the maximum water levels and so the accuracy of the position measurements is not greater than about one inch. The h values fluctuated within a range of about 0.2 ins. In Figure 3 the experimental results can be compared with the wave positions calculated from Eq. (1), where the velocities have been calculated using $V = Q/(bd)$, and it is seen that there is close agreement.

For predicting wave positions the simple momentum relationships appear to give satisfactory results but for calculating wave heights a much more thorough investigation is required as the shape of the junction boundaries will then be an important factor. Drowning of the cross waves can be achieved by raising the depth downstream until the flow through the junction becomes subcritical. Jumps are then formed in one or both of the upstream channels and momentum equations can be used to determine the relationship between the three depths assuming hydrostatic pressure along the wall DE.

References

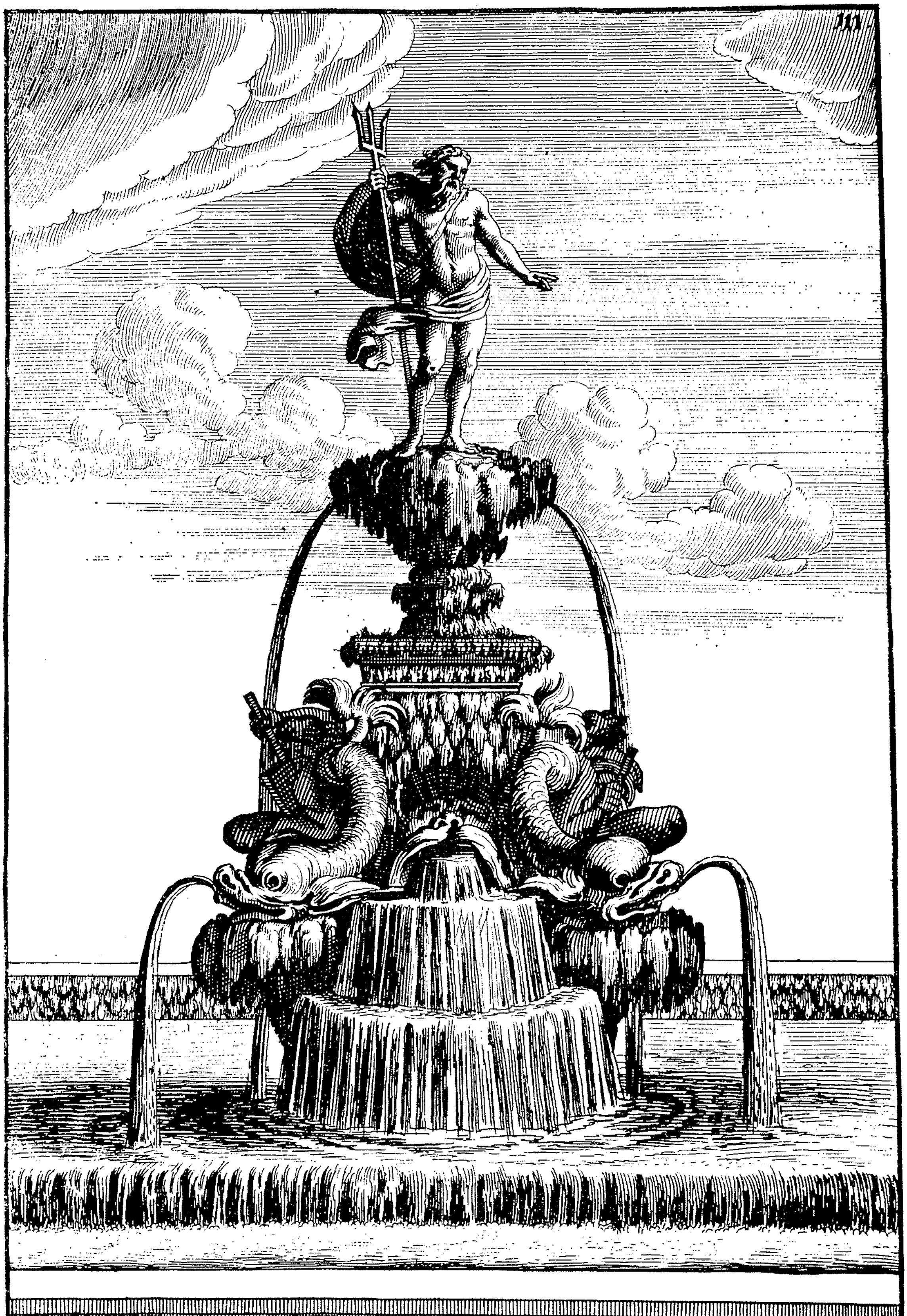
BOWERS (C. E.). — Hydraulic Model Studies for Whiting Field Naval Air Station. St. Anthony Falls Hydraulic Laboratory. Project Report No. 24, Part V, (1950).

Symbols

Q discharge;
V mean velocity;
d depth;
b width of channels;
$\mathcal{F}$ Froude number $= V/\sqrt{gd}$;
α angle between lateral and main channel;
D depth ratio $= d_1/d_2$;
ω specific weight of water;
g acceleration of gravity.

Suffices $_1$ and $_2$ refer to the main channel and lateral respectively.

Gravure du XIXe siècle



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