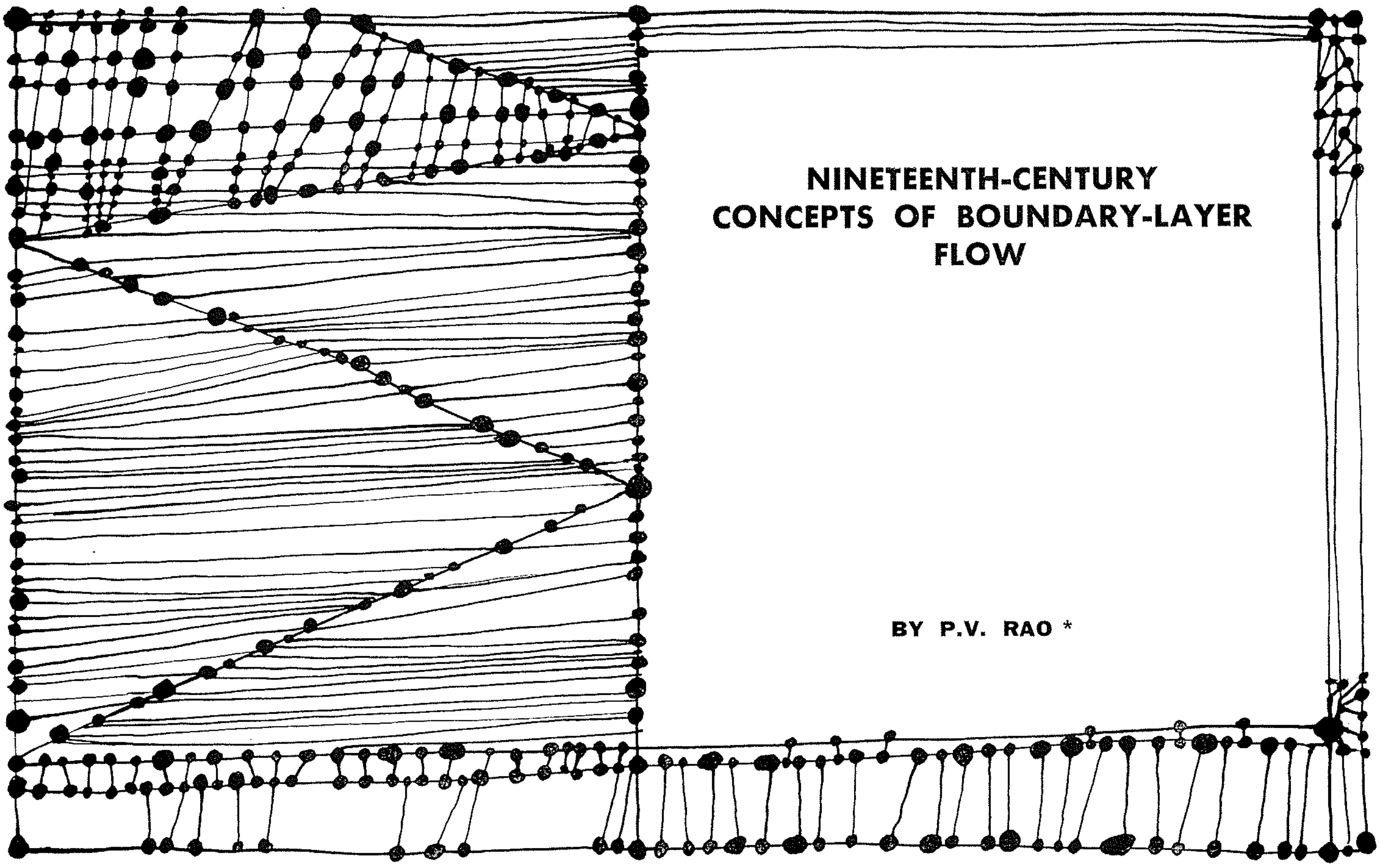

NINETEENTH-CENTURY CONCEPTS OF BOUNDARY-LAYER FLOW

BY P.V. RAO *

In 1954 Hugh Latimer Dryden [1] ** gave a very interesting account of the history of the boundary layer in theory and experiment since 1904, when Prandtl first presented his flume experiments and advocated the boundary layer approach to fluid-dynamics problems. The developments in boundary-layer theory since 1954 can readily be traced from the *Applied Mechanics Reviews* published monthly by the American Society of Mechanical Engineers. However, the early history of the boundary layer is not well documented in the existing literature of fluid mechanics. The nineteenth-century physicists and engineers in France, Germany, and England had many right conceptions as well as fallacious ones regarding the role of a boundary layer in the mechanics of real fluids. The various notions prevalent until 1904 are recorded and commented upon in this paper. Those may well be of considerable historical interest to the fluid dynamicist today.

Prehistoric man must have labored long and hard by the primitive means at his disposal to increase the carry of his arrows and spears, and the efforts of the Roman and the Egyptian mariners must have been similar in designing ships which could sail with fair speed over the Mediterranean and Red Seas [2]. Before the advent of air and space navigation, the naval architects of the last century were largely interested in exploring ways and means to reduce the drag of ocean bound vessels.

It is proper here to recall the pertinent developments in the mathematical description of fluid motion before and during the early period of the nineteenth century. Newton in 1687 was perhaps the first scientist to introduce the concept of viscosity in trying to explain the fluid resistance exerted on solid bodies. He propounded the linear relationship between the rate of the deformation and the shear stress. The equations of motion for ideal (inviscid) fluids were obtained by Euler in 1735. D'Alembert showed in 1752 that the solution of Euler's equations for steady flow would yield zero longitudinal force, which was contrary to observation. The inadequacy of Euler's equations to explain real-fluid situations was thus established. In 1822 Navier obtained the general differential equations of motion valid for real (viscous) fluids. Navier stated that the internal stresses in a viscous fluid were due to the nonequilibrium dynamic condition of fluid molecules. His hypothesis that the force between any two molecules is directly proportional to the relative velocity between them was not approved by his contemporaries and later investigators. This led many of them, such as Cauchy (1828), Poisson (1831), and Stokes (1845), to develop more accurate proofs of the differential equations of motion of a viscous fluid. It was, however, Saint-Venant [3] who obtained the equations in generalized form applicable to all types of flow.

Navier was a reputed bridge engineer in his time. He derived the equations of motion of a viscous fluid while designing a bridge on the River Seine. He submitted his derivation to the French Academy of Sciences in seeking its membership. Strangely enough, the bridge, which was the first engineering piece designed according to the equations of motion, failed in the same year it was constructed. Navier was, however, elected as a member of the Academy of Sciences.

During the intervening period from 1822, when Navier formulated the equations of motion in correct form, to 1904, when Prandtl reported on his boundary-layer research, a few engineers and physicists in England and Europe had unmistakably sensed the presence of a boun-

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** Numbers indicate references listed in the order in which they appear in the text and given at the end.

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dary layer and its significance, while investigating experimentally or analytically the complex phenomena of surface resistance of ships of flow through pipes. Some of them had obtained solutions of the Navier- Stokes equations for simple boundary geometries. Nevertheless, progress in the science of real fluids lagged for nearly a century.

Two reasons seem to be plausible. First, the physicists were apparently puzzled as to how the fluid behaves in contact with a solid surface: does the fluid slip along the surface or does it stick to it ? To some extent the engineers who provided measurements of pipe flow were to blame for this confusion. Secondly, the Navier-Stokes equations were too new for the mathematicians of those times to do anything other than improve their derivation or study their properties or uniqueness.

Daniel Bernoulli [4, 5] as early as 1738 recognized the fact that a real fluid could not slip on a solid surface. He came to this conclusion as a result of the large discrepancy he observed between ideal-fluid theory and measurement. As evident from d'Alembert's paradox and later developments which led to the rift between experimental hydraulics and classical hydrodynamics, it seems that due attention was not paid to this important discovery. In contrast to Bernoulli's reasoning, Franz Ernst Neumann [6] in 1835 concluded from his analysis of efflux through plane circular orifices that the fluid might have a definite velocity at the wall. Through application of continuity and energy principles and accounting for no energy loss between the plane of a circular orifice and the vena contracta, Neumann obtained a value of 0.7071 for the coefficient of contraction. In obtaining this value, Neumann assumed tho no-slip boundary condition at the edge of the orifice. It agreed with Newton's 1713 measured value of $1/\sqrt{2}$, which was then known to be too high; its correct value was 0.61. In order to decrease the value thus computed from 0.7071 to 0.61, Neumann admitted a finite velocity of flow at the edge of the orifice, and this velocity of slip works out to be approximately 13 percent of the central velocity. Though irrelevant to some extent, it may be noted here that G. Kirchoff [2], the celebrated student of Neumann, showed in 1845 by conformal analysis that the value of the coefficient of contraction is equal to $\pi/(\pi + 2)$ for rectangular slots. It is interesting that Daniel Bernoulli and a century later Franz Neumann should have arrived at contradictory conclusions through comparison of ideal-fluid theory and measurement.

The experimental observations by the French Engineers Du Buat (1786), Coulomb (1800), and Girard (1813, 1814, 1815) had strongly influenced the 19th century trends of thinking of the boundary condition at the wall. Du Buat [2] concluded from his experiments with open-channel flow that a layer of stagnant fluid exists at the boundary, and the remaining fluid slides over it. Coulomb's experiments with oscillating discs and oscillating cylinders in water showed the absence of any slip between the fluid and the boundary of the solid. Prony [4] had also noticed a similar but thicker stagnant layer. Girard [4] from his experiments on the flow of water through copper pipes, was led to the same conclusion as Du Buat's, but the layer was very thin and of constant thickness for the same boundary material. This had misled the contemporary physicists and engineers to assume that the liquid slips on a solid boundary with a finite velocity.

Navier believed Girard, and he accepted slip at the wall. He assumed that the shear stress between the liquid and the boundary is directly proportional to the velocity of slip. He integrated the equations for pipe flow accordingly, and arrived of course at a wrong formula, $Q \sim r^3h/l$ instead of $Q \sim r^4h/l$. Here Q is the rate of flow in a pipe of radius r, and h is the drop in piezometric head over the length l of the pipe. In 1832 Poisson suggested that Navier's boundary condition should be applied at the outer edge of the stagnant layer and not at the wall.

Although Stokes [2, 7] had derived the equations of motion as early as 1845 and integrated them for the case of pipe flow, he was not sure about the boundary condition at the wall until 1851, when he advocated the no-slip condition in the following memorable words, "the existence of slip would imply that the friction between solid and fluid was a different nature from, and infinitely less than, the friction between two layers of fluid".

Even though the experiments of Hagen [8] in 1839 and of Poiseuille [9] in 1841 on flow of viscous fluids through round pipes showed perfect adherence of water to glass, Girard's measuments with large tubes had misled virtually all the investigators during the nineteenth century to believe that the fluid would slip on a solid surface. Hagen first (in 1839) stated that the velocity increases from zero at the wall to a maximum value at the axis of the pipe, but later [10] (in 1854) he admitted the stagnant-layer hypothesis of Girard. Couette [11] in 1888 expressed his belief that the measurement of the diameter of the pipes by Girard may not be correct. Here it should also be noted that Girard's measurements with the same pipes showed complete adherence of water to the boundary for values of the rate of flow below a certain limit. Based on the experiments of von Piotrowski with oscillating copper spheres in water, Helmholtz [4] in 1860 had also put forth the possibility of the slipping of water on copper surfaces. The slip was also admitted by Kirchhoff [11] in 1877. It is of interest to note here that Hagenbach [12] in 1856 and Neumann [6] in 1859, who had obtained analytically the Hagen-Poiseuille equation for the flow of a viscous fluid through a round tube, assumed the boundary condition that the velocity of flow at the wall is zero.

The following points summarize the above discussion. The boundary condition at the contact of the fluid and the solid surface was very much debated by the scientists and the engineers during the early part of the nineteenth century. The measurements by Girard and others had shown a stagnant layer of fluid in contact with the pipe wall over which the rest of the fluid slid. The integration of the Navier-Stokes equations with a no-slip boundary condition yielded results which did not agree with the pipe-flow measurements by Girard and others even though they would have agreed with the measurements by Hagen and Poiseuille.

The German engineer Hagen [10] showed in 1854 for the first time the existence of the two regimes of pipe flow and the transition between them. However, the boundary condition for the second regime (turbulent flow) was not discussed by any one till 1860, when Hagenbach [12] for the first time adopted the boundary condition that the velocity of flow at the wall is zero for the turbulent flow (vibrating resistance) in a pipe. Unfortunately, this fact was not recognized by the researchers during the last century. Reynolds [13] reestablished in 1883 by his famous color-band experiments the existence of the two regimes of flow. However, he did not discuss the boundary condition at the wall for the case of the turbulent flow. In his doctoral dissertation Couette [11] in 1890 had dis-

HENRI DARCY
(1803-1858)

HAGENBACH
(1833-1910)

cussed in detail the boundary condition at the wall for the turbulent flow. He had clearly distinguished the shape of the velocity profiles for the laminar and turbulent flows in a round pipe in the neighborhood of the wall. He said that the parabolic velocity distribution corresponds to the measurements by Hagen and Poiseuille, and the velocity profile for the turbulent flow is uniform across the pipe with rapid variation of the velocity in the vicinity of the wall. The later corresponds to the measurements by Girard and Darcy. The hydraulicians and the physicists during the last century had naturally mistaken the steep gradient of the velocity at the wall to represent a finite velocity of slip of the fluid on the surface of the boundary, although in reality the fluid in contact with the solid boundary always remains at rest with it. In a way it can be considered that the hypothesis given by the French school of the hydraulicians early in the nineteenth century suggests a primitive concept of a boundary layer at rest relative to the surface of the solid. Following Coulomb's suggestion, if one assumes that the shear stress at the outer edge of the boundary layer is proportional to the velocity of flow, and if one grants that the velocity at the wall is equal to zero, then it amounts to the modern concept of a laminar sublayer.

Since the formulation of the Navier-Stokes equations, researchers were occupied during the remaining part of that century with the following problems: a rigorous proof of these equations; the nature of the boundary condition at the wall; the techniques of measuring viscosity; and accurate measurement of the viscosity of common liquids and air. So far as the search for exact solutions of these equations was concerned, the problems attempted during this period were too few. For example, viscous flow through a round pipe (1845), flow between concentric cylinders (1845), flow past a sphere (1851), flow near an oscillating plate (1851), and very slow motion between parallel plates (1898) were solved by Stokes [14]. Theoretical analysis of viscous flow between two concentric cylinders was also given by Saint-Venant in 1847, and by Boussinesq in 1873 [11]. In the same year Helmholtz expressed the equations of motion in dimensionless form, and in 1894 Reynolds extended these equations to describe turbulent flow [2].

A problem of considerable interest which would have demonstrated the physical existence of a boundary layer is the steady and incompressible flow between two convergent plates. For Reynolds numbers greater than 5000 the velocity distribution across the two plates remains uniform except near the plates, where the velocity decreases steeply from its maximum value to zero at the plate. The uniform velocity distribution in the core corresponds exactly to the potential-flow solution. The utility of such boundary geometries in practical hydraulics had already been discussed by Venturi [2] in 1797. Surprisingly enough this problem was not attacked until 1915, when G.B. Jeffrey [15] gave its solution. A solution of this problem through the boundary-layer approach was first obtained by K. Pohlhausen [15] in 1921. Had the mathematicians of the last century been interested in obtaining the exact solutions of the Navier-Stokes equations for more practical problems, they would surely not have failed to observe the phenomenon of boundary-layer flow, which would have advanced research in fluid mechanics by fifty years.

The French hydraulicians Du Buat (1734-1809) and Darcy (1803-1858), as well as Hagenbach (1833-1910), professor of physics at the University of Basel, had described the boundary-layer phenomenon in their works. The English engineer William Froude (1810-1879) had clearly noticed the role of the boundary layer in causing the surface resistance of solid bodies towed in water, and he had aroused the curiosity of the British naval architects about this phenomenon. Hele-Shaw (1854-1941) was the first to demonstrate visually the formation of a boundary layer on the surface of a solid body by means of color-band experiments.

In his analysis of the velocity distributions that he had observed for open-channel flow and pipe flow, Du Buat [2] claimed to have noticed in 1786 a stagnant layer of fluid in contact with the boundary surface, and he assumed that the remaining fluid glided over this layer. Du Buat's tests with different boundary materials such as glass, lead, iron, wood, or earth showed no appreciable effect on the resistance. If the laminar sublayer is several times thicker than the absolute height of roughness of a boundary surface, then the nature of the boundary material is now known to have no effect on the resistance. Du Buat's data were unfortunately limited to this narrow range of what are customarily called hydrodynamically smooth surfaces. Considering that the status of hydraulics at the time of Du Buat was such that even Chezy's equation was not available to him [2], Du Buat deserves to be credited for distinguishing a layer of fluid in contact with the surface of the body from the rest of the fluid, and it may be regarded as a primitive concept of a laminar sublayer although he did not visualize the dual role of viscosity or the boundary condition at the wall.

In 1848 Darcy [16] conducted as many as 198 tests on pipe flow using different materials like steel, steel coated with asphalt, cast iron new and old, glass and lead, and different sizes of the pipes varying from 1½ cm to 50 cm. He measured the velocity distributions and the resistances of the pipes, and published his work [17] in 1857. He was the first to describe the role of the boundary layer and the surface resistance of the pipes [2]. Departing from his predecessors, Du Buat, Prony and others, Darcy concluded from his tests that the resistance coefficient depends on the nature and state of the boundary surface and the diameter of the pipe. He reasoned that part of the resistance was due to the boundary roughness, even though the scale of the roughness might not be perceptible to the eye, and that part of the resistance was produced by the layer of fluid close to the boundary. Darcy was wrong in making this distinction. However, it is noteworthy that he proposed a resistance formula which could be used for smooth and rough pipes. A remarkable discovery by Darcy was the velocity-defect expression for the distribution of velocity in a pipe as a function of the size of the pipe and the energy gradient. His formula had a dimensional coefficient which can, however, be interpreted as a relative-roughness parameter [2]. Darcy had also noticed the influence of viscosity on the velocity distribution in a pipe.

Hagenbach, professor of physics at the University of Basel, the alma mater of the great Bernoullis and Euler, was known to the hydraulicians for his analytical derivation of the Hagen-Poiseuille law and for his calculation of the correction for the kinetic energy [2], even though he calculated this correction wrongly for an inviscid fluid. For larger pipe diameters and for larger velocities of flow, Hagenbach [12] observed the shock resistance (Erschütterungswiderstand) which was proportional to the square of the velocity. He found that this type of flow was characterized by eddies and that the velocity was uniformly distributed as if the parabolic profile as a whole were compressed. He further remarked that the resistance in this case depended also on the roughness of the material of the pipe wall. As mentioned earlier in this report, Hagenbach in 1860 was the first to use the no-slip boundary condition at the wall even for the turbulent flow in a pipe. In the analysis of the experimental data of turbulent pipe flow, he introduced a new parameter a which was a characteristic of the pipe wall only and was independent of the radius of the pipe. When a was very large, the velocity distribution in the central part of the pipe was determined by the shock resistance (turbulent flow) while the velocity distribution near the wall was determined by the fluid friction (viscosity).

This hypothesis is reminiscent of the modern concept of a boundary layer for turbulent flow in a smooth pipe [18], because preponderant inertial resistance in the core of the pipe and preponderant viscous resistance in the wall layer are assumed for a smooth pipe at present. The parameter a, in fact, represents the thickness of a fluid layer moving with the same velocity as the central fluid and having the same energy content as the boundary layer. It represents virtually the same as the present-day energy-defect thickness of a boundary layer. Noteworthy are the remarks by Hagenbach that the value of a should be determined by experiment, and furthermore, that there is only one value of this parameter for one particular pipe which will yield accurate results. It is unfortunate that this important discovery by Hagenbach in the middle of the last century was totally ignored by contemporary scientists. In the history of hydraulics, credit should go to him for having been the first to describe a turbulent boundary layer in a quantitative way.

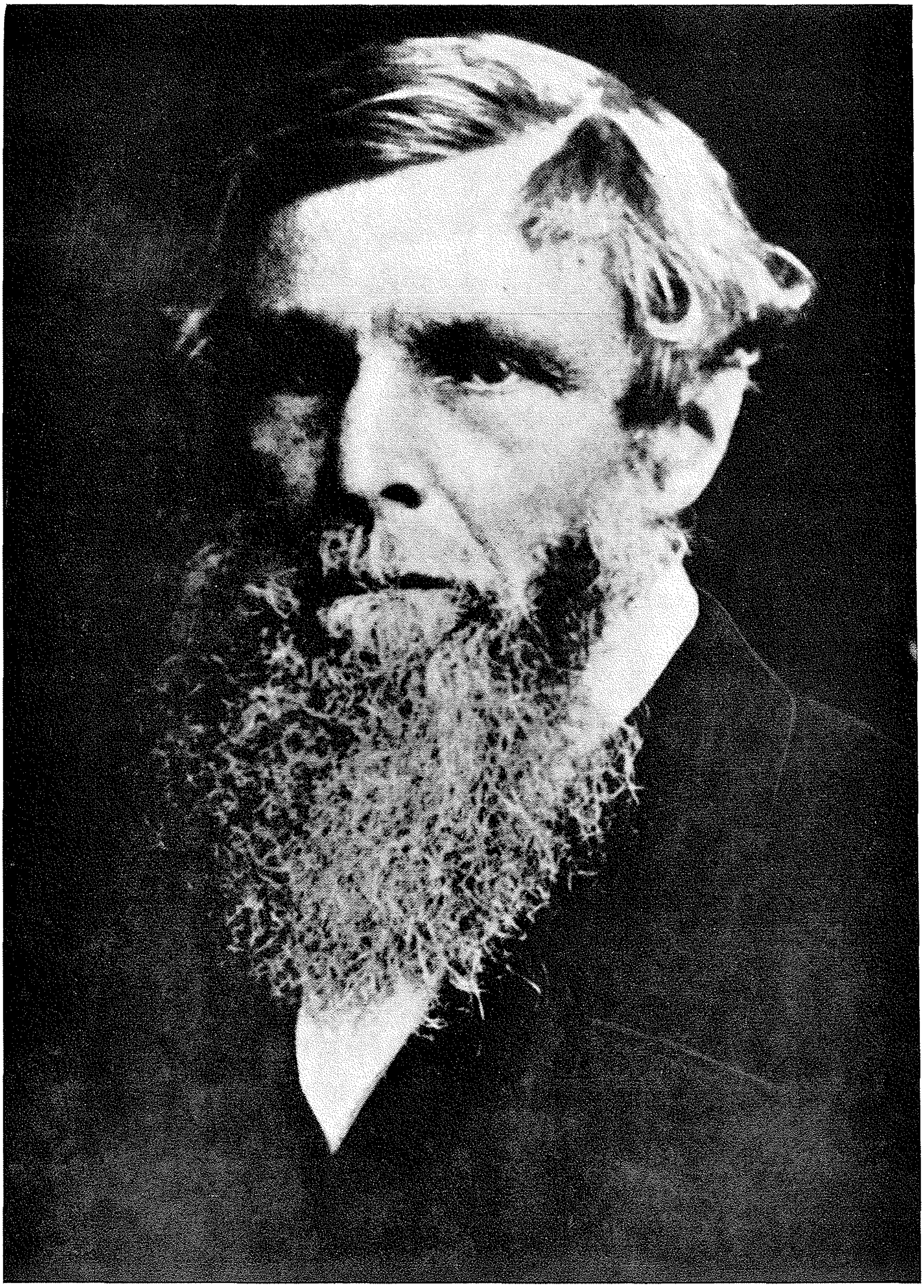
Hagenbach's contributions were followed chronologically by those of two British engineers who also discussed the significance of the boundary layer and the surface resistance of bodies based upon experimental observations. Froude [19, 20, 21] studied the drag of ships by means of scale models in a 250-foot-long towing tank for the British Admiralty. He used planks of different lengths varying from 10 inches to 50 feet, and the surface finish included both varnish and sand-grain roughness. He measured with a dynamometer the drag force of the submerged planks when towed with uniform velocity in the tank. He found that the resistance varied as the 1.85 power of the velocity for smooth surfaces, and as the 2.0 power for very rough surfaces. Froude recognized that the mean resistance decreased with the increase in length, because the boundary shear had to decrease along the length of the plank in the direction of the fluid flow [20]. He also observed that the boundary shear reached a constant value after some length so that the observed mean resistance of planks longer than this remained constant. Froude's interpretation of the observation on the surface resistance of the planks was in terms of modern boundary-layer theory [2]. He was the first to recognize in 1874 [21] that the surface resistance was but one part of the total resistance of a ship and depended on the surface condition. He also showed how to estimate the surface resistance at prototype scale.

Froude's discovery, that the relative motion of water close to the boundary of a solid body produced the frictional resistance of the body, exerted a great influence on the naval architects of those days, and they soon realized how important it would be to know more about what was happening in the layer of water close to the surface of a solid body. Hele-Shaw, professor of engineering at Liverpool University, is a familiar name in experimental hydraulics for the ingenious apparatus invented by him and known by his name [22]. The apparatus consists of two parallel glass plates separated by a thin film of liquid flowing around an arbitrarily shaped body held tightly between the two glass plates. It helps in plotting the streamline pattern for flow past a two-dimensional body of any arbitrary shape. It was when he was perfecting this apparatus at Liverpool that he observed with curiosity a clear border line or a thin film adjacent to the surface of the obstruction, and it appeared in all his experiments. Encouraged by his friends in the British Navy, Hele-Shaw proceeded to study in detail the surface resistance of solid bodies and pipes with reference to the flow of fluid in the boundary zone using his apparatus.

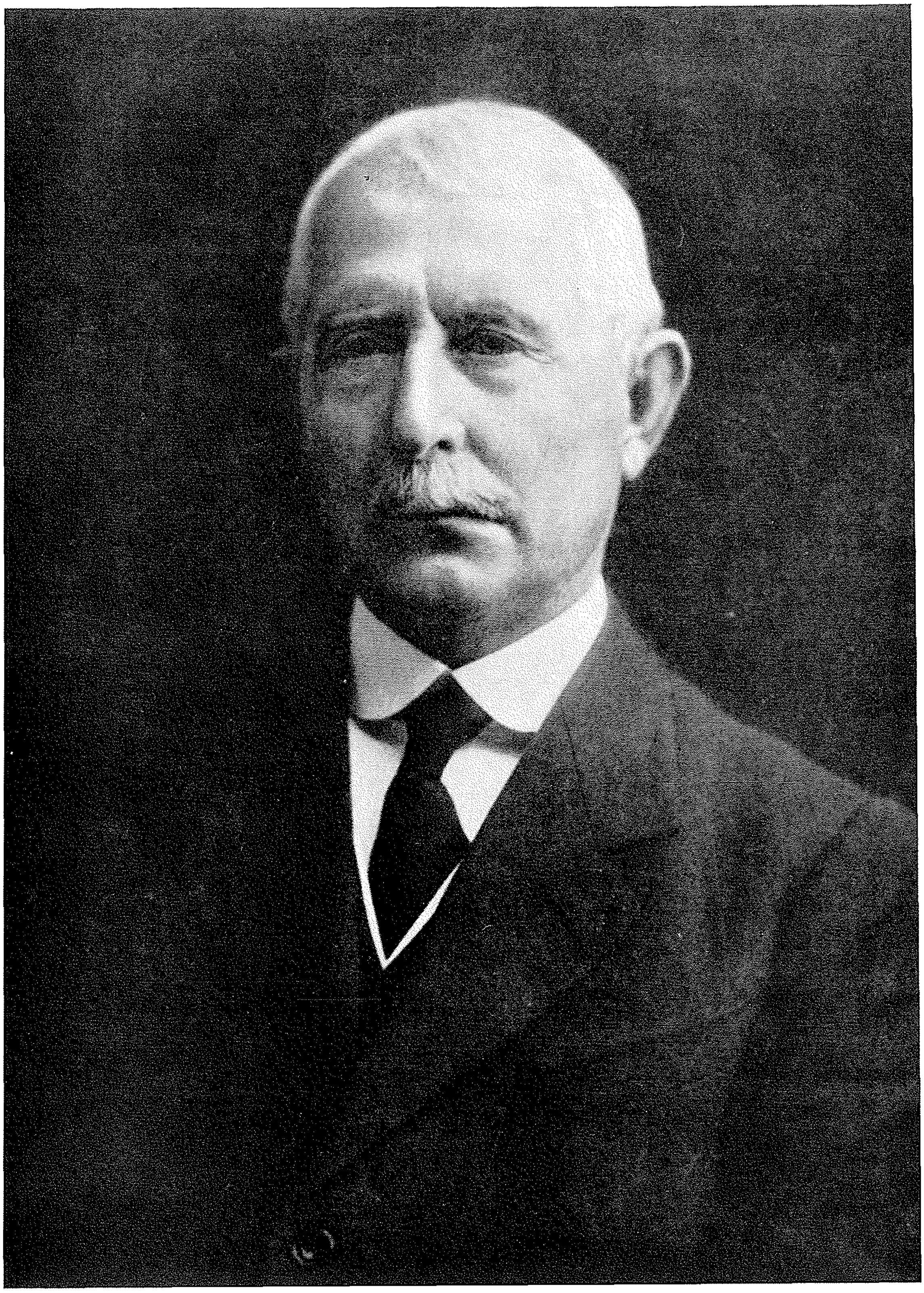
He presented his experiments and findings before the Institution of the Naval Architects in 1898 [23]. He had found that, when dye was introduced on the surface of the solid body, the deepest color always remained near the surface of the body, showing that the velocity within the layer of fluid close to the surface of the body was very slow and gradually increased outwards. It is remarkable that he found different thicknesses of this layer at different points along the body, and that the thickness of the layer at a point on the body could be varied by varying the velocity of flow [23]. He concluded that the flow within this layer represented a state of parallel motion (laminar flow), while outside this layer the fluid was in a state of sinuous motion (turbulent flow). It may be noted here that the boundary-layer flow was regarded only as laminar till 1914, when Prandtl [1] discovered that it could also be turbulent.

Hele-Shaw further stated that shear stresses exist within this border layer. His failure to relate the thickness of the layer with any of the flow parameters is evident from his statement that its thickness would be of the order of many thousands of molecules. He agreed with Froude that the resistance of a body or pipe was directly related to this layer. He carried out similar tests for flow past a cylindrical pier and flow in a circular pipe when the surfaces were made rough [23, 24]. It is significant that he observed in either case that the layer was much thicker than for the smooth boundary. His curiosity as to the swift motion of a fish in water led him to inject into the border layer fresh bile, which, he thought, would simulate the exudation from the skin of the fish. As he expected, the resistance dropped immediately to a low value and thereafter again increased. Bile is a non-Newtonian liquid. It is interesting to recall in this connection that the injection of a non-Newtonian liquid into the boundary layer as a means to reduce the surface resistance is one of the topics of current research [25].

The beginnings of any new science are marked with difficulties and imperfections in concepts. This was evidently also true of the boundary-layer theory. Hence credit should be given to the works of the many researchers who contributed in any way to the understanding of the science, though their contributions remained unrecognized during their times. The intention here is certainly not to discount the wonderful conception of the boundary layer by Prandtl at the beginning of this century, but to highlight the contributions made by others before him. As Tollmien [26], a celebrated student of Prandtl's, had rightly pointed out, Prandtl's contribution was the simplification of the Navier-Stokes equations, which are elliptic in nature, into the parabolic type, which are easy to handle.



WILLIAM FROUDE
(1810-1879)



HELE-SHAW
(1854-1951)

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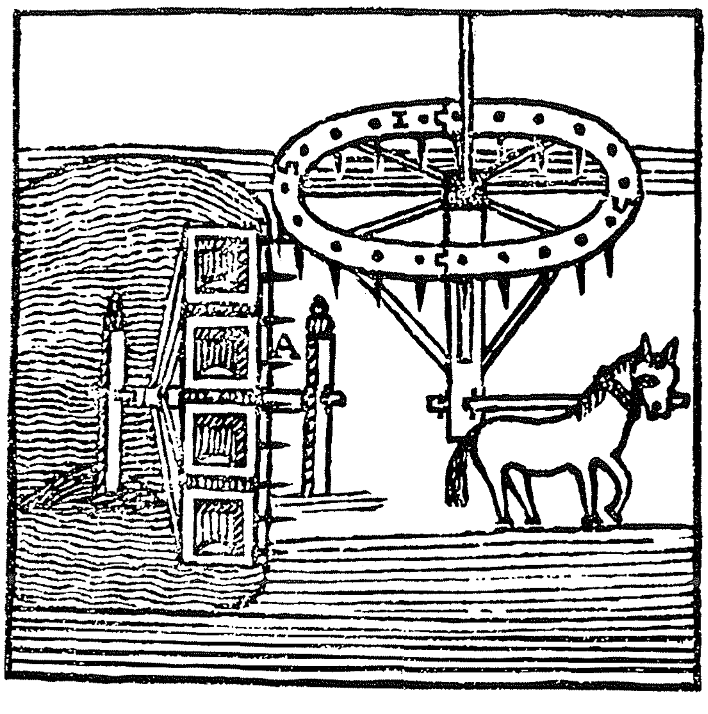

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