Introduction

Hydraulic structures like river and canal outlets and culverts sometimes work under submerged conditions. For designing safe but economical energy dissipators for these outlets structures, it is necessary to have a good knowledge of at least the mean flow characteristics below them. If the outlet jet is far removed from the confining boundaries, it could be analysed as a free turbulent jet [1, 5, 13] (1). If it occupies the full width of the downstream channel and issues tangential to its bed, then it could be treated as a plane turbulent wall jet [4, 5, 6, 11]. If the full-width jet has an abrupt drop at the entrance, the flow could be analysed as a plane turbulent reattached wall jet [8]. If it is a part-width jet with no drop, it could be analysed as a restricted form of three dimensional wall jet [7]. The case of part-width rectangular jets with drop has recently been studied by Rajaratnam and Muralidhar [10]. This paper presents the results of an experimental study of the diffusion below certain non-rectangular outlets under submerged flow conditions.

Experiments and results

The experiments were conducted in a rectangular flume 18 inches wide, 36 inches deep and 16 feet long with smooth bed and side walls. A detailed description of the experimental arrangement could be found in [8]. The non-rectangular shapes studied included basically a triangle, semi-circle, circle and some variations of the above which are shown in Figure 1.

With the exception of the circle, all the other outlets were short conduits and had a length of about 30 inches. The circular outlet was a nozzle with an area ratio of about 2.8 and a length of transition of 10 inches. On the whole, nine series of experiments were made and the important details of these experiments are given in Table 1. In Table 1, B is the width of the downstream rectangular channel, b is the characteristic width and h is the height of the outlet, \( U_o \) and \( U_a \) are respectively the maximum and average velocities at the outlet, \( r' \) is the characteristic length of the outlet defined as the ratio of the area to the fluid boundary perimeter of the outlet jet and \( h' \) is the height of abrupt drop at the efflux section.

On the whole 24 experiments were conducted. In all these experiments, the mean velocity distribution in the forward flow in the central plane of the outlet was measured by means of a 3 mm external diameter Prandtl-type Pitot-static tube. The average outlet velocity \( U_o \) was obtained by dividing the measured discharge by the outlet area and the maximum velocity \( U_a \) was obtained by making a velocity traverse as close as possible to the efflux section. In the first two series, the centerline bed shear stress was measured by means of a
3 mm external diameter Preston tube [2, 3]. All these experimental data are presented and discussed in the following sections of this paper.

**Triangular outlets**

For all the triangular shapes tested, the vertex angle was kept at about 90 degrees. If \( b \) is the base width of the triangle, with its base sitting on the bed, the ratio \( b/B \) was varied from 0.224 to 0.826. Figure 2 shows a typical centerplane velocity distribution in which \( u \) is the turbulent mean velocity at a normal distance of \( y \) from the bed and \( x \) is the longitudinal distance from the outlet. From Figure 2, it is seen that the almost-symmetrical parabolic velocity distribution at the efflux section transforms to that of a wall jet at \( x \) greater than about 21.0 inches.

The wall jet profiles of the first series are tested for similarity in Figure 3, in which \( u/u_{m} \) is plotted against \( y_{1}/y \), where \( u_{m} \) is the maximum value of \( u \) at any section and \( y_{1} \) is the value of \( y \) where \( u = (u_{m}/2) \) and \( \partial u/\partial y \) is negative. Sometimes \( u_{m} \) and \( y_{1} \) are referred to respectively as the velocity and length scales. Figure 3 shows that the velocity distribution for all the five runs of the first series agree fairly well with that of the classical wall jet, that is, the plane turbulent wall jet growing on a smooth boundary in an infinite expanse of the same fluid under zero pressure gradient.

The variation of the velocity and length scales are studied in Figure 4. In Figure 4 a, \( u_{m}/U_{o} \) is plotted against \( x/l^{*} \) along with the curves of the classical wall jet, denoted as CWJ and of the rectangular wall jet in wider channels, denoted as WJWC. The present data lie somewhat lower than the above two curves and a dotted line has been drawn to represent the same. The length scale data in Figure 4 b is described fairly well by the WJWC curve for \( x/l^{*} \) up to about 40 beyond which they approach the CWJ curve.

Figure 5 a shows few typical variations of the centerline bed shear stress \( \tau_{0} \). A dimensionless plot of the variation of the shear stress is shown in Figure 5 b, in which \( \tau_{0m} \) is the maximum value of \( \tau_{0} \) and \( \theta \) is the value of \( x \), where \( \tau_{0} = \tau_{0m}/2 \) and a single curve could reasonably be drawn for all the five experiments of the first series. It was found that the shear stress scale \( \tau_{0m} \) could be taken as equal to 0.0044 \( \tau_{0m}/2 \) and the length scale \( \theta \) was found to be equal to about 26.2 \( r^{*} \).

**Semicircular outlets**

Four experiments were done with the semicircular shape and with its diameter as the base width \( b \), the ratio \( b/B \) was varied from 0.22 to 0.67. A typical velocity distribution is shown in Figure 6, which shows the initial parabolic distribution gradually changing to that of a wall jet. The velocity distribution data are tested for similarity in Figure 7. It is found that for the first experi-
Velocity distribution. Similarity plot (triangular outlets).
Répartition des vitesses. Similitude des courbes.

Norm. — The origin has been vertically shifted for each curve. Les origines des courbes ont été déplacées dans le sens vertical.

Velocity and length scales (triangular outlets).
Echelles des vitesses et des longueurs (cas des orifices triangulaires).
WJWC : jet de paroi dans les canaux de plus grande largeur.
CWJ : jet de paroi classique.

Bed shear stress studies (triangular outlets).
Etudes de la contrainte de cisaillement au fond (orifices triangulaires).

Typical velocity distribution (semicircular outlets).
Répartition type des vitesses (cas des orifices semi-circulaires).

Velocity distribution. Similarity plot (semicircular outlets).
Répartition des vitesses. Similitude des courbes (orifices semi-circulaires).
Two experiments were done, one each with an inverted semicircle (run 4a) and an inverted triangle (run 5a), sitting on the bed of the channel. In both the cases, for sufficiently large values of $x$, the velocity distribution is described well by the curve of the CWJ, as could be seen from the lowermost curves of Figures 12 and 13. The variation of the velocity scale is studied in Figure 14a where it is seen that for both these inverted shapes, the data agree fairly well with the curve of the WJWC. But the length scale variation studied in Figure 14b, shows that for each of these two shapes, the data lie on different curves, distinct from those of the CWJ and WJWC.

The variation of the velocity scale is studied in Figure 8a, and the mean curve for the semicircular outlets is somewhat lower than the curves of the CWJ and WJWC. The length scale data for the first three runs, shown in Figure 8b agree fairly well with the WJWC line. A study of the centerline bed shear stress data for the semicircular outlets was made and these data agreed well with the results of the triangular outlets.

Circular outlets

The circular outlet was provided by a nozzle of 3.05 inches diameter and only two experiments were done. The typical velocity distribution in Figure 9 shows that here again the forward flow in the centerplane behaves like a plane wall jet. The dimensionless plot of Figure 10 confirms the above observation. Figure 11a shows that the velocity scale data agree well with the curve of the WJWC but the length scale data in Figure 11b lie somewhat higher than the curves of the CWJ and WJWC.
Inverted semicircular and triangular outlets with abrupt drop

For the inverted semicircular and triangular shapes, it was desired to study the effect of an abrupt drop at the efflux section on the forward flow velocity distribution. For the semicircular outlet of basic width of 6.0 inches, the height of the drop was made equal to 1.03, 2.42 and 3.42 inches (runs 6a, 6b and 6c). The velocity distribution measurements, plotted in the conventional dimensionless manner in Figure 12, shows that for all these runs, the distribution in the reattached forward flow agrees (of course, with some scatter) with the curve of the classical wall jet. But in the case of the inverted triangular outlet with the drop height equal to 0.97, 2.50 and 3.58 inches (runs 7a, 7b and 7c), the velocity distribution in the reattached forward flow differed greatly from that of the CWJ as could be seen from Figure 13.

In Figure 15a, the velocity scale data has been plotted for these two shapes and a mean curve is drawn, which is much lower than the curves of the CWJ and WJWC for x/r' greater than about 40. In Figure 15b, the length scale data for only the semicircular outlet are shown and their variation appears to be very complicated. The corresponding data for the inverted triangle are not studied since the velocity distribution was considerably different from that of the classical wall jet.

11/ Velocity and length scales (circular outlets).
Echelles des vitesses et des longueurs (orifices circulaires).

12/ Velocity distribution. Similarity plot (inverted semicircle).
Répartition des vitesses (cas du demi-cercle inversé).

13/ Velocity distribution. Similarity plot (inverted triangle).
Répartition des vitesses. Similitude des courbes (triangle inversé).

14/ Velocity and length scales (inverted semicircle and triangle).
Echelles des vitesses et des longueurs (cas du demi-cercle et du triangle inversé).
Circular outlets with abrupt drop

The circular nozzle was tested for four values of the drop height equal to 1.0, 2.60, 3.60 and 5.08 inches (runs 8a, 8b, 8c and 8d) and the last experiment was conducted with a circular pipe outlet of the same diameter with a drop height of 2.10 inches. The velocity distribution patterns of these experiments (series 8), plotted in Figure 16 (a to d), show qualitatively that as the height of drop increases, the velocity distribution in the centerplane of the forward flow changes from that of a plane wall jet to the circular free jet. The maximum velocity \( u_m \) at different sections for these
General discussion

In the preceding sections of this paper experimental results have been presented concerning the diffusion below certain non-rectangular outlets. For the present, confining the discussion to the non-rectangular outlets without a drop at the efflux section, it could be said that in general the velocity distribution in the centerplane (for \( r > 12 r' \)) agrees with that of the classical wall jet. This indeed is an interesting observation. Secondly, regarding the velocity scale, the variation for each particular shape seems to be different and the data for all the runs are shown together in Figure 18. It is seen from Figure 18 that the data is roughly contained by the WJWC curve on top and a possible lower limit curve is also shown. Similarly, the length scale data for all the non-rectangular outlets is shown in Figure 19 and for certain preliminary design purposes, the WJWC curve itself could be assumed to represent the data over the range studied.

With these observations, it should be possible to predict the mean velocity distribution in the centerplane below submerged non-rectangular outlets. The centerplane is believed to be plane of maximum velocity. Further studies regarding the flow characteristics on either side of the centerplane have to be made. The bed shear stress distribution on the bed should also be studied for all these shapes. If these outlets are located with an eccentricity with respect to the centerline of the downstream channel, using the results of [10], it could be predicted that the effects of eccentricity on the velocity distribution, velocity and length scale would be small. In the case of non-rectangular outlets with an abrupt drop, only some preliminary results have been obtained. The exploratory experiments with the circular nozzle for four heights of drop indicate that it would be very interesting to study the transformation of the plane wall jet profile to that of the circular free jet, as the height of drop increases continuously from zero.

Another result of practical value is that for the non-rectangular outlets studied, the ratio of the average to maximum velocity at the efflux section is about 0.84.

Conclusions

Based on the experimental results presented in this paper, the following conclusions could be drawn.

For the submerged non-rectangular outlet shapes studied, when there is no drop at the efflux section, the forward flow velocity distribution in the centerplane of the outlet could satisfactorily be described by the corresponding curve of the classical wall jet, beyond a certain minimum distance from the outlet equal to about 12 times the characteristic length \( r' \). The length scale could be predicted fairly well using the curve of the rectangular wall jet (i.e. WJWC). But the velocity scale data varies in a distinct manner for each shape, which could be obtained from the corresponding figure. The centerplane bed shear stress has been studied only for the triangular and semicircular outlets. When there is a drop at the efflux section, the flow becomes more complex and only some exploratory results have been obtained.

Acknowledgements

The work reported in this paper was conducted in the Hydraulics Laboratory of the Civil Engineering Department of the University of Alberta, Edmonton and the authors are thankful to the National Research Council of Canada for the financial assistance provided.

| TABLE 1 |
| Non-rectangular outlets (experimental details) |

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</tr>
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<tr>
<td>( h_{10} )</td>
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<tr>
<td>( h_{10} )</td>
</tr>
<tr>
<td>( b/B )</td>
</tr>
<tr>
<td>( r'/r' )</td>
</tr>
<tr>
<td>( U_1 )</td>
</tr>
<tr>
<td>( U_1 )</td>
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<td>( h_{10} )</td>
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<tr>
<td>w</td>
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NOTE: Temperature of water about 70°F.
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<th>$u_a$ ft/s</th>
<th>$\delta_1$ ft</th>
<th>$\frac{u_a}{U_a}$</th>
<th>$\delta_1/r'$</th>
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<td>0.681</td>
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<td>7.70</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$x_{in}$</th>
<th>$u_a$ ft/s</th>
<th>$\delta_1$ ft</th>
<th>$\frac{u_a}{U_a}$</th>
<th>$\delta_1/r'$</th>
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**TABLE 2**

Detailed data
Non-rectangular outlets

The following symbols are used in this paper:

- $b$: width of outlet;
- $B$: width of channel;
- $h$: height of abrupt drop;
- $h_0$: height of outlet;
- $m$: suffix to denote abrupt drop value;
- $r'$: characteristic length of outlet;
- $u$: turbulent mean velocity;
- $u_w$: velocity scale;
- $U_e$: maximum velocity at efflux section;
- $U_0$: average velocity at efflux section;
- $x$: longitudinal distance from outlet;
- $y$: normal distance from the channel bed;
- $\delta_x$: length scale for velocity plot;
- $\eta$: dimensionless ordinate;
- $\theta$: length scale for shear stress plot;
- $\rho$: mass density of the fluid;
- $\tau_{0w}$: bed shear stress;
- $\tau_{max}$: maximum value of $\tau_{0w}$.

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