Typical or standard beach profiles if established for a given wave climate and sediment characteristic, could prove useful as a yardstick of stability. The balance of volumes in a storm and swell profile would permit the assessment of imminent beach degradation. The many formulae derived for littoral current and littoral drift need attention, as also the many practical variables involved in their measurement. Greater cognisance should be taken of sediment transport offshore from the surf zone.

**Beach profiles**

In order to assess the probable movement of sediment it is necessary to compute the amplitude and velocity of water particle motion near the bed. To do this the depth of water must be known. Hydrographic charts supply a reasonably accurate profile of the deeper zones offshore, which change imperceptibly over a number of decades. For depths less than 10 fathoms, however, current survey data are required.

Before detailing attempts to define beach profiles in terms of wave and sediment characteristics it is pertinent to question the need for such information. The answers should provide a lead for future research into this matter.

Shoreline stability.

If a theoretical or empirical relationship could be developed for a given wave climate and sediment characteristic, in terms of a stable underwater profile, a measuring stick would be available with which to gauge any shoreline. Should a beach section be significantly steeper than the "standard," reasons for erosion should be sought and necessary remedial measures taken. Should the profile be flatter than normal, transient accretion may be indicated, such as humps traversing the coast [1].

For oceanic margins the wave climate in this respect is closely associated to the persistent swell, but in enclosed seas storm waves are the sole source of wave energy. In the latter case the profile beyond the permanent offshore bar must be related to a widely varying spectral width. Some relationship may be derivable which employs average energy per unit area of ocean, together with mean sediment characteristics [2].

Swell to storm profile.

One of the major concerns of the coastal engineer is the probable loss of beach in a future storm sequence and the depth to which the eroded surface may fall. If this knowledge were available he could advise on the safe limit for beach encroachment for commercial purposes and the depth to which pipelines and cables must be laid through the beach zone. Most beaches are subject to storm waves at some time, generally on a winter summer sequence. These may be accompanied by higher than normal water levels, which must be allowed for in any forecasting procedure. Reference [3] details the long term effects in the erosion process,
whereas the present discussion refers to any specified swell-built beach profile.

A typical transposition from swell to storm-wave profile is depicted in Figure 1 A, in which the secondary undulations can be smoothed to straight lines as indicated, whilst preserving equal areas of the various sections. If the swell profile can be defined, up to the average beach level $R^*$, plus the storm wave beach depth $d_s$ and the overall width $Z$, then manipulation of the profile can be carried out to give equal volumes of beach erosion and submarine accretion (See Figure 1 B). As seen, a probable elevation of the normal water level $d'$ during the storm cycle can be seen to displace the beach line back an amount $Z'$.  

Theoretical analyses.

Stiarz [4, 5] has developed equations, based upon energy principles, for the profile beyond the breaker line to the limit of bed disturbance. He has also determined the width of the swell and storm-wave surf zones, and the cross-sectional area of the water in the surf zone.

The offshore profile is given by:

$$ x = ay^2 $$

and:

$$ a = \frac{0.95}{(S - 1)^{1/2}D^{3/2}} $$

(1)

where:

$x$ is distance seawards from the breaker line;

$y$ is depth below SWL;

$S$ is ratio: density of sediment to seawater ($\approx 2.6$);

$D$ is median sediment diameter at some appropriate depth (mm);

$H$ is wave height just offshore from breaker zone (metres).

From equation (1) it can be seen that the milder slopes occur with larger values of $(a)$, which in turn result from finer sediment or smaller waves. The usual value of $S = 2.65$ is applicable to most sand situations.

It appears strange that wave period does not enter this relationship, since on this depends the amplitude of water particle oscillation, as much as wave height. Similarly the mass transport "velocity" within the boundary layer also is dependent strongly on the period. In the overall picture it is this factor that determines the refraction pattern and hence the proportion of the deep-water wave height occurring near the breaker zone.

The width of the surf zone $Z$ is given by:

$$ Z = \frac{BH^{1/2}}{(S - 1)^{1/2}D^{1/2}} $$

(2)

where $H$ and $D$ are of dimensions similar to those in equation (1). Factor $B = 43.5$ for a swell profile and $75.0$ for a storm profile. In the latter case the height $H$ could well be represented by the significant height $(H_{1/3})$, [6, 7], which is defined as the average of the third highest waves in the spectrum [8]. The diameter of sediment $D$ (mm) would be that at the beach face and hence would differ from $D$ in equation (1).

The cross-sectional area of water in the surf zone is given by:

$$ A = \frac{3.44H^{1/2}T}{(S - 1)^{1/2}D^{1/2}} $$

(3)

where:

$H$ is significant wave height (metres);

$T$ is significant period [8] (seconds);

$D$ is median sediment diameter (mm);

$g$ is $9.81$ metres/sec$^2$;

$S$ is sediment to seawater density ratio.

Division of equation (3) by (2) gives the mean depth of the surf zone ($d_n$ in Figure 1). This could
be compared to the depth of $H_{1/3}$ which approximates the mean depth as suggested by Bruun [6].

The slope of the built storm beach face is best taken from prototype measurements [9, 10] which are summarised in Figure 2 for three conditions of exposure. The slope in a storm beach face is of little consequence compared to the other relevant dimensions (See Figure 1). The height of the beach above SWL should be measured where possible, since the calculation from wave and sediment characteristics involves too many variables to serve any practical purpose.

Other workers [11, 12, 13] have attempted to relate the offshore bed profile to wave climate, but much more wave data are needed on coasts before specific equations can be quoted. The order of differences that exists between such relationships can be gauged by that for the formulae by Sitarz [4, 5] equation (1) and by Larras [12] as follows:

$$\frac{x}{L_0} = K \left( \frac{x}{L_0} \right)^m$$

where:

$$K = \frac{H_0}{L_0} + 0.039 \frac{D^{1/2}}{(S - 1)^{1/3}}$$

and:

$$m = 11.5 \frac{H_0}{L_0} + 0.275 \frac{1}{(S - 1)^{1/3}} - 0.05$$

where:

- $m$ is the beach slope;
- $H_0$ is the deep-water wave height [9];
- $L_0$ is the deep-water wave length, and other variables as before.

The constants refer to $H_0$ and $L_0$ in metres and $D$ in mms.

Substituting $D = 1$ mm, $T = 12$ sec, and $H = 1$ metre in equations (1) and (4) results in $x = 0.75 y^2$ (Sitarz) and $x = 0.025 y^{1.8}$ (Larras). The former is much more uniform from the $x = 0$ at the breaker line, whereas the latter probably refers more to the zone some distance from this origin. The anomalies indicate the need for further data on these profiles, so that standards could be derived to test for beach stability.

One difficulty in using such measurements, which include a wave height and wave length (or as noted above a wave steepness) is that these vary from hour to hour. With each change the sand ripples associated therewith are altered and so, in given time, is the profile. During these transition periods the sediment in transit is at a maximum. Thus an equilibrium profile would take some days to establish itself even with a steady input of wave energy with uniform characteristics.

Adding to the above dynamics of the beach profile, is the seasonal winter and summer sequences of storm and swell. The storm waves transfer portions of the beach offshore to form a bar. Subsequent swell returns this to the beach, but this task may take several weeks or even months. During this time the profile recorded would not be in equilibrium, since material is being transmitted through it to the beach.

Thus, to obtain a “standardised” profile for any given wave climate and sediment characteristics, measurements should be taken at the end of a lengthy period of swell. Also the wave climate should be expressed in average conditions over the part of the year when this swell incidence occurs. Any criteria should be derived for both oceanic and enclosed sea conditions, since the latter will experience mainly storm type waves rather than swell.

### Littoral drift

Waves arriving obliquely to a shoreline are refracted as they traverse the Continental Shelf, thus tending to move more nearly normal to the bed contours. However, on breaking, the crests are still angled slightly to the beachline, so that a component of their energy is directed along the shore. This produces a littoral current which, together with the suspension of sediment due to the turbulence present, effects a transport of material known as “littoral drift.”

Since the water momentum generating this current can be assessed from knowledge of the wave energy, and its longshore component, it is possible to relate deep-water wave characteristics to both the current and the volume of sediment carried. There are factors such as energy dissipation, bottom percolation and wave reflection which are difficult to account for, but information is being accumulated which indicates general values. But as indicated by Sonu et al. [14] the several formulae they examined [15, 16, 17, 18, 19, 20, 21, 22] did not agree with the measurements made over some 6 months. They concluded: “Under natural conditions, the nearshore topography participates in the longshore current mechanism as a dynamic variable, not only redistributing the breaker influx into different positions along the shore but also itself undergoing displacements and transformation due to the waves and the currents thus affected.” Any measurements taken during a transition period from the storm-wave profile to that of the swell profile is certain to be a failure, even though the deep-water wave characteristics may be steady and well defined.

Further formulae have become available, the theoretical aspects of which disagree amongst themselves and previous papers [23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. These differences arise from using either momentum or energy fluxes of the breaking wave and employing theory for the sinusoidal, cnoidal or solitary waves in the transition and shallow water zone. Most workers have based their computations on a single sinusoidal wave which could well represent the persistent swell situation on an oceanic margin, if its characteristics are suitably chosen. Bruun [6, 7] has specifically dealt with the storm beach profile, but the transient condition in respect to wave height, period and direction makes application in this case extremely subjective.

The step from littoral current to littoral drift involves empirical factors, provided either from models or prototype data. In order to indicate the variables involved the solution obtained by Casalho [30] will be outlined, since this is the result
of a comprehensive study, but has not been report­
ced widely. Using the solitary-wave theory he ob­
tained:

\[ G = \frac{E_p}{(1 - 1/S) \tan \varphi} \]  

(5)

where:

- \( G \) = weight of sediment moved per second across a plane normal to the beach;
- \( E_p \) = energy dissipated

\( \text{longshore energy component} \)

\[ = \sin \alpha_f \left[ \frac{mH_b}{K' \tan \alpha_o L_0} \right] \]

where:

- \( m \) = beach slope;
- \( H_b/L_o \) = \( H_b/C_bT \)
  
  \[ = H_b/\sqrt{1.78gdT} \]
  
  \[ = H_b/\sqrt{1.78 g (1.3) H_b T} \]

- \( K' \) = roughness factor;
- \( P \) = wave power per unit length of shore;

\[ = 2.2 \, wH_b^3 \sin \alpha_o \cos \alpha_o /T \]

\[ = wH_b^3L_0 \sin \alpha_o \cos \alpha_o /16 \]

where \( w \) is the specific weight of sea­
water;

- \( \varphi \) = angle of internal friction of the sediment.

By substituting certain values suggested by Cas­
tanko for sandy shores, namely:

\[ mH_b/K' L_0 = 0.12; \quad \varphi = 35^\circ; \quad (1 - 1/S) = 0.623 \]

(6)

The value of \( \alpha_o \) depends upon \( \alpha_o \) and the wave
steepness \( H_b/L_o \) \[33, 34\], so that both \( E_p = f(0.12/
\tan \alpha_o) \) and \( \sin \alpha_o \cos \alpha_o \) have been graphed
against \( H_b/L_o \) in Figures 3 & 4. Since equation (6) is
dimensionally homogeneous any appropriate units
may be employed in solving for \( G \).

Inman and Frautschy \[35\] have derived an empi­ical relationship between immersed weight per
second passing any point and wave power per unit
length of coast. In terms of equation (5) this is:

\[ G = P/4 (1 - 1/S) \]  

(7)

and is graphed in Figure 5, together with data
supplied from references \[37, 38, 39, 40\] for labo­
atory tests and references \[41, 42, 43\] from field
measurements. The hatched zone indicates the
location of the bulk of the laboratory results.

Accepting the value of \( \tan \varphi = 0.7 \) previously
employed, equation (7) implies that \( E_p = 17.5\% \).
As seen from Figure 3 this should apply to waves
steeper than 1/20, which is the limit of the figure.
Inman and Frautschy \[35\] commented on the likely
differences in laboratory and field investigations as
follows: "The equations for wave power are usually
written for the root-mean-square wave height, which
is the wave parameter most commonly mea­sured in the laboratory. However, the power com­
putation for the field data appears to be based on
significant wave height and thus may be too high
by a factor of 2. On the other hand, the practice
of representing the entire power spectrum in terms
of a single significant wave may result in the omission
of important energy contributions from waves
of different frequency, causing the computations
to be too low. A rigorous evaluation of this rela­
tion is essential to beach planning."

In this respect the substantial difference between
storm waves and the more prolonged swell condi­tion
should be kept in mind. With the former the
concept of significant wave or energy of the spec­
trum is important, whereas with swell waves an

\[ \begin{align*}
\frac{dT}{dH} &= \frac{E_p}{\sin \alpha_o \cos \alpha_o} \\
\sin \alpha_o \cos \alpha_o &= \frac{E_p}{\sin \alpha_o \cos \alpha_o}
\end{align*} \]  

(8)

(9)
average height and period over a lengthy period could prove adequate. The wave climate of a coastal area is important to consider, in terms of the total wave energy (or more correctly the longshore component of this total) arriving in the form of storm or swell waves. Not only will these have different losses (E_r) but they will also act on different profiles, as discussed previously and illustrated in Figure 1.

Castanho [30] gave examples of two beaches in which E_r was 7% and 16% respectively. In Figure 5, lines are drawn representing a range of E_r, from which it can be seen that the above values fall within the scatter of data from which equation (7) was derived. It would seem that relationships presented by Castanho [30] could lead to a general solution of this problem. The non-wave variables which determine E_r, namely beach slope (m) and friction factor K', might be related in some manner to sediment size.

**Overall aspects**

Having discussed details in respect to beach profiles and sediment transport in the surf zone, it is useful to consider the coastal length as a whole. This will not necessarily be a lengthy stretch of straight beach to which the above calculations can apply ad-infinitum. In fact, where littoral drift is strong due to obliquity of the deep water waves to the continental margin, the sedimentary sections will be strung between the headlands in the form of crenulate shaped bays [44].

These bays have been shown to contain a tangent and a curved section, the latter approximating to a logarithmic curve [45], as depicted in Figure 6. The former will tend to become parallel to the crests of the incoming waves as the bay approaches an equilibrium shape. Around the curved zone the waves are being refracted and diffracted about the upcoast headland. They will break at some angle to the beach, depending upon the offshore contours.

Consider a reasonable length of coast where a number of such bays occur successively and where the waves at the edge of the Continental Shelf are sensibly the same in energy content and direction of travel. If there is a negligible discharge of sediment from rivers in this length, and no substantial accretion or erosion is taking place, it could be presumed that the rate of longshore transport is sensibly the same along it. Many such lengths of Continental margin do remain static over decades as gauged from hydrographic charts produced in the last century.

However, the conditions for transferring this constant rate of material along the coast differ greatly within the bays previously mentioned. Both the wave height and the angle of breaking will vary around the whole periphery. The slope of the beach could vary due to longitudinal sorting of sediment. In the clefted headland zone no beach will exist at all, but the sediment is still being transported. This all highlights the fact that the surf zone is not the only highway for longshore drift, but that it and the offshore zone combine to transmit the load available.

For the bay as a whole, it has been shown by tracer tests [46] that from the upcoast headland most of the sediment is transmitted across the bay to the tangential section, without entering the curved section of the coast. Thus, any littoral drift measured or computed in this sheltered zone should not be applied to the straight stretch of beach even a little distance downcoast. The short-cut transfer of material across the bay can be explained in terms of wave reflection expediting its passage to a point well beyond the headland (See Figure 6) and the mass-transport of the swell sweeping it across even deep zones of the bay. This has important implications in the design of marine structures within embayments.
Conclusions

1. The profile of the seabed beyond the breaker line might be related to sediment and wave characteristics, in order that a measuring stick can be available to test the stability of any given shoreline.

2. The computation of a beach profile for storm waves and swell should permit the prediction of transient erosion during any storm intensity and accompanying surge.

3. Many formulae have been derived relating longshore current and littoral drift in the surf zone to wave power per unit length of coast. With some rationalisation a simple relationship may evolve connecting wave and sediment characteristics.

4. Application of any equations so derived as in (3) above should be made with due recognition of the natural variations in energy source, bed material and coast orientation.

5. More attention needs to be concentrated on the macroscopic view of the coast than on the surf zone alone, since material can be transported in the offshore zone, especially in cliffed zones where reflection aids this mechanism.

List of symbols

- $a$: parameter in beach profile equation;
- $A$: cross-sectional area of water in surf zone;
- $B$: constant in equation (2);
- $C_0$: celerity of breaking wave;
- $D$: diameter of sediment;
- $d_b$: depth of wave breaking;
- $d_w$: storm wave beach depth;
- $d_r$: increased beach depth during storm sequence;
- $E_r$: ratio of energies as defined in equation (5);
- $g$: acceleration due to gravity;
- $G$: weight of sediment moved per second (littoral drift);
- $H_b$: height of breaking wave;
- $H_d$: deep-water wave height;
- $K$: parameter in equation (4);
- $K_r$: roughness factor in equation (5);
- $D_L$: deep-water wave length;
- $L_d$: length of breaking wave
  \[ (= C_0 T = \sqrt{g d_b} T) \]
- $m$: beach slope;
- $P$: wave power per unit length of shoreline;
- $R$: average beach height above SWL;
- $S$: sediment to seawater density ratio;
- $T$: wave period;
- $w$: specific weight of seawater;
- $x$: horizontal distance from breaker line in beach profile equation (4);
- $y$: depth to bed from SWL in beach profile equation (4);
- $Z$: width of storm-wave surf zone;
- $Z'$: shoreward displacement of water line in storm wave beach profile at abnormal SWL;
- $\alpha_b$: angle of waves to shore on breaking;
- $\alpha_d$: angle of waves to shore in deep-water;
- $\varphi$: angle of internal friction of sediment.

References


