VARIATION OF THE DRAG COEFFICIENT OF A SPHERE ROLLING ALONG A BOUNDARY

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Introduction

The resistance to relative motion between a body and a real fluid is usually expressed in terms of the drag coefficient which combines the characteristics of the body and the fluid with the resistance.

Considerable information is available concerning the variation of drag coefficient with Reynolds number for a body moving in an infinite fluid. However, there is very little information about the variation of drag coefficient when the body moves either in contact with a boundary or in the vicinity of a boundary. A sand particle resting on or rolling along the bed of an alluvial stream, movement of vehicles used for under-sea exploration, ball bearing set-up and such other problems come under this category. In such cases, the problem gets complicated due to the presence of rolling resistance about which very little is known at present.

Investigation on drag coefficient of a sphere rolling on a smooth plane boundary was conducted by Carty [1] 1957. However, similar studies with sphere rolling on rough boundary have not yet been reported. The present investigation was conducted to study the fluid dynamic forces on sphere rolling along an inclined smooth and rough boundary and the variation of drag coefficient with (*a*) the roughness parameter, D/K where D is the diameter of the sphere and K is the diameter of the roughness elements and (*b*) the flow and fluid characteristics.

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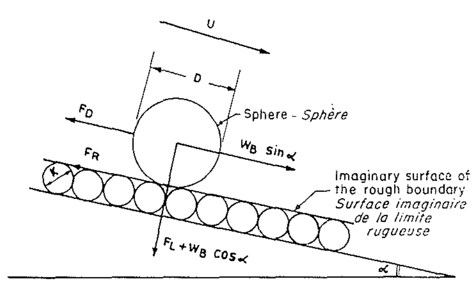
Theoretical considerations

For a smooth boundary, the rolling resistance can be assumed to be negligible as was done by Carty. Hence, the following equation of motion can be written for a sphere rolling down a smooth boundary at a constant velocity :

$$C_D A \rho_f \frac{U^2}{2} = \left[\frac{\pi D^3}{6}(\gamma_s - \gamma_f)\right] \sin\alpha \qquad (1)$$

Figure 1 shows the various forces acting on the sphere rolling on a rough boundary. Rough bound-



1/ Sketch defining the forces on a sphere rolling along a rough boundary.

Croquis de définition des forces sollicitant une sphère roulant sur une limite rugueuse.

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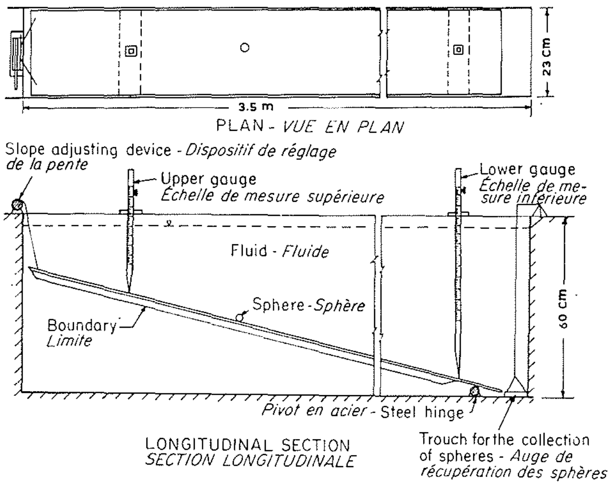


2/ Experimental setup.
Installation expérimentale.

ary in this study consisted of spheres staggered and packed closely on a plate. Balancing the forces that act on the sphere after the attainment of constant velocity, on can write

$$C_D A \rho_f \frac{U^2}{2} + F_R = \left[\frac{\pi D^3}{6}(\gamma_s - \gamma_f)\right] \sin \alpha \qquad (2)$$

The lift is caused due to the circulatory motion being superimposed on rectilinear motion past the body, the phenomenon being called Magnus effect, Hoerner [2] refers to the investigations carried out by Maccoll on a sphere rotating in an infinite fluid and his results indicate C_L to be equal to 0.58 C_D for the present case (i.e. where the ratio of rotational to rectilinear velocity is unity). The effect of lift in this particular case will be to increase the effective friction and thus to reduce the drag coefficient for a given velocity. Effect of lift force is neglected in this study.

On the basis of the preceding arguments, the equation of motion for smooth and rough boundaries reduce to

Smooth boundary:

$$C_D = \frac{4}{3} \frac{gD}{U^2} \frac{\rho_s - \rho_f}{\rho_f} \sin \alpha \qquad (3)$$

Rough boundary:

$$C_D \rho_f \frac{U^2 \pi D^2}{8} + F_R = \frac{\pi D^3}{6} (\gamma_s - \gamma_f) \sin \alpha \qquad (4)$$

If the rolling resistance is included in the fluid resistance, the consequent drag coefficient, C'_D will be given by the equation:

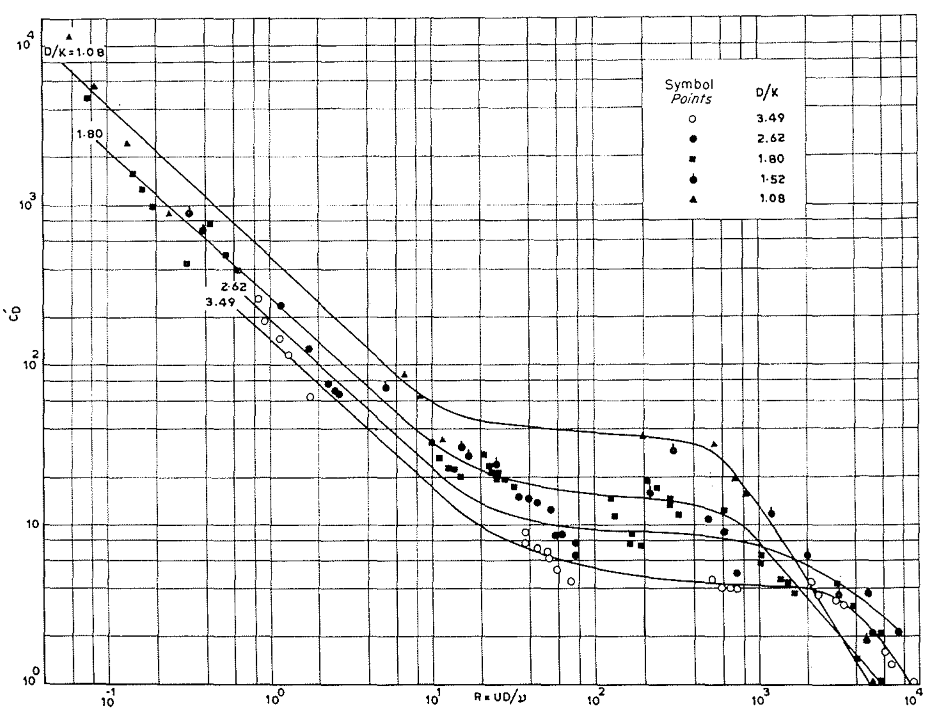
$$C'_D = \frac{4}{3} \frac{gD}{U^2} \frac{\rho_s - \rho_f}{\rho_f} \sin \alpha \qquad (5)$$

which is valid only for rough boundary.

It can be shown by dimensional analysis that C_D is a function of Reynolds number, R and relative roughness, K/D.

Experimental equipment and procedure

The spheres used in this investigation were of glass, steel and plastic and were of varying diameters. For each sphere, diameter, weight in air and specific gravity were determined. Properties of fluids that required determination were their spe-



3/ Variation de C'_D with R and D/K (rough boundary).
Variation de C'_D en fonction de R et D/K (limite rugueuse).

cific gravity and kinematic viscosity. Viscosity of each fluid was determined in two ways, one by the use of Redwood viscometer and another by the use of a capillary tube viscometer. The values obtained by the latter acted as an effective check on those got from the former. The sphere diameter varied from 9.22 mm to 25.33 mm and the kinematic viscosity of the fluid was varied between 10^{-6} and 7.15×10^{-4} m²/sec.

Figure 2 indicates the layout and the dimensions of one of the closed glass walled flumes in which the tests were conducted. The inclined plane used for this purpose was hinged at its lower end. A drum with wire rope arrangement was used at the upper end for slope adjustment which permitted an accuracy in the variation of the level of the upper end of 3 mm. Two gages that could read upto 0.01 cm and with a fixed intermediate distance were installed for the purpose of the measurement of the slope of the boundary. Artificial roughness elements that were used for the investigation consisted of 7.25 mm diameter wax beads, staggered in rows and affixed firmly to an aluminium plate. Rolling of the sphere was timed over a fixed horizontal distance. With the help of the slope already measured, velocity of the sphere could be easily computed. Suitable precautions were taken to ensure that constant velocity is reached before the recording of the observations. Average velocity of each sphere was obtained over sufficient number of observations. In addition the slope of the boundary and temperature of the fluid were also recorded.

Attempt was made to directly measure the rolling resistance. The setup consisted of an aluminium plate studded with artificial roughness elements with a brass, frictionless pulley attached to one end. A thin wire was led over the pulley and had at its lower end a very light pan and to its upper end was fastened a copper ring that could be towed along the boundary by placing weights on the pan. Initially, the weight required to move the ring alone at a constant velocity was found. Then the sphere was placed inside the ring and weights added to the pan to move the ring and the sphere at constant velocity. Difference between the two weights, W_1 gave the rolling resistance because of the fact that the drag due to air works out to be negligible. The range of Reynolds number covered in this method was between 40 and 170.

Variation of C'_D with R for rough boundary

Figure 3 shows the relation between C'_D and Reynolds number for spheres rolling on a rough boundary where C'_D is the drag coefficient that takes into account both fluid drag and rolling resistance. As was expected, the value of C'_D comes out to be larger than C_D at a given Reynolds number. A feature of the plot is that the drag coefficient is dependent on Reynolds number and the ratio, D/K. The lines for different D/K*s* become fairly parallel between Reynolds numbers 10^{-1} and 10^3 beyond which all the curves tend to converge into one.

It can be seen that for a given D/K value the C'_D — R curve shows a fairly rapid decrease in C'_D beyond a certain Reynolds number. This Reynolds number increases with increase in D/K. This phenomenon may be due to the transition from the laminar to turbulent boundary layer as observed in the case of roughned spheres and cylinders in an infinite fluid.

Analysis for rolling resistance

Since the rolling resistance happened to be an important factor in the motion of the sphere over a rough boundary, a thorough analysis of the collected data was made to bring forth the rolling resistance in its true perspective. With the rolling resistance determined directly in the manner explained above, attempts to relate rolling resistance, F_R or the rolling resistance coefficient μ_R to the ratio D/K did not prove successful as the curves were not unique. Hence, a method for the indirect determination of rolling resistance was evolved.

This indirect method is built on the following three assumptions :

1. For Reynolds number smaller than two, it was assumed that the drag coefficient C_D is inversely proportional to the Reynolds number. This assumption is not strictly valid for the rough boundary. However, as long as the two consecutive Reynolds numbers differed by a small amount, this did not introduce a serious error.

2. For large values of Reynolds numbers exceding about 500, the drag coefficient, C_D was assumed to be the same for two close Reynolds numbers.

3. For the same D/K, the rolling resistance is constant for a given Reynolds number. With these assumptions, substitution of appropriate quantities of two close Reynolds numbers in Equation 4 resulted in two equations and the simultaneous solution of them yielded the rolling resistance, F_R. Rolling resistance coefficient, μ_R could be computed as:

$$\mu_R = \frac{F_R}{W_B \cos \alpha_{av}} \qquad (6)$$

Variation of rolling resistance coefficient

Figure 4 shows the variation of μ_R with R_{av} with D/K as the third variable, where R_{av} was the average of two consecutive Reynolds numbers. Data collected in air to determine rolling resistance directly have also been plotted in Figure 4. The relation between μ_R, R_{av} and D/K was found to obey an equation of the form:

$$\mu_R = 0.4\,(D/K)^{-0.5} - 0.06 \log_{10} R_{av} \qquad (7)$$

As is apparent from Figure 4, the increase in the rolling resistance with the reduction in Reynolds number seems to be due to the corresponding reduction in the velocity causing a longer contact of the sphere with the roughness elements. On the other hand, at large Reynolds numbers the sphere, due to increased velocity just skips over the roughness elements with the consequent reduction in the rolling resistance. During the experiments it was actually observed that at higher velocities (and hence, higher Reynolds numbers) the sphere lost contact with the bed more often.

Another feature was that with increase in D/K ratio the rolling resistance reduces. With increase in D/K ratio, the ratio of the contact area of the sphere with the roughness elements to the area of cross-section of the sphere reduces. This may be the reason for the above trend of decrease in rolling resistance coefficient with increase in D/K.

It was observed after the completion of the analysis and plotting of results that, in the Stokian range the plot C_D versus R for the rough boundary obeyed an equation of the form:

$$C_D = \frac{\text{constant}}{R^{1.3}}$$

instead of the equation of the form:

$$C_D = \frac{\text{constant}}{R}$$

assumed. But modification in the equation of motion pertaining to one of the two consecutive Reynolds numbers to take into account this change in the exponent of Reynolds number did not affect appreciably the values of μ_R already computed.

Variation of C'_D with R for rough boundary

Figure 5 shows the variation of drag coefficient with Reynolds number for a rough boundary, the values of C_D having been computed from Equation 4 after substituting the appropriate rolling resistance, F_R as obtained from Figure 3 or Equation 7. One feature of this plot is that the drag coefficient for different D/K ratios tend to fall on a single curve. Hence, a mean curve has been drawn to indicate the variation of C_D with Reynolds number for a sphere resting on a rough boundary.

At low values of Reynolds numbers the curve obeys a straight line relation on a log-log chart. This corresponds to that of Stokes range for a sphere falling in an infinite fluid where the linearity extends upto a Reynolds number of 0.1. But here, the Stokian range extends upto 1 and follows an equation of the form:

$$C_D = \frac{82.5}{R^{1.3}} \tag{8}$$

The extension of the zone of predominance of viscosity is due to the presence of the boundary. A similar feature has been referred to by Carty [1] while reviewing the work done by McNow on cylindrical boundaries. The critical Reynolds number signifying the formation of the turbulent boundary layer occurs at a much earlier stage compared to the sphere falling in an infinite fluid.

Apparently there is a relatively less scatter of points at Reynolds number less than one, thereby indicating that, in this range the effect of D/K on C_D has been adequately taken into account. On the other hand, for higher Reynolds numbers there is a greater scatter. Even though there is no systematic variation of C_D with D/K in this range, one might expect that in the range of Reynolds number where C_D becomes nearly constant, C_D should decrease with decreasing values of D/K. The reason for this trend is that, smaller the value of D/K for a given D, smaller will be the area of the sphere exposed to the flow due to the projection of roughness elements.

Another observation that can be made is that the drag coefficient remains constant between Reynolds numbers 40 and 10^3. On the other hand in the case of a sphere falling in an infinite fluid the drag coefficient is almost constant (in fact it slightly increases) within the range of Reynolds numbers, 10^3 to 2×10^5 beyond which it suddenly drops due to the change in the boundary layer from laminar to turbulent. The reason for this difference is not known at present.

Variation of C'_D with R for smooth boundary

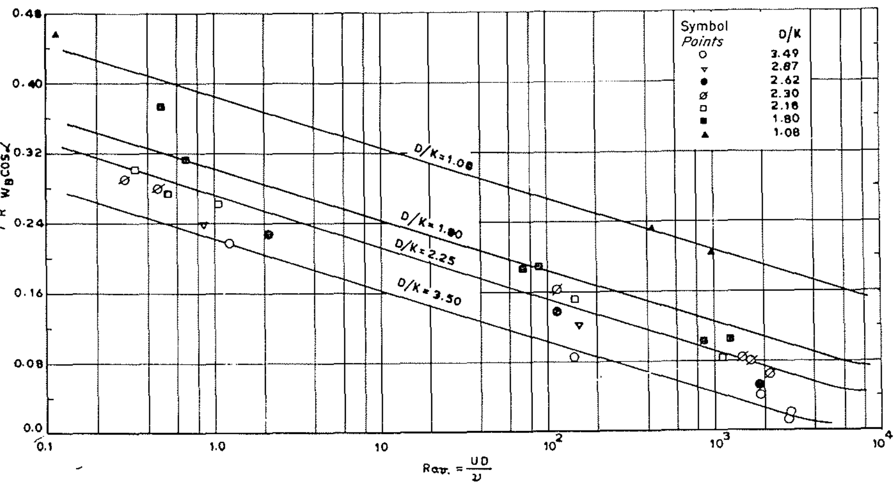
Figure 6 shows the variation of C_D with Reynolds number for the smooth boundary which was found to be similar to that for spheres in an infinite fluid. The Stokian range extends upto a Reynolds number of 20 and in this range C_D — R curve obeys a straight line variation of the form:

$$C_D = \frac{420}{R^{1.07}} \tag{9}$$

Carty's curve for the sphere rolling on smooth boundary is also shown in Figure 6. He has found the laminar range to obey a law of the form:

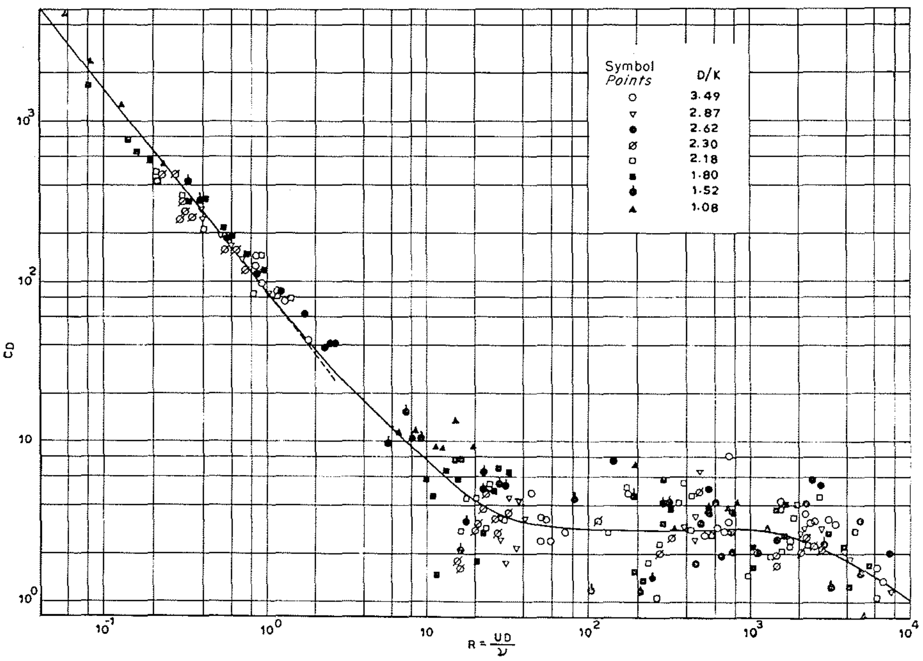
$$C_D = \frac{215}{R^{0.957}} \tag{10}$$

Whereas the curves of the author and Carty agree well in the range of Reynolds numbers greater than 8 000, they differ appreciably in the laminar range. Although the reason for this difference is not known, it can be mentioned here that the equipment used for the present study was two and half times longer than that used by Carty, with the obvious advantage that it permitted larger lengths both for the attainment of uniform velocity and for the purpose of timing the motion of the sphere. Hence, it is felt that the data collected in the present study is more reliable.



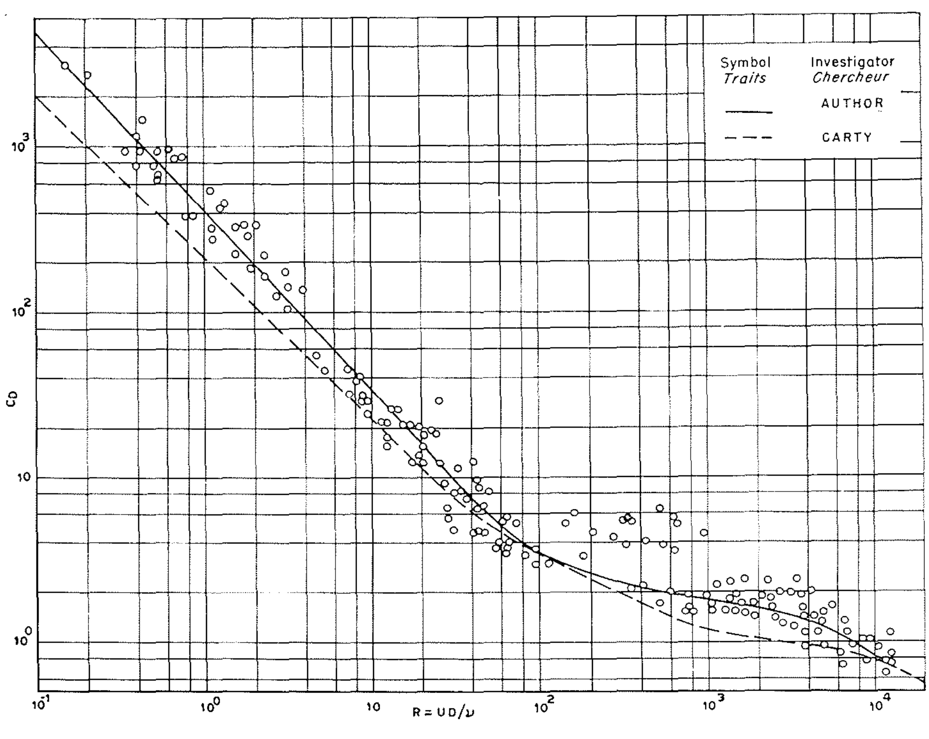
4/ Variation of rolling resistance coefficient.

Variation du coefficient de résistance au roulement.



5/ Variation of C_D with R (rough boundary).

Variation de C_D en fonction de R (limite rugueuse).



6/ Variation de C_D with R (smooth boundary).

Variation de C_D en fonction de R (limite lisse).

Conclusions

The drag coefficient for a sphere moving on a smooth or a rough boundary is found to be much different from the drag coefficient of a sphere in an infinite fluid. This fact is important and must be taken into account in the analysis of movement of sediment particles in alluvial channels. Figure 5 shows rather large scatter in the range of Reynolds numbers 10 to 10^4. This indicates that there is a scope for improvement in the estimation of rolling resistance.

List of notations

A :	Projection of the cross-sectional area normal to the direction of velocity.......	L^2
C_D :	Drag coefficient with fluid resistance only...........	dimensionless
C'_D :	Drag coefficient with rolling resistance included in fluid resistance...............	dimensionless
C_L :	Coefficient of lift..........	dimensionless
D :	Diameter of the sphere....	L
F_D :	Drag force...............	MLT^{-2}
F_L :	Lift force................	MLT^{-2}
F_R :	Rolling resistance offered by the rough boundary.......	MLT^{-2}
g :	Acceleration due to gravity.	LT^{-2}
K :	Diameter of the spheres forming the rough boundary.	L
R :	Reynolds number........	dimensionless
R_{av} :	Average Reynolds number corresponding to rolling resistance................	dimensionless
U :	Relative velocity between the fluid and the sphere..	LT^{-1}
W_B :	Buoyant weight of the sphere.....................	MLT^{-2}
W_1 :	Net weight in the rolling resistance method........	MLT^{-2}
μ :	Dynamic viscosity of the fluid....................	$ML^{-1}T^{-1}$
μ_R :	Rolling resistance coefficient.	dimensionless
ρ_f :	Mass density of the fluid...	ML^{-3}
ρ_s :	Mass density of the sphere.	ML^{-3}
α :	Angle of slope of the boundary in degrees............	—
α_{av} :	Average angle of slope of the boundary................	—
γ_f :	Specific weight of the fluid.	$ML^{-2}T^{-2}$
γ_s :	Specific weight of the sphere....................	$ML^{-2}T^{-2}$

References

[1] Carty (J.-J.). — *Resistance Coefficients for Spheres on Plane Boundary*, B.S. Thesis, Massachusetts Institute of Technology, 1957.

[2] Hoerner (S. F.). — *Fluid Dynamic Drag*, published by the author, 1965, p. 7-20.