PLUNGING PHENOMENON OF DENSITY CURRENTS IN RESERVOIRS

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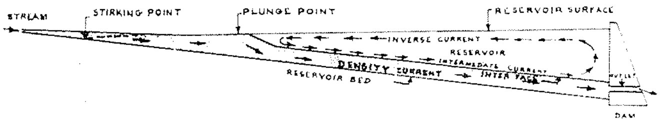
Introduction

The construction of a dam across a stream and the creation of a reservoir causes the sediments brought by the stream to be deposited within the reservoir body, thus reducing the useful life of the reservoir. The formation of higher density stream water into a density current, its travel through the reservoir length and emission out of the dam by proper location of sluices would pass at least a part of the sediment load downstream of the dam. A successful utilization of this concept requires an understanding of the conditions necessary for formation of density currents, and the characteristics of density currents, if formed. An objective of the present study was to obtain an understanding of the "plunging phenomenon" i.e. the phenomenon in which the stream flow takes a plunge from the reservoir surface and flows as an underflow or a density current at the bottom of the reservoir. The "plunge point" is the end point of the stream flow and the starting point of the density current. Figure 1 shows the definition sketch of a density current, and a plunge point.



1/ Definition sketch of a density current in reservoir.

Though considerable work has been done on the stability of density currents, the authors were unable to find in the published literature conditions for the establishing of a stable plunge point which is usually attributed to the formation of a delta like deposit or "alluvial cone" in the head reaches of the reservoir. As will be seen below the findings of the present study are contrary to this assumption.

Experimental set up

Two-dimensional experimental studies of the plunging phenomenon were carried out in the main flume shown in Figure 2. Clear tap water was used to form a reservoir in the flume, while coloured salt water was used as stream flow and was prepared in a mixing tank. A circulating system was provided to pump the salt water from this tank to be introduced as stream flow into the reservoir. It flowed though the reservoir as a density current, and returned through outlets back to the tank. The measurement of reservoir depths and density current depths was carried out by fixing scales on the glass side of the flume to an accuracy of 0.1 cm. The inflow and outflow discharges were measured by two V notches fixed for the purpose. The density measurement of the salt solution was carried out by a conductivity bridge (Phillips Model No.PR/9500) and an immersion type cell (Phillips Model No.PR/9510). The cell was used either in the flume itself or outside, in a sample taken from the flume. The density of salt solution was precalibrated with electrical

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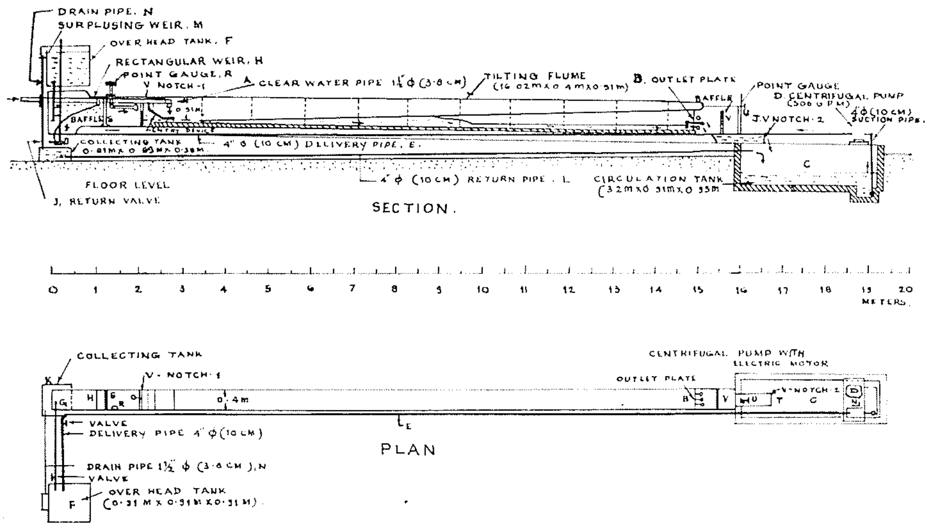
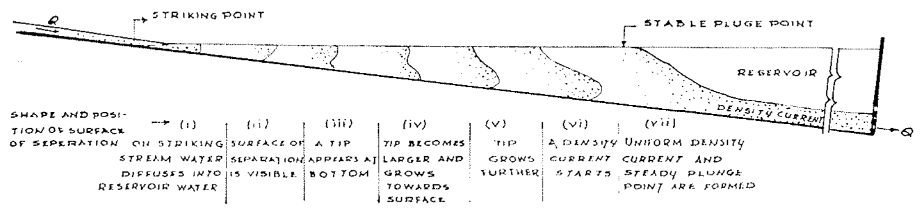
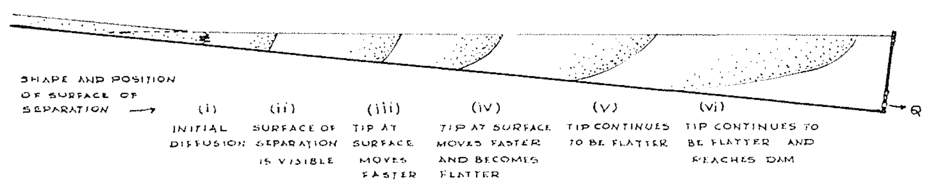


2/ Experimental set-up of main flume for study of plunging phenomenon.



3/ Observed phenomenon for formation of a stable plunge point.



4/ Striking of stream water (coloured) with reservoir water of same density and the resulting phenomenon.

resistance at different temperatures. To obtain velocity distribution curves point velocities were measured by a suspended bead device. The device consisted of a bead (density 1.41 gm/cc, dia = 0.78 cm) suspended from a 1 m long nylon thread. The bead was suspended in the flow at the desired depth, and the deflection of the nylon thread was measured on a horizontal scale fixed above the flow surface. The deflection was precalibrated with velocity by using suspended drops formed of two liquid chemicals P-Xylene (Sp. Gr. 0.86) and Di-Butyl pthalate (Sp. Gr. 1.047), adjusted to have practically the same density as the liquid layer in which the velocity was measured.

Development of the plunge point

The experimental study of plunging phenomenon included visual observations of the process of formation of the plunge point from the instant of introduction of stream water to the stage of formation of a uniform density current and a steady plunge point. After a steady plunge point was established, location of the plunge point, depth of water at the plunge point, inflow discharge and density and characteristics of uniform density current were measured and recorded.

The process of development of the plunge phenomenon as observed in the present experiments is shown diagrammatically in Figure 3. Immediately on striking, the salt solution flowing as a stream diffused in the reservoir water and no surface of separation was visible (Fig. 3 *i*). However, as more stream water continued to flow in, the reservoir water was pushed forward and in a short time, a surface of separation was visible as shown (Fig. 3 *ii*). The initial shape of the surface of separation looked like a half parabola with an apex at the surface. With further in-flow, the surface of separation continued to move forward; at the same time it started to get distorted. A tip appeared at the bottom (Fig. 3 *iii*) and continued to grow in size, to form a bulge moving ahead of its contact point at the surface (Fig. 3 *iv-v-vi*). At this stage, an incipient plunge point was seen but it was not steady and the plunge point as well as the bulge were moving forward. Finally, the bulge at the bottom was converted into a uniform density current and the plunge point became steady. Due to molecular diffusion at the interface and the escape of some reservoir water through outlets along with the density current, the density of salt water in circulation did not remain constant for more than about 30 minutes. All the observations had, therefore, to be completed within this time.

In some runs having a combination of low discharge and high density, the plunge was observed in two stages instead of one direct plunge.

To evaluate the influence of density in formation of the plunge, the striking of a stream of coloured water having the same density as the reservoir water was also studied. The visual observation is shown diagramatically in Figure 4. In this case, the surface of separation formed after initial diffusion, was parabolic with an apex at the surface. But instead of formation of a tip and bulging near the bottom, it continued to flatten near the surface till it reached the outlets.

The formation and stability of the plunge point was studied within the following range of variables:

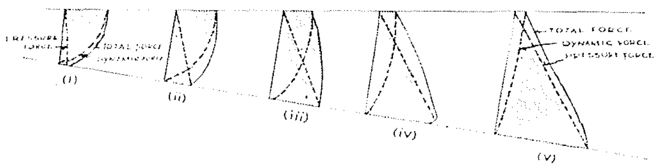
(*i*): Stream discharge of 0.5 cc/cm/sec to 135 cc/cm/sec;
(*ii*): Flume slope of 0.005 to 0.02;
(*iii*): Density difference of 0.0005 gm/cc to 0.013 gm/cc;
(*iv*): Density current depths of 1.5 cm to 17.0 cm;
(*v*): Plunge depth of 3.0 cm to 22.5 cm.

The physical concept of plunging

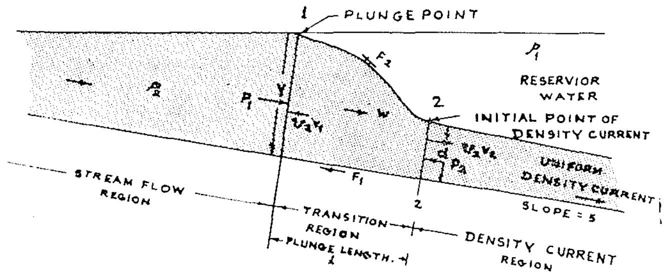
When the higher density stream water meets the relatively lighter and static reservoir water, it has got a higher density as well as a higher momentum. The resulting phenomenon can therefore be diffusion or displacement. Neglecting the lower order molecular diffusion, either eddy diffusion or displacement will be the deciding criterion. At the instant of striking, the stream water would have some level of turbulence which would be momentarily increased by the process of striking. It would seem to be very probable that this initial level of turbulence would have sufficiently large magnitude to cause eddies on either side of the surface of striking, to cross it, and thus to cause eddy diffusion as seen in Figure 3 *i*. However, as the stream water continues to flow in, the process of displacement also takes place. As the striking surface moves downstream, the depth of flow increases causing average velocity and hence fluctuations to decrease. A stage would soon be reached, when eddies would no longer be able to cross the striking surface and hence a surface of separation would be seen as in Figure 3 *ii*. This means that at this section the process of eddy diffusion has given way to the process of displacement. The surface of separation would thence-forth be simply pushed forward or distorted depending upon the forces acting.

The surface of separation would be acted upon by two driving forces and one resisting force. The driving forces are (*i*) the dynamic force exerted by the stream water due to its higher momentum and (*ii*) the static force due to pressure gradient across the surface of separation. The shear stresses along the surface of separation and the bed provide the resisting force. The net effect of these forces would determine the shape and movement of the surface of separation. Its stable position would be achieved when these forces (neglecting minor forces like air resistance, surface tension, etc.) are balanced.

The dynamic force at a point along the surface of separation would be proportional to mV_r, m being the mass flowing per unit time ($\rho_2 V_r$) and V_r the relative velocity of flow. Therefore it will vary as V_r^2 from surface to bottom. The pressure force at a point would be proportional to $(\rho_2 - \rho_1)\, gh$, h being measured from the surface. The magnitude of each driving force and the net effect of these forces at successive cross sections as envisaged by the authors is shown diagramatically in Figure 5. Initially, the dynamic force predominates over the pressure force on account of smaller depth and higher velocity. However, as the surface of separation moves downstream,



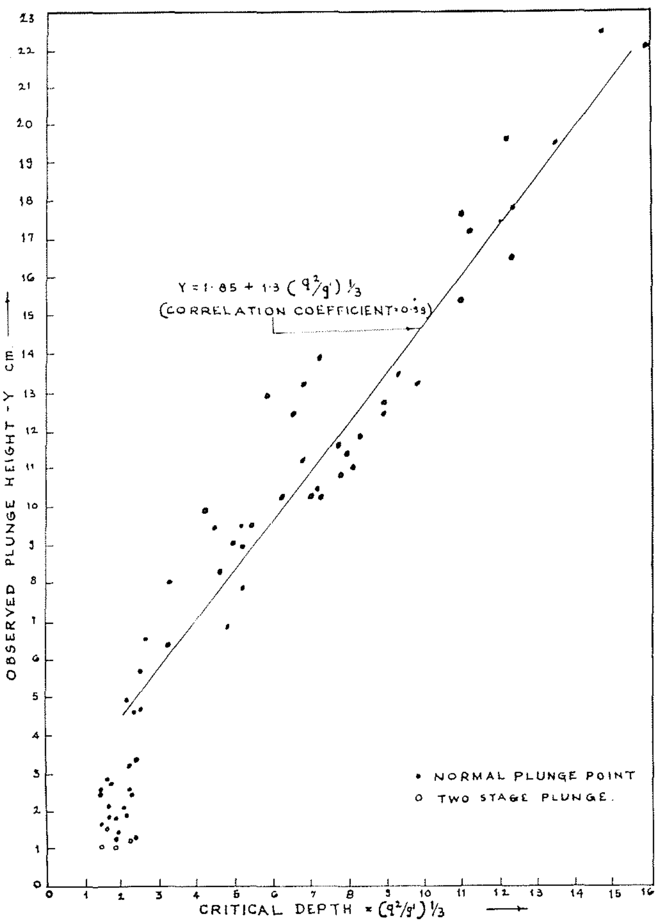
5/ Action of dynamic force and pressure force to form an underflow and a plunge point.



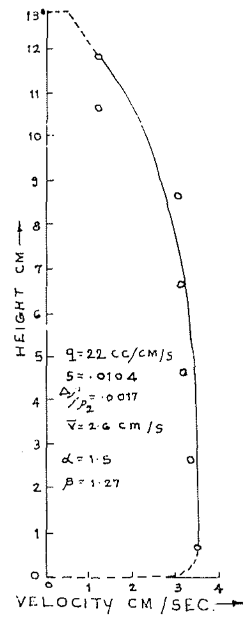
6/ Stability of plunge point.

the flow depth increases and the velocity of flow decreases. Therefore, the effect of dynamic force will go on reducing while that of pressure force will go on increasing. The pressure force will have zero magnitude at surface and will be maximum at the bottom. The dynamic force would have just the reverse trend, being zero at bottom and attaining a maximum value at the surface, following the velocity distribution curve. The relative movement of the flow at bottom with respect to that at the surface would therefore be faster. When the pressure force at bottom would become significant, it would cause a bottom flow in form of a tip as seen in Figure 3 *iii* or Figure 5 *iv*. As the pressure forces continue to grow most rapidly near the bottom, the tip becomes larger and larger and finally turns into an underflow or density current. Even when a density current has formed, the velocity at its surface would have a relatively small magnitude and the point of contact of the surface of separation with the free surface would still be moving forward till equilibrium is reached. This point on the surface would now be seen as a plunge point because of the density current flowing at the bottom.

Once a density current has started, the surface of separation would become an interface and the density current depth would be determined by discharge and density of flow, and bed and interfacial friction. The stability of the plunge point would then depend on the stable position of the interface, which in turn would depend on the balance of forces within the flow field, formed by the free surface and the interface as upper boundary and the bed as lower boundary. The position of the interface in the stable density current region is fixed with respect to bed independently on basis of the forces on the density current. Density current flow is in fact quite similar to flow in an open channel, and is established by the given discharge and channel characteristics. If sufficient length exists, a uniform density current will be formed representing a balance between the driving gravity forces and the resisting friction forces. Therefore, if the forces between section 1-1 at the plunge point and section 2-2 at the initial point of density current, as shown in Figure 6 are balanced, the interface location in the transition region between the two sections, would be fixed in position. This would mean that the plunge point would also be stable in position. At



7/ Plot of observed Y Vs critical depth $(q^2/g')^{1/3}$.



8/ Velocity distribution at plunge point: V.D.P.2.

this stage, the free surface upstream of the plunge point would be adjusted to a steady position by formation of a back-water curve.

If there was no density difference between the stream water and the reservoir water i.e. if there were no pressure forces, the movement of surface of separation would be caused by dynamic forces only. In this case, therefore, there would be no under-flow at the bottom and the flow will take place with maximum velocity at the surface as observed and shown in Figure 4.

Plunge depth relationships

The physical phenomenon of plunging studied above indicates that the plunge depth.

$$Y = f(q, g, \Delta\rho, \rho_2, \mu_2, S) \qquad (1)$$

where:

- q : discharge per unit width;
- $\Delta\rho$, ρ_2 : density difference and density of the density current respectively;
- μ_2 : dynamic viscosity of stream water;
- S : bed slope.

By dimensional analysis:

$$Y = F\,[(q^2/g)^{1/3},\ (\Delta\rho/\rho_2)^a,\ (\mu/q\rho_2)^b,\ S^c] \qquad (2)$$

As underflow problems are known to be governed by effective gravitational acceleration $g' = g\,\Delta\rho/\rho_2$, we can write:

$$Y = F\,[(q^2/g')^{1/3},\ R^b,\ S^c]$$

R being the Reynolds number of the density current. The parameter $(q^2/g')^{1/3}$ which is analogous to critical depth was considered as the most pertinent parameter. Plots of $Y/(q^2/g')^{1/3}$ against R and S separately were made and the variation was found negligible for the experimental range of R = 600 to 11,000 and S = 0.0056 to 0.0215. The Reynolds number and the slope do not seem to have significant influence on the plunge depth. A plot of Y *Vs.* $(q^2/g')^{1/3}$ was therefore made as shown in Figure 7. The observations of normal plunge points are shown by dots while those for two stage plunges are indicated by circles. For combinations of small q and large $\Delta\rho$ the plunge was seen in two stages. Such cases where $(q^2/g')^{1/3}$ was less than 2.0 cm were not considered for analysis, as they displayed distinct characteristics and are in any case outside the range of usual density currents. A straight-line fit by least mean square method gave:

$$Y = 1.85 + 1.3\,(q^2/g')^{1/3} \qquad (3)$$

in cm units.

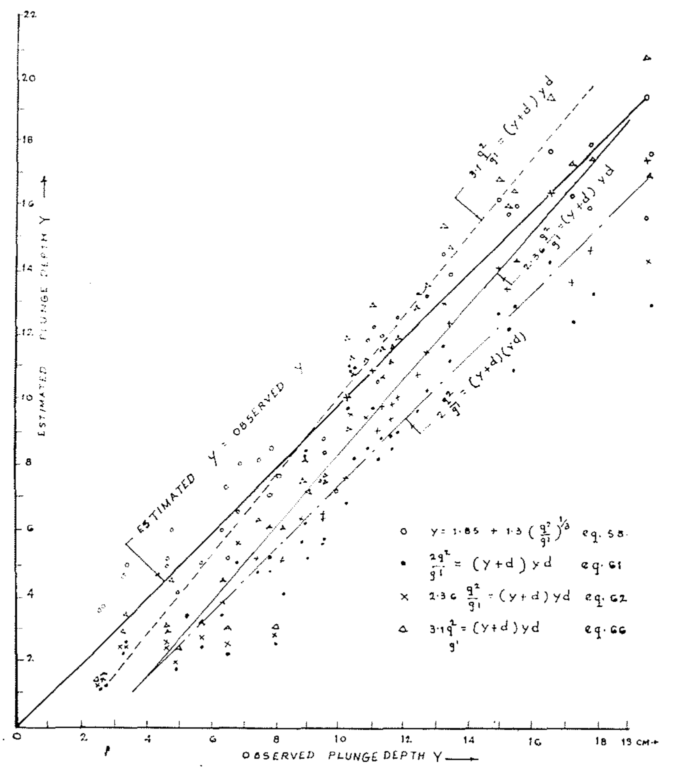
For large values of $(q^2/g')^{1/3}$, 1.85 becomes negligible and the equation can be written in dimensionless form:

$$Y/(q^2/g')^{1/3} = 1.3 \qquad (4)$$

Thus the plunge depth Y can be estimated approximately from the known values of q and ρ_2.

Referring to Figure 6, the rate of change of momentum between sections 1-1 and 2-2 will be balanced by the net force on the fluid mass between the two sections. That is:

$$q\rho_2\ (\beta_2 V_2 - \beta_1 V_1) = P_1 + W_s - P_2 - F_1 - F_2 \qquad (5)$$



9/ Comparison of estimated plunge depth with observed plunge depth.

In equation (5), the left hand side represents the rate of change of momentum, β_2 and β_1 being the momentum coefficients and V_2 and V_1 the mean velocities at the two sections respectively. P_1 and P_2 are the pressures at the two sections while F_1 and F_2 are the shear resistances at the bed and the interface. W_s is the tangential component of the weight of the fluid between the two sections.

The following assumptions are made:

(*i*) : Pressure distribution is hydrostatic at both sections;
(*ii*) : Flow direction at every point on sections 1-1 and 2-2 is parallel to the bed;
(*iii*) : Plunge length l and bed slope S are small so that total depth of water upto the reservoir surface at section 2-2 equals that at section 1-1 which is the plunge depth Y;
(*iv*) : Friction force F_2 at the interface acts parallel to the bed.

For simplified analysis F_1, F_2 and W_s can be neglected, as being small in magnitude in comparison to the other forces. On this basis;

$$q\rho_2 (\beta_2 V_2 - \beta_1 V_1) = \frac{\rho_2 g Y^2}{2} - \left(\frac{\rho_1 g Y^2}{2} + \frac{\rho_2 - \rho_1}{2} g d^2\right)$$

from which we get:

$$\frac{2q^2}{g'}\left(\frac{Y\beta_2 - d\beta_1}{Y - d}\right) = \left(Y + d\right)\left(Yd\right)$$

If β_1 and β_2 are assumed unity:

$$2q^2/g' = (Y + d)(Yd) \qquad (6)$$

which is a typical equation for conjugate depths of a submerged hydraulic jump. The phenomenon, however, differs from a hydraulic jump in so far as there is appreciable loss of head in the expanding jet of the latter while the plunging results in contracting flow with head loss due to friction only. The application of the momentum principle is, of course, equally pertinent in both cases.

The value of β_1 was computed from actual measured velocity distribution curves. A typical velocity distribution curve at a plunge point is shown in Figure 8. An average value of $\beta = 1.18$ was computed.

If:

$$\beta_1 = \beta_2 = 1.18$$

$$2.36\, q^2/g' = (Y + d)\, Yd \qquad (7)$$

The friction forces F_1 and F_2 would be difficult to define unless the resistance law for the interfacial friction as well as the geometry of the interface are known. In the absence of such knowledge, the analysis can be attempted on basis of experimental data. Taking F_1 and F_2 into consideration, neglecting W_s and assuming:

$$\beta_1 = \beta_2 = 1$$

the momentum equation can be written as:

$$q\rho_2 (V_2 - V_1) = P_1 - P_2 - F_1 - F_2$$

Writing:

$$F_1 = \frac{f_1 \rho_2 l}{8}\left(\frac{V_1 + V_2}{2}\right)$$

and:

$$F_2 = \frac{f_2 \rho_2 l}{8}\left(\frac{V_1 + V_2}{2}\right)$$

f_1 and f_2 being bed and interface resistance coefficients respectively, and readjusting:

$$\frac{2q^2}{g'} = \left(Y + d\right)\left(Yd\right) - \frac{q^2}{g'}\left(\frac{f_1 + f_2}{16}\, l\, \frac{(Y + d)^2}{Yd\,(Y - d)}\right)$$

The total resistance coefficient:

$$f = f_1 + f_2$$

was found to have an average value $= 0.13$ for uniform density currents. The average plunge length, l, was equal to $30\,(Y - d)$ from experimental observations.

Now:

$$\frac{(Y + d)^2}{Yd} = \frac{Y}{d} + 2 + \frac{d}{Y}$$

and will have a value of 4 to 4.5 for the experimental variation of $Y/d = 1$ to 2. An average value of this parameter was taken as 4.3 for the experimental data.

With these substitutions, the expression:

$$\frac{f_1 + f_2}{16}\; \frac{l}{Y - d}\; \frac{(Y + d)^2}{Yd}$$

will have a value of 1.1.

Therefore the momentum equation becomes:

$$3.1\,\frac{q^2}{g'} = (Y + d)(Yd) \qquad (6)$$

A plot of observed plunge depth Y and estimated plunge depths from equations 3, 6, 7 and 8 is given in Figure 9. A line at 45° to X axis is drawn to represent complete agreement between observed and estimated plunge depths.

As Equation (3) is derived from observed points, it coincides witt the 45° line and no separate line is shown for Equation (3). The improvement offerred by momentum coefficient and friction forces can be seen by corresponding lines coming closer to the 45° line.

Discussions of the results

The plunging phenomenon thus explains the basic mechanism of formation of density current. For a given elevation of the free surface of reservoir, the position of the plunge point would remain fixed for a given discharge and density of the stream water. However if the free surface elevation is itself changing, as would be the case when the outflow discharge is different from the inflow stream discharge, the position of the steady plunge point would move till the required plunge depth is available. A plunge point is possible for a very small difference of density, and a stable density current does not seem to require any minimum density difference. But in some cases, the plunge depth required for steady plunge point and density current formation would be so large that, for a given reservoir depth, it may not be available. In such cases the stream flow would go on mixing with the reservoir water rather than form a density current. Therefore, even though there is no minimum density for formation of density current on basis of the above treatment, the maximum available reservoir water depth does, in fact, impose a limit on minimum density difference $\Delta\rho$ for forming a stable density current and a plunge point. For such steady plunge point and density current depth, the equations 3, 6, 7 and 8 can be used, to give approximate values of plunge depth Y. Actually, there is appreciable deposition from the inflowing stream water upto the time it reaches the plunge point and the estimate can be improved if a correlation between inflow stream density and density at the plunge point can be obtained.

The plunging phenomenon also explains the formation of an alluvial cone in the bed of the reservoir. The flow will be expanding from the striking point to the plunge point and hence the deposition process would continue upto the plunge point. Immediately after a steady plunge point, a uniform density current will be formed and a stable concentration of suspended load would be maintained by the velocity of uniform density current. Thus deposition will go on increasing upto the plunge point, and then stop, thus forming an alluvial cone under the plunge point. Thus the plunge point is responsible for the formation of an alluvial cone rather than the usually accepted reverse theory viz that deposition in the alluvial cone causes a plunging of the stream inflow. If the latter were true, there should have been no plunging in experiments with brine, in which, of course, there is no deposition and no alluvial cone formation.

Field observations

Published field data of sufficient accuracy and detail to enable check of the proposed plunge depth equations, is limited. However, the existence of "plunging" in reservoirs where density currents form, is well established.

In case of Lake Mead in the U.S.A. turbid outflow through outlets (i.e. formation of density currents) was observed [1] when low discharges were combined with high densities, which would mean small values of q^2/g'.

At Treska reservoir in Yugoslavia, it was observed [2] for a flood of 9.5 m^3/s, that the silt concentration at the plunge point was 0.3 gm/lit. The critical depth $(q^2/g')^{1/3}$ for a concentration of 0.3 gm/lit would be 2.8 m. The estimated plunge depth as per Equation (4) would be $1.3 \times 2.8 = 3.64$ m against the actual observed plunge depth of 7.0 m. However, using Equation (8), and estimating the density current depth from the actual river bed slope of 0.05, and an assumed value of friction coefficient $f = 0.13$ in the equation;

$$q = \sqrt{\frac{8\,g'}{f}}\sqrt{dS}\;.\;d$$

the estimated plunge depth comes to 5.1 m. Considering the uncertainties involved the estimated value is close enough to that actually observed.

Conclusions

The present study provides an explanation of the plunging phenomenon and a basis for an approximate estimate of the plunge depth. Knowing the slope and cross-section of the river valley under the reservoir, and the density and discharge of the incoming stream, the density current depth, *d*, can be estimated from the usual flow relationships. Equation (8) can then be used to compute the plunge depth Y. If this much water depth is available in the reservoir, well upstream of the dam to accomodate plunge length and subsequent uniform density current for some distance, a density current would form-otherwise not.

References

[1] Grover (N.C.) and Howard (C.S.). — "The passage of turbid water through Lake Mead". *Trans. A.S.C.E.*, vol. 103, 1938.
[2] Bata (G.L.) and Bogich (K.). — "Some Observations On Density Currents in the Laboratory and in the Field". I.A.H.R., Minnesota Convention, 1953.