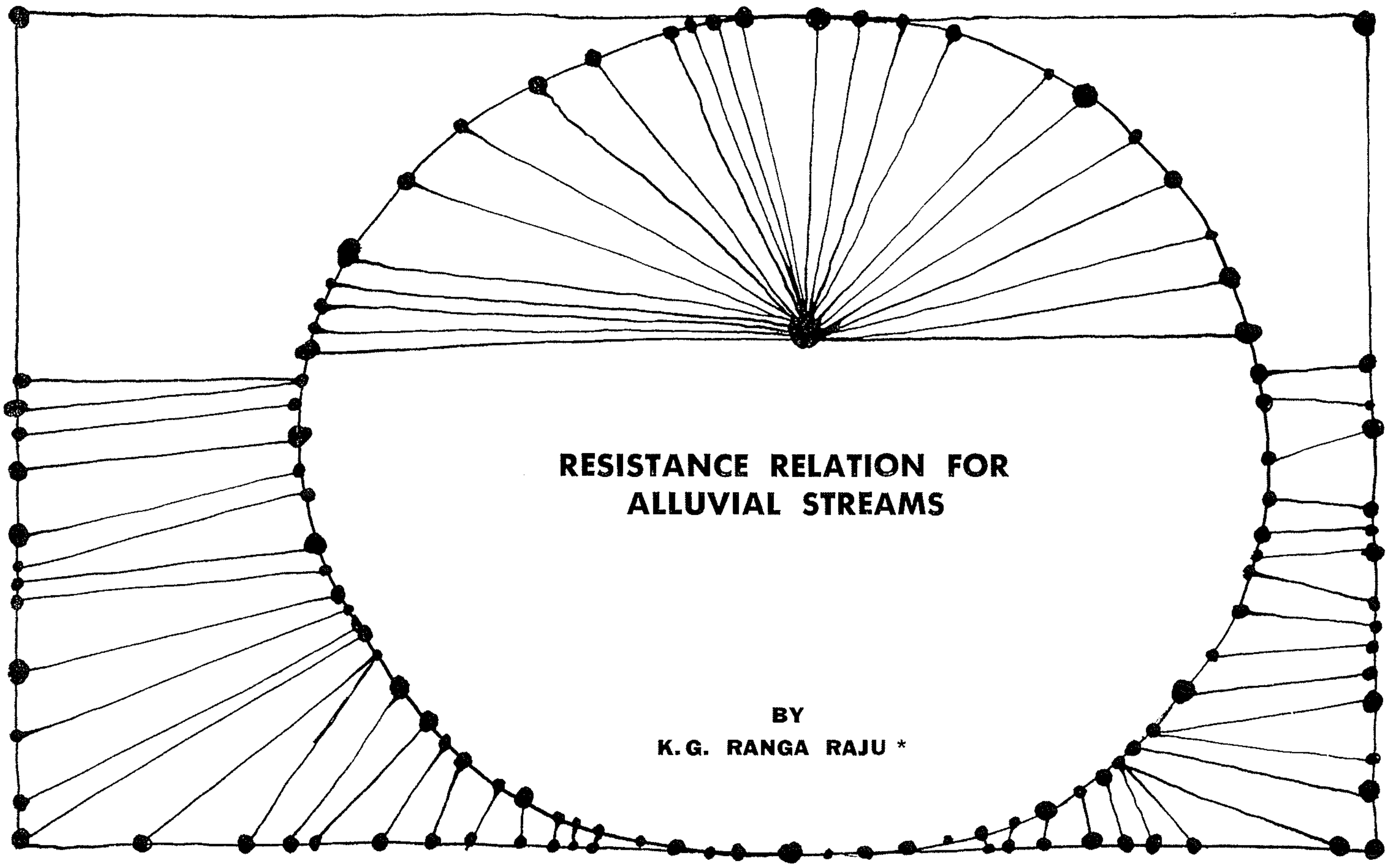

RESISTANCE RELATION FOR ALLUVIAL STREAMS

BY

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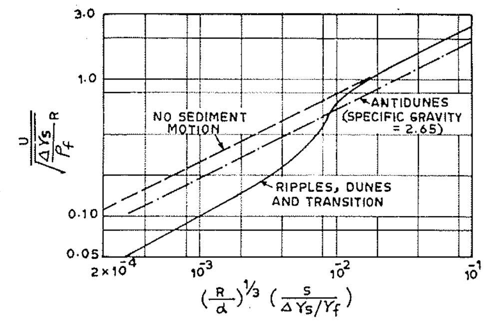
Introduction

The important role played by a resistance relation in problems connected with alluvial streams and the complex mechanism of resistance of these streams are well known and have been discussed previously by Garde and Ranga Raju [1]. In 1966 [1], they critically reviewed the existing resistance relationships (as of 1966) and proposed a new relationship which showed better agreement with the available data than did the other relations. The analysis carried out in that paper [1] resulted in the resistance relationship shown in Figure 1. As can be seen from this figure, three different laws were proposed, one for the 'plane bed with no motion,' another for the 'antidune' regime and a third one for 'ripples, dunes and transition' regime. (The classification of regimes as mentioned here is the one given in 1963 [2].) In using Figure 1, it was suggested that the regime be determined by the R/d and $S/(\Delta\gamma_s/\gamma_f)$ regime criterion proposed by Garde and Ranga Raju [2]. Here R is the hydraulic radius, S is the slope of the water surface, d is the median size of sediment and $\Delta\gamma_s = \gamma_s - \gamma_f$, where γ_s is the specific weight of the sediment and γ_f is the specific weight of the fluid.



1/ Resistance relationship proposed by Garde and Ranga Raju, 1966.
Loi de résistance proposée par Garde et Ranga Raju, 1966.

Another resistance relationship has been proposed by Engelund recently [3]. The accuracy of this relationship was tested by Mittal [4] and it was found that Engelund's relationship predicts the mean velocity of flow with the same degree of accuracy as does the law proposed by Garde and Ranga Raju [1], namely Figure 1. This conclusion was drawn by comparing observed and predicted velocities for several natural streams.

Since a large amount of additional data from flumes, canals and rivers were available, an effort was made to test Figure 1 with these data and modify it suitably to improve the accuracy of prediction of mean velocity. The results of that investigation are presented here.

Modification of resistance law proposed earlier

The resistance relationship shown in Figure 1 was obtained by starting from the functional relationship for the problem in the form :

$$U/\sqrt{(\Delta\gamma_s/\rho_f)\,d} = f_1\,(R/d,\ S/(\Delta\gamma_s/\gamma_f),\ g^{1/2}d^{3/2}/\nu) \quad (1)$$

which was obtained by carrying out dimensional analysis of the problem of resistance in alluvial channels. Here U is the mean velocity of flow, g is the gravitational acceler-

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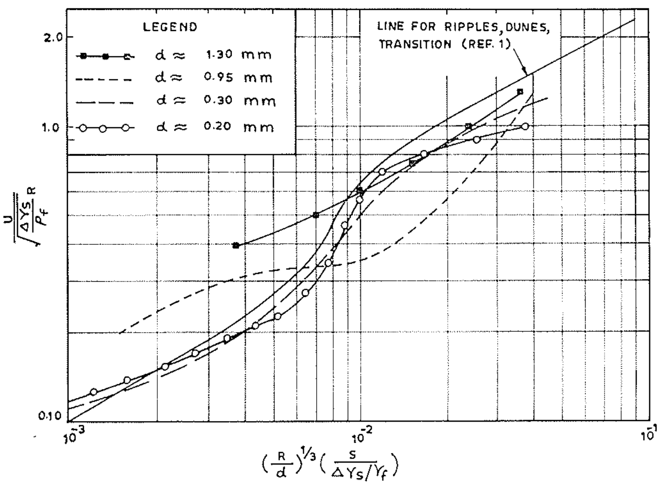


2/ Variation of F_R with S.

Variation de F_R en fonction de S.

ation, ν is the kinematic viscosity of the fluid and ρ_f is the mass density of the fluid. The above equation can be modified as:

$$U/\sqrt{(\Delta\gamma_s/\rho_f)\,R} = f_2\,(R/d,\ S/(\Delta\gamma_s/\gamma_f),\ g^{1/2}d^{3/2}/\nu) \qquad (2)$$

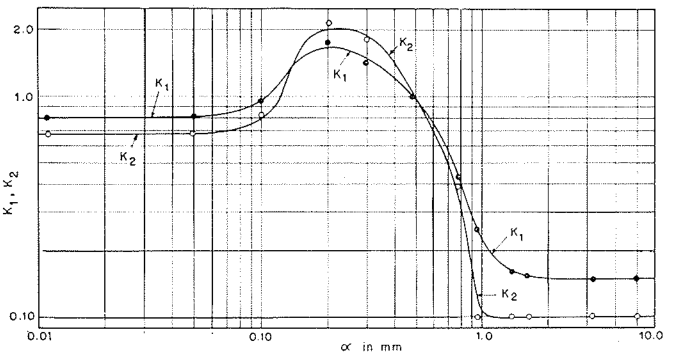
Figure 1 was developed on the basis of Equation 2, relating the parameters:

$$\frac{U}{\sqrt{(\Delta\gamma_s/\rho_f)\,R}} \quad \text{and} \quad \left(\frac{R}{d}\right)^{1/3}\left(\frac{S}{\Delta\gamma_s/\gamma_f}\right)$$

because the parameter $g^{1/2}d^{3/2}/\nu$ did not seem to indicate any systematic influence on this diagram. In fact Garde and Ranga Raju showed [5] that different lines can be drawn for different sediment sizes (i.e. different values of $g^{1/2}d^{3/2}/\nu$) on Figure 1 (in a different form, though) and these differed from the average lines drawn on Figure 1. Preliminary analysis could not however, lead to a method of unifying these lines.

However it was felt that a parameter involving sediment characteristics should be significant. This belief arose from the observation that the characteristics of ripples and dunes, the washing away of dunes, and the relative importance of grain resistance in total resistance depend to a large extent on the size of the bed material.

Since recently a large amount of reliable flume data of U.S.G.S. [6, 7] became available to the author this investigation was taken up to study the effect of the parameter $g^{1/2}d^{3/2}/\nu$ on the resistance relationship. Since the variation of the kinematic viscosity was comparatively small in the available data, the variation of $g^{1/2}d^{3/2}/\nu$ really represents the effect of variation of the sediment size in the data used. Hence it was decided to consider the sediment size only in the analysis instead of the dimensionless parameter $g^{1/2}d^{3/2}/\nu$. Further it was felt desirable to obtain a single resistance relation for all regimes involving sediment movement and hence the data from the "antidune" regime were analysed along with those for "ripples, dunes and transition" regime.



3/ Variation of K_1 and K_2 with sediment size.

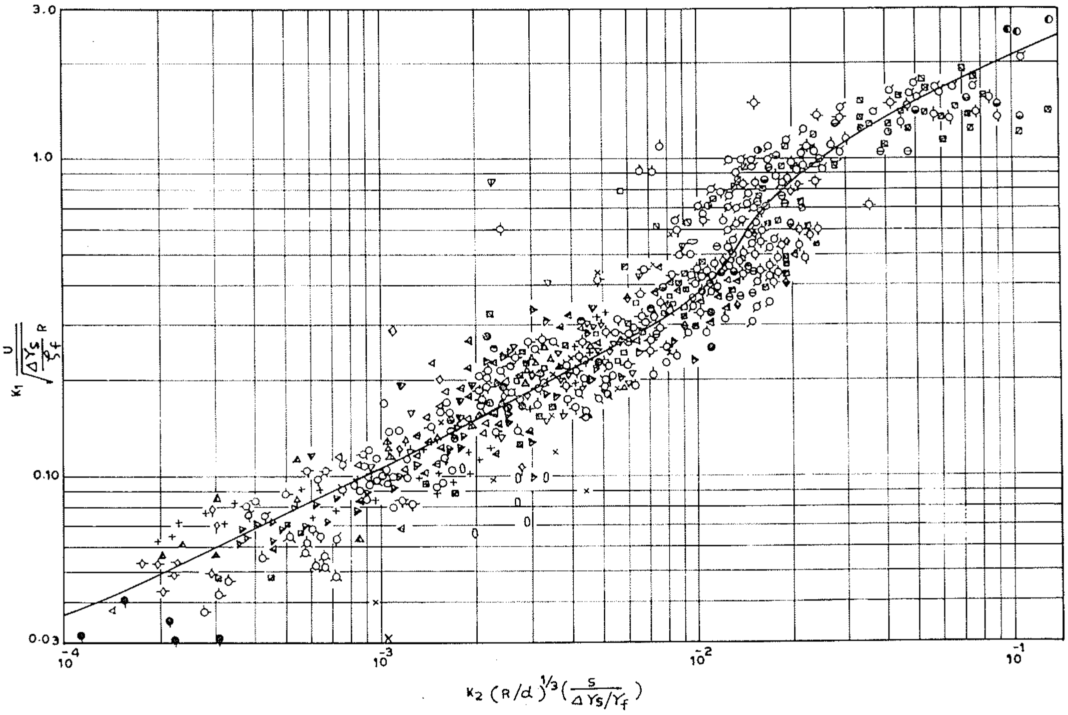
Variation de K_1 et K_2 en fonction de la granulométrie des matériaux.

Variation of $\mathcal{F}_R = \dfrac{U}{\sqrt{(\Delta\gamma_s/\rho_f)\,R}}$ with $\mathcal{S} = \left(\dfrac{R}{d}\right)^{1/3}\left(\dfrac{S}{\Delta\gamma_s/\gamma_f}\right)$ for different sediment sizes

All the available data from flumes, canals and natural streams were grouped under different sediment size ranges. It may be mentionned that an additional twenty five sets of data were used along with the data used in 1966 [1]. The variation of $\mathcal{F}_R$ with $\mathcal{S}$ was studied for each of these size ranges. Figure 2 shows the mean lines for a few of the size ranges investigated along with the line proposed earlier [1] for ripples, dunes and transition. In general these curves reveal two straight portions, connected by a fairly steep curve. The value of $\mathcal{S}$ at which the sudden change in curvature occurs also appears to be different for different sediment sizes. Since most of the data on the lower portions of the curves correspond to the ripple and dune regime, it may be supposed that the transition from the ripple and dune regime to the plane bed and/or antidune regime occurs at different values of $\mathcal{S}$ for different sediment sizes. It is also seen that some of the lines drawn on Figure 2 depart considerably from the mean line proposed earlier. This is explained partly by the fact that all the antidune data have also been plotted on Figure 2, while the mean line shown on Figure 2 and which was proposed earlier was not intended to be applicable for the antidune regime; however, some of the recent data, from ripple, dune and transition regimes, do depart from the relationship proposed earlier. There is thus a necessity for introducing a correction (which should be a function of sediment size) to improve the existing resistance law. The mean lines for various sediment sizes did not, at first glance, show any systematic departure from the line proposed earlier. But it was found by careful examination that a unique relation could be obtained between $K_1\mathcal{F}_R$ and $K_2\mathcal{S}$ where K_1 and K_2 are numerical constants and are functions of the sediment size.

Relation between $K_1\mathcal{F}_R$ and $K_2\mathcal{S}$

The empirical coefficients K_1 and K_2 were determined by the following procedure: The values of K_1 and K_2 were assumed to be unity for sediment size group having a mean of 0.47 mm. The lines for other sediment sizes were shifted so as to agree with the mean line for material having a sediment size of approximately 0.47 mm. The values of K_1 and K_2 which cause the above shift were then determined. The variation of K_1 and K_2, so determined, with the median size of sediment is shown in Figure 3. The points can be well approximated by smooth curves as shown in the figure. It is seen that for material finer than 0.06 mm K_1



4/ Resistance law for ripples, dunes, transition and antidunes.

Loi de résistance des rides, des dunes, du régime transitoire et des antidunes.

Table 1 *Range of variables covered by data used in the analysis*

PARAMETER	RANGE
Hydraulic radius	0.076 ft. to 56.09 ft.
Velocity	0.191 fps to 11.30 fps
Slope	3.64×10^{-5} to 2.56×10^{-2}
Sediment size	0.011 mm to 5.20 mm
Specific gravity of sediment	1.052 to 4.22

Table 2 *Summary of data used in the analysis*

N°	SOURCE OR INVESTIGATOR	*d* in mms	SPECIFIC GRAVITY	SYMBOL
1	Kalinske-Hsia	0.011	2.65	Q
2	Laursen	0.04	2.65	-○-
3	Kennedy	0.088 to 0.549	2.65	ơ
4	Laursen	0.10	2.65	□
5	Vanoni-Brooks	0.137	2.65	◆
6	Chabert-Chauvin	0,17	2.20	ϕ
7	Barton-Lin	0.18	2.65	⌕
8	Pien	0.18	2.65	◑
9	U.S.G.S.	0.19	2.65	⧄
10	Vanoni-Hwang	0.206	2.65	◆
11	Hwang	0.23	2.65	◩
12	U.S.G.S.	0.27	2.65	◒
13	Einstein-Chien	0.274	2.65	◐
14	U.S.G.S.	0.284	2.65	ɸ
15	U.S.G.S.	0.32	2.65	⊠
16	U.S.G.S.	0.33	2.65	⧅
17	Chabert-Chauvin	0.397	2.65	◭
18	U.S.G.S.	0.45	2.65	⌖
19	Chabert-Chauvin	0.45	2.65	▼
20	U.S.G.S.	0.47	2.65	●
21	U.S.G.S.	0.54	2.65	⊖
22	Plate	0.545	2.65	⦶
23	Liu	0.69	2.65	◓
24	Chabert-Chauvin	0.72	2.65	▼
25	Gilbert	0.786	2.65	♀
26	Shen	0.80	1.36	▽
27	U.S.G.S.	0.93	2.65	+
28	Chabert-Chauvin	0.96	2.65	◭
29	Shen	1.12	1.055	◁
30	Chabert-Chauvin	1.28	1.326	-Q
31	Chabert-Chauvin	1.31	1.31	ơ
32	U.S.G.S.	1.35	2.65	▶
33	Gilbert	1.71	2.65	◀
34	Chabert-Chauvin	2.48	2.65	◪
35	Chabert-Chauvin	2.72	1.32	ϙ
36	Chabert-Chauvin	3.00	1.075	Q
37	Shen	3.17	1.052	△
38	Gilbert	3.17	2.65	⊕
39	Gilbert	4.94	2.65	-♀
40	Meyer-Peter	5.20	1.25	▽
41	Meyer-Peter	5.20	4.22	△
42	Rio Puerco river	0.20 to 0.51	2.65	⬭
43	Einstein-Barbarossa	0.191 to 1.28	2.65	×
44	Nile river	0.25	2.65	◇
45	Niobrara river	0.28	2.65	-◇
46	West goose creek	0.287	2.65	♁
47	Elkhorn river	0.29	2.65	⌖
48	Tiber river	0.30	2.65	-♁
49	Riogrande river	0.30	2.65	⌕
50	Middle loup river	0.30	2.65	ϕ-
51	Mississippi river	0.31	2.65	¤
52	Colorado river	0.33	2.65	⌕
53	Big sand creek	0.33	2.65	◇-
54	Tisza river	0.36, 1.0	2.65	◊
55	Pigeon Roost Creek	0,40	2.65	♀
56	Waikato river	0.51	2.65	♁-
57	Enoree river	0.70	2.65	0
58	Mountain creek	0.90	2.65	-♀-
59	Indus river	0.97	2.65	-◇-
60	Luznice river	2.40	2.65	●
61	Sind canals	0.014 to 0.196	2.65	○
62	West Bengal Canals	0.025 to 0.346	2.65	⊞
63	U.P. Canals	0.047 to 0.365	2.65	▽
64	CHOP	0.081 to 0.378	2.65	▶
65	Simons-Bender	0.096 to 0.805	2.65	◁
66	Ackers	0.153 to 0.372	2.65	◀
67	Punjab canals	0.17 to 0.43	2.65	△
68	Tsubaki et. al.	1.26, 1.30	2.65	⚲
69	San Luis valley canals	20.1 to 82.0	2.65	●

and K_2 attain values of 0.80 and 0.667 respectively; also for material coarser than 2 mm, $K_1 = 0.15$ and $K_2 = 0.10$. It may be mentioned that analysis of data with different specific gravity indicated that K_1 and K_2 do not depend on the specific gravity of the sediment.

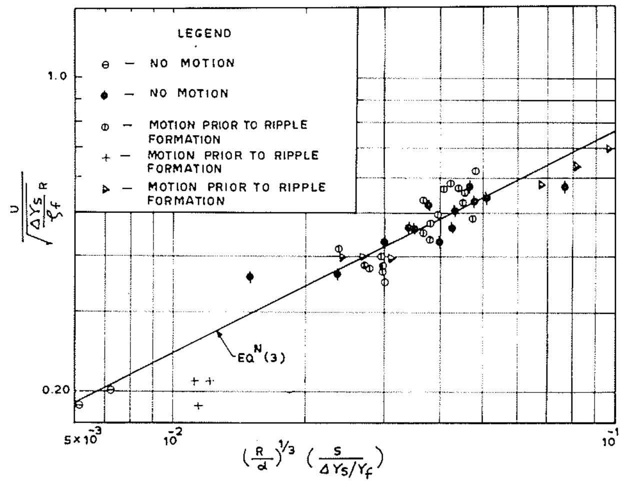
Figure 3 was used to compute the values of $K_1\mathcal{F}_R$ and $K_2\mathcal{S}$ for all the available data. Table 1 shows the range of parameters of the data used in the analysis and Table 2 gives the sediment size and symbol adopted for all sets of data. Figure 4 is a plot of $K_1\mathcal{F}_R$ versus $K_2\mathcal{S}$ for all the data. It is seen that the data fall along a unique curve with a reasonable amount of scatter. It was found that only 10 % of the data showed errors greater than ± 30 % in the predicted mean velocity. This scatter is less than that obtained when the correction is not applied. The scatter of points appears to be greatest near the steep portion of the curve connecting the relatively flat portions.

Data from the ripple, dune, transition and antidune regimes have been plotted on Figure 4. Very few runs having plane bed with motion prior to ripple formation are available. Some of these runs with coarse material showed agreement with the resistance law proposed in Figure 4, while some other runs indicated smaller resistance than predicted by Figure 4. Thus it is proposed that Figure 4 be used for alluvial channel flow with sediment movement, except for the very rare case of plane bed with motion prior to ripple formation. For a plane bed with no motion, the Manning-Strickler equation is known to predict the resistance quite well for hydrodynamically rough boundaries for which viscous effects are absent. It is seen from Figure 5, that the limited amount of data for plane bed with motion prior to ripple formation show agreement with the Manning-Strickler equation, namely:

$$\mathcal{F}_R = 7.66\,\mathcal{S}^{1/2} \qquad (3)$$

Some data corresponding to plane bed with no motion are also plotted on this figure for comparison. The agreement with Equation 3 seems to be good.

Thus Figures 4 and 5 can be used to predict the mean velocity and also construct stage-discharge curves for alluvial streams. It may be mentioned that the hydraulic radius corresponding to the bed may be used in cases where the bed and sides have different roughnesses. It may be noticed that plane bed with no motion or plane bed with motion prior to ripple formation are seldom obtained in the field and thus Figure 4 would suffice for all practical problems.



5/ Resistance law for plane bed with no motion and for plane bed with motion prior to ripple formation (for coarse material).
Loi de résistance valable pour un fond plan sans mouvement et pour un fond plan avec mouvement, avant la formation des rides (matériaux grossiers).

Conclusions

Analysis of a vast amount of flume, canal and river data has been carried out to obtain a resistance relation for alluvial channel flow. The resistance relation relates:

$$K_1\mathcal{F}_R = K_1 \frac{U}{\sqrt{(\Delta\gamma_s/\rho_f)\,R}}$$

to:

$$K_2\mathcal{S} = K_2\left(\frac{R}{d}\right)^{1/3}\left(\frac{S}{\Delta\gamma_s/\gamma_f}\right)$$

where K_1 and K_2 are unique functions of the sediment size and this relation is valid for the ripple, dune transition and antidune regimes—the regimes usually obtained in practice. For the case of plane bed with no motion and plane bed with motion prior to ripple formation, the Manning-Strickler equation seems to be satisfactory.

Acknowledgement

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Notation

$d = d_{50}$ = Median sediment size, for which 50 % of the material, by weight is finer;
g = Acceleration due to gravity;
K_1, K_2 = Coefficients used in the proposed relation;
R = Hydraulic radius of the stream;
S = Water surface slope;
U = Mean velocity of flow;
γ_f = Specific weight of fluid;
γ_s = Specific weight of sediment;
$\Delta\gamma_s = \gamma_s - \gamma_f$;
ρ_f = Mass density of fluid;
ν = Kinematic viscosity of fluid.

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