

THE DRAG COEFFICIENT OF A STATIONARY SPHERE ON A BOUNDARY OF SIMILAR SPHERES (*)

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Introduction

Garde and Sethuraman [1] have experimentally developed a drag coefficient function for a sphere rolling down an inclined boundary of closely packed spheres, the whole being submerged in a quiescent fluid. In these experiments, the fluid was stationary relative to the boundary, whereas the sphere was moving relative to both boundary and fluid. The sphere was moving through a fluid that had neither ambient turbulence nor a velocity gradient normal to the boundary. Circulation was induced by the rolling of the sphere. Under these conditions, drag coefficients were substantially higher, for a given Reynolds number, than those for a sphere moving in an infinite fluid.

This paper describes the experimental determination of the drag coefficient function for a stationary sphere on a horizontal bed of closely packed identical spheres. The experiments were done in a water tunnel. In these experiments the fluid was not quiescent but was moving relative to both the sphere and the boundary. The flow displayed both ambient turbulence and a velocity gradient normal to the boundary. The sphere was stationary relative to the boundary, and not rolling. Hence, the circulation effects present resulted only from the velocity gradient in the approach flow, and not from rotation of the sphere.

(*) A contribution from the USDA Sedimentation Laboratory, Southern Branch, Soil and Water Conservation Research Division, Agricultural Research Service, U. S. Dept. of Agriculture, Oxford, Mississippi in cooperation with the Mississippi Agricultural Experiment Station and the University of Mississippi.

The purpose of this paper is to illustrate the difference between the drag coefficient functions for a sphere resting on a rough boundary and for a sphere rolling on a rough boundary. The drag coefficient data obtained by the author have been cited previously [2], in much less detail and in a different context, in a paper on the incipient motion of alluvial particles.

Definitions

Figure 1 shows the cases of a sphere settling in an infinite fluid, rolling down an inclined rough boundary, and resting on a rough horizontal boundary. The drag, lift, and submerged weight forces are F_D , F_L , and W , respectively, and u is the velocity of the fluid relative to the sphere. Both F_D and u are defined as acting through the center of gravity of the particle. This definition is slightly in error for a sphere on a boundary, because in this case the skin friction forces which make up part of the total drag are not symmetrically distributed over the sphere.

For a sphere at rest on a horizontal boundary, the so-called lift force component has been found by both Coleman [2] and Rao [3] to be acting downward at particle Reynolds numbers lower than about 100, and acting upward at higher Reynolds numbers. In the experiments described here, the drag force component was measured directly, and the lift component did not affect the determination of the drag coefficient.

The particle Reynolds number is defined as:

$$R = \frac{uD}{\nu} \quad (1)$$

and the drag coefficient is:

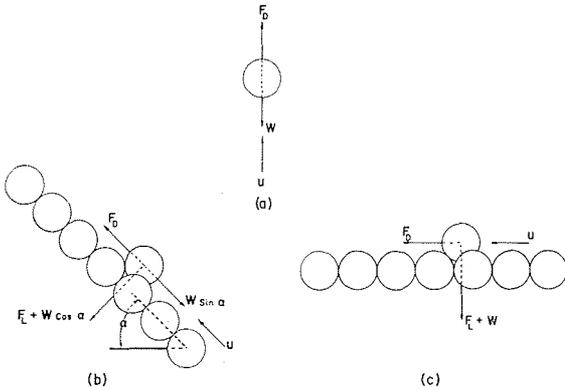
$$C_D = \frac{2 F_D}{\rho A u^2} \quad (2)$$

where D is the sphere diameter, A is the projected sphere area, and ρ and ν are, respectively, the density and kinematic viscosity of the surrounding fluid. In the data given by Garde and Sethuraman [1] for the sphere rolling down a rough boundary, C_D was corrected by the authors to remove the effect of rolling resistance.

Apparatus

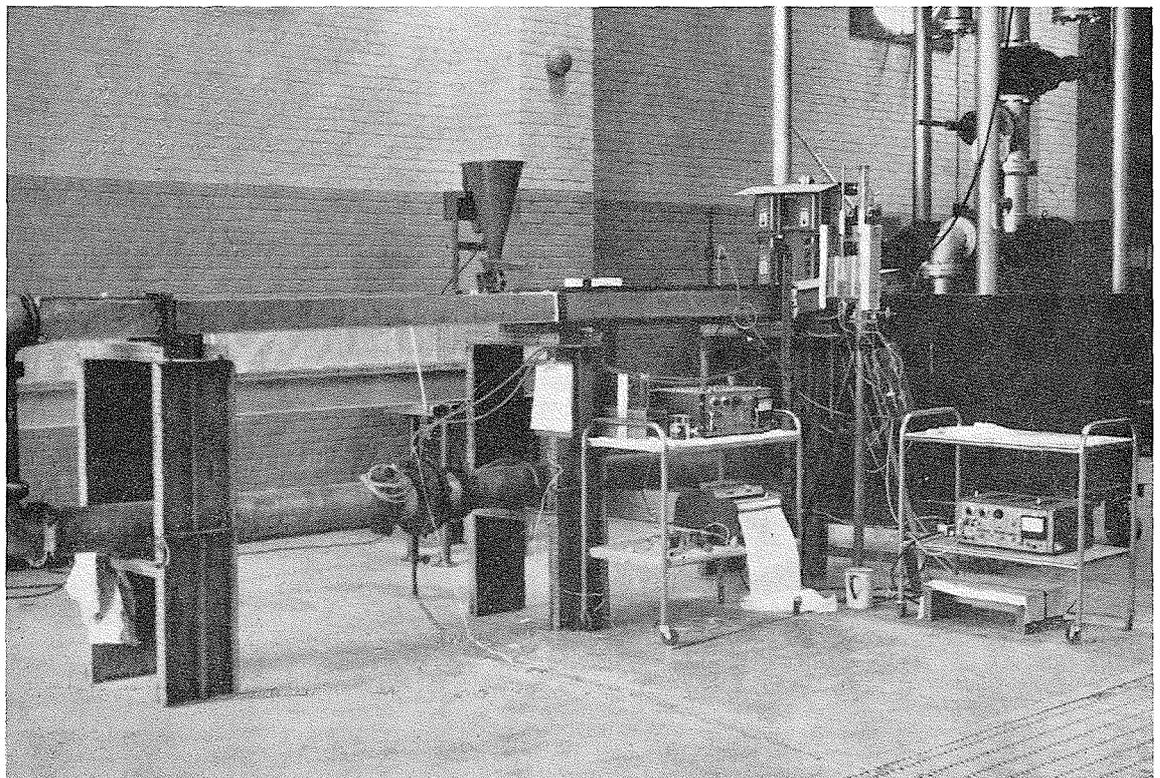
The water tunnel used for these experiments is shown in Figure 2. The test section of this tunnel was a rectangular plexiglass conduit 10 inches wide by 4-3/4 inches high. A bed of closely packed 1/2-inch plastic spheres was glued to the floor of the conduit. This bed was 3 feet, 8 inches long. The sphere on this bed, upon which drag force measurements were made, was a part of the drag force transducer shown in Figure 3. This transducer was located 8 inches from the downstream end of the fixed bed. Figure 4 is a view of the inside of the water tunnel with the transducer installed.

As shown in Figure 3, the upper ball on the transducer was attached to a brass rod extending through the transducer casing. This ball was raised slightly above its normal support points on the underlying balls so that the rod could deflect when a drag force was applied. Two semiconductor strain gauges, glued to the rod and wired in a half-bridge circuit, measured the drag force. The transducer was connected to a commercial strain indicator for reading out the drag force. The system was calibrated using a specially constructed balance to apply known forces to the transducer.

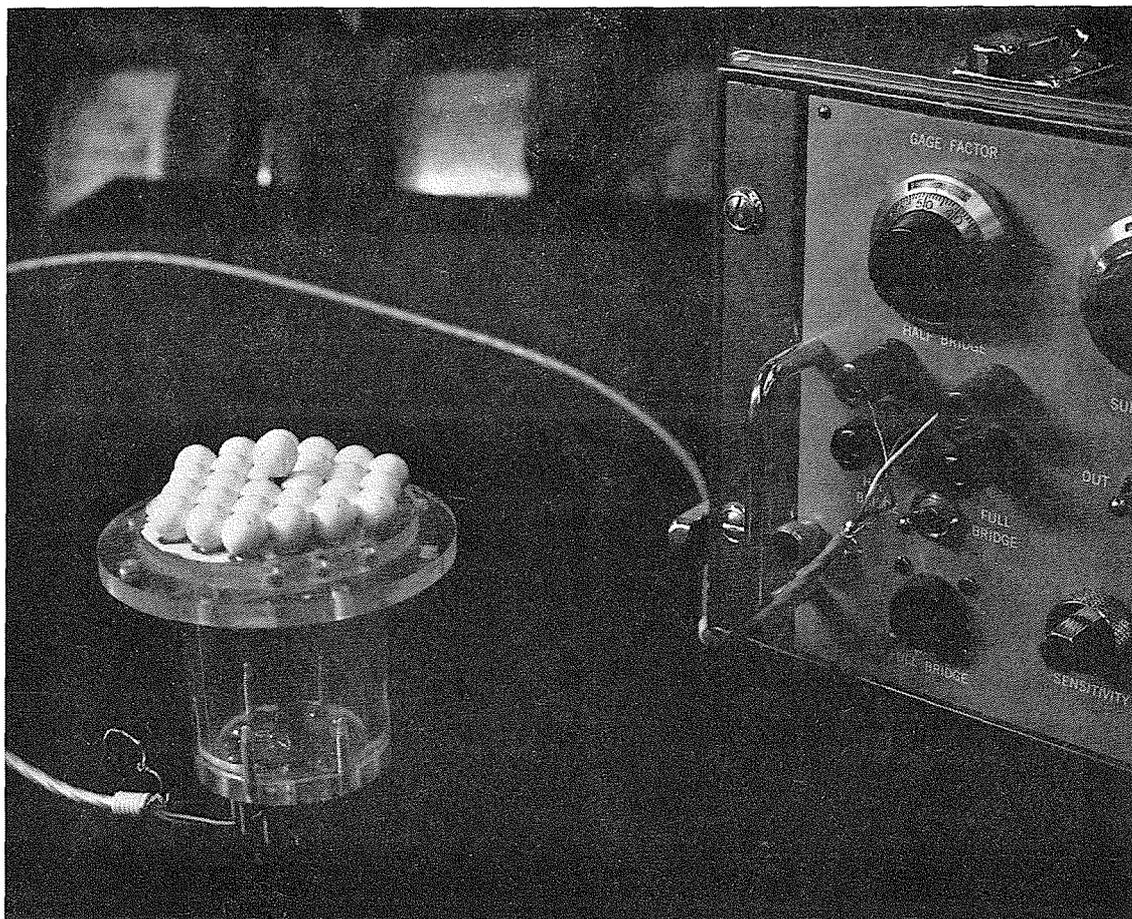


1/ The forces on a sphere, with rolling friction neglected in case (b).

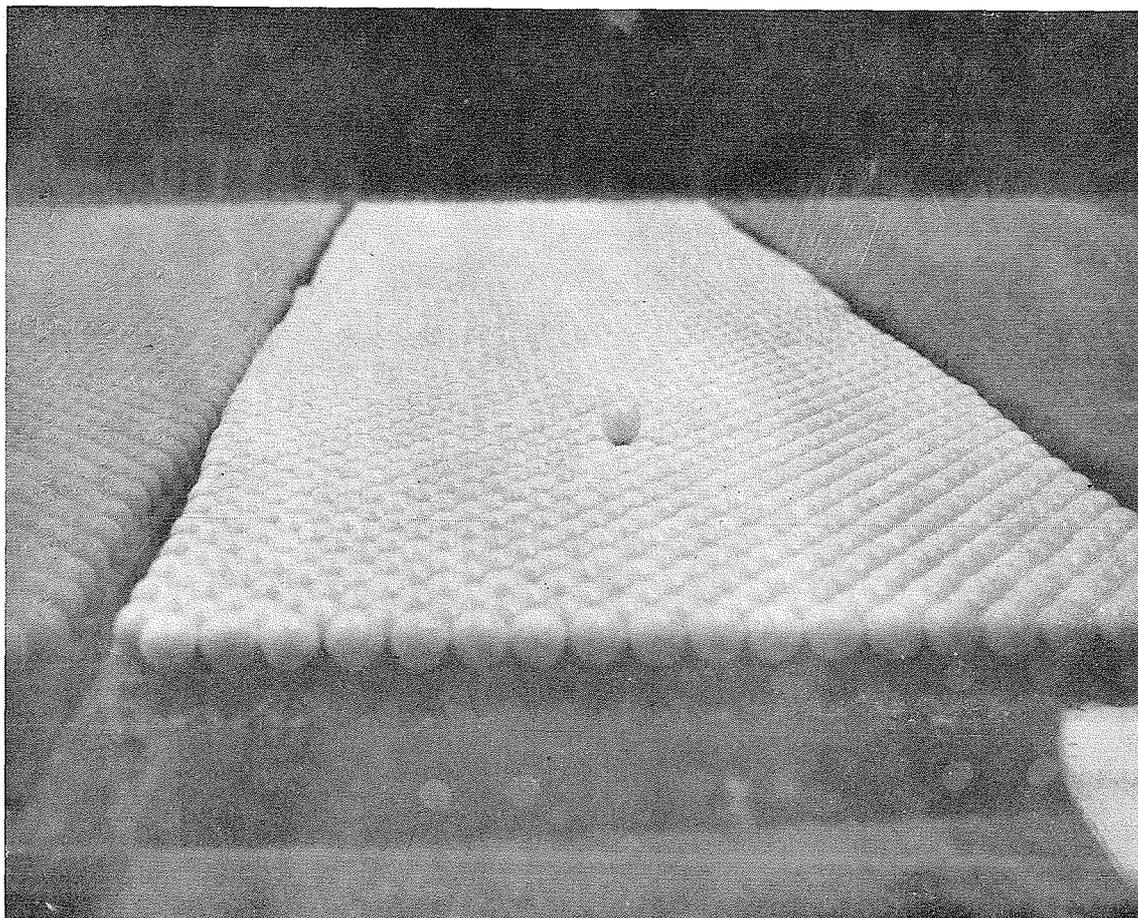
Les efforts sollicitant une sphère. Pour le cas (b), nous avons négligé les phénomènes de frottement au roulement.



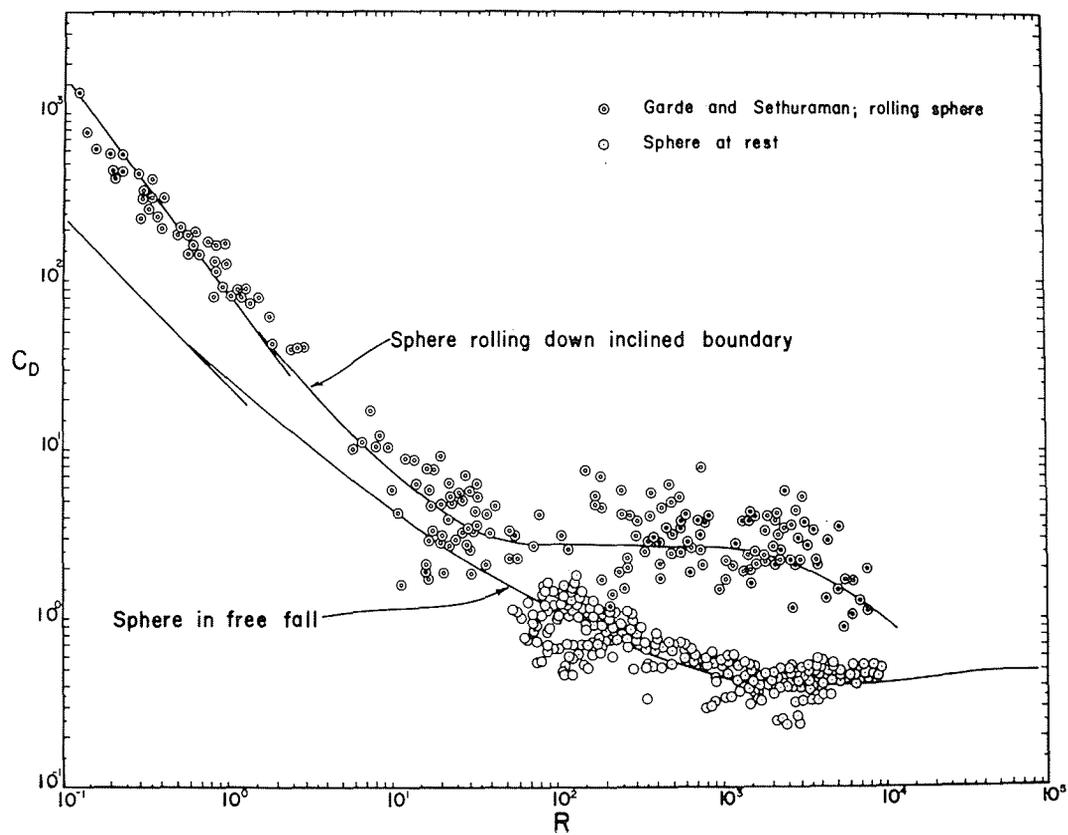
2/ The water tunnel.
Le tunnel hydrodynamique.



3/ The drag force transducer.
Sonde de mesure des efforts de traînée.



4/ The drag force transducer installed in the water tunnel.
Implantation de la sonde de mesure des efforts de traînée dans le tunnel hydrodynamique.



5/ Comparison of drag coefficients for rolling sphere, stationary sphere and falling sphere.

Comparaison des coefficients de traînée correspondant à une sphère en roulement, à une sphère fixe, et à une sphère en chute libre.

The velocity u was measured with a small total head tube and a piezometer tap located in a section of the tunnel just upstream from the ball on the transducer. The total head tube moved through a packing gland in the top of the conduit. It could be lowered to the elevation of the ball center for measuring the velocity u , and then retracted so as not to interfere with the flow during drag force measurements. The tube and the piezometer tap were connected to a differential pressure transducer and strain indicator.

The force and velocity readout devices were connected to a two-channel millimeter recorder.

The water tunnel was operated with either water or solutions of sodium carboxymethylcellulose, so that kinematic viscosities from 1.0×10^{-5} to 40×10^{-5} ft²/s were obtained. The apparatus provided a working range of particle Reynolds numbers from about 50 to 9,400, the lower limit being set by the sensitivity of the measuring equipment, and the upper limit by the capacity of the water tunnel.

Experimental procedure

The first step in performing experiments was to mix a solution of sodium carboxymethyl-cellulose. The temperature-viscosity relation for the fluid was found using a falling ball viscosimeter in a controlled temperature water bath. Fluid densities were measured with a Westphal balance.

Before each series of experiments with a given working fluid, the force transducer was calibrated and installed in the water tunnel. At the beginning of each experiment, the force and velocity indicators were balanced, and the recorder was started to record initial no-flow zero readings. The flow was then started and increased gently to some desired discharge, where it was held constant while a one-minute record of drag force was made. The total head tube was then introduced, and a one-minute velocity record was made. The temperature of the experimental fluid was also observed at this time. At the end of the velocity recording period the flow was stopped, and the recorder was allowed to record final no-flow zero values of force and velocity.

The above process was repeated three to five times for each selected discharge, beginning at the lowest flow for which force and velocity measurements could be obtained, and proceeding by increments up to the highest flow attainable. At the end of each experimental series, the working fluid was discarded, and a new fluid prepared.

Analysis and results

The observed data consisted of the flow velocity, drag force, and fluid temperature during each experiment, and the temperature-viscosity relation for each fluid.

The velocity and force records were analysed for the time-average velocity and force by measuring the areas under the respective chart traces with a planimeter, and

dividing the areas by the record length. The Reynolds number R and drag coefficient C_D were then calculated from equations (1) and (2).

Figure 5 contains the classical drag coefficient function as given by Rouse [4] for a sphere settling in a quiescent infinite fluid, the Garde-Sethuraman [1] data for a sphere rolling down a rough boundary in a quiescent fluid, and the water tunnel data for a stationary sphere on a rough boundary in a moving fluid. The drag coefficients for a rolling sphere follow a function higher than that for a sphere in free fall. In particular, the difference is greatest for Reynolds numbers from about 70 to 10,000. In contrast, the drag coefficients for a stationary sphere, as obtained from the water tunnel experiments, cluster about the free fall function, and this result occurs in the same Reynolds number range where the data for a rolling sphere shows the greatest deviation.

In the absence of detailed studies of the local flow patterns around spheres on rough boundaries, it is only possible to speculate that the marked difference between drag coefficient functions for a rolling sphere and a stationary sphere is a result of additional circulation induced around the rolling sphere. In view of the inherent asymmetry of the flow around a stationary sphere on a boundary, the close correspondence between the drag coefficient function for this case and for the sphere in free fall is remarkable.

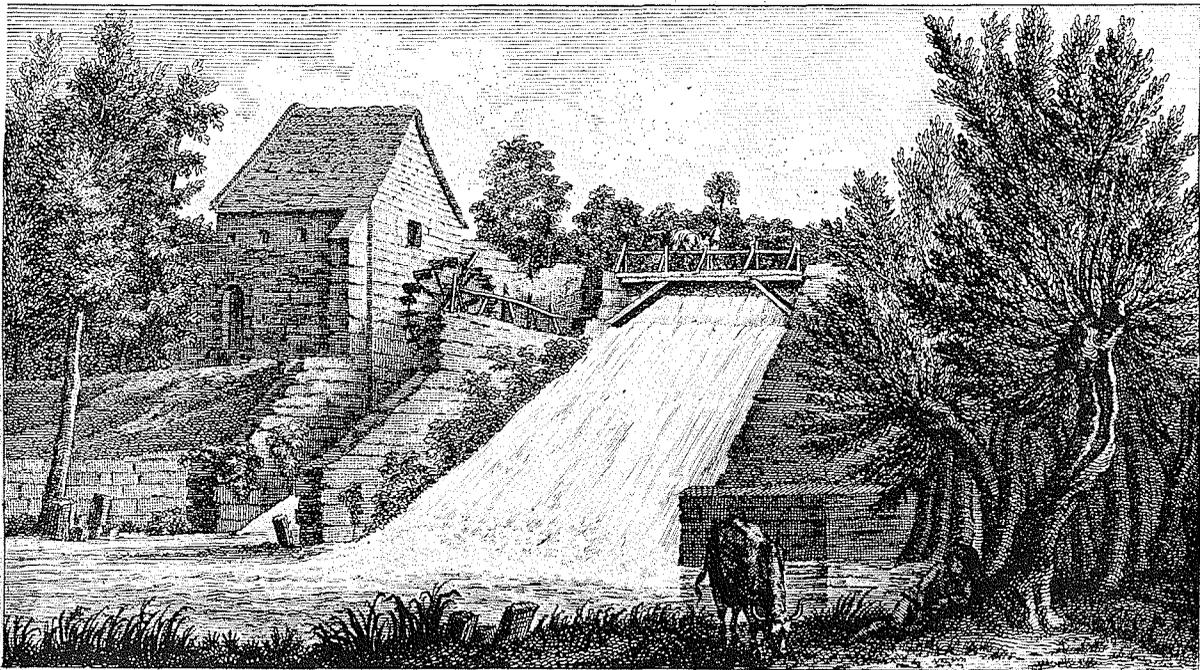
Conclusions

The drag coefficient function for a sphere rolling on a rough boundary has been defined by other experimenters [1]. It predicts drag coefficients that are higher than those predicted by the classical function for a sphere in free fall.

In contrast to the above findings, the water tunnel experiments reported here show that the drag coefficient function for a stationary sphere on a rough boundary tends to correspond with the function for a sphere in free fall.

References

- [1] GARDE (R.J.), and SETHURAMAN (S.). — Variation of the Drag Coefficient of a Sphere Rolling Along a Boundary. *La Houille Blanche*, n° 7 (1969), p. 727-732.
- [2] COLEMAN (N.L.). — A Theoretical and Experimental Study of Drag and Lift Forces Acting on a Sphere Resting on a Hypothetical Streambed. Proceedings of the Twelfth Congress of the International Association for Hydraulic Research, Fort Collins, Colo., (september 1967), vol. 3, p. 185-192.
- [3] RAO (M.V.P.). — Effects of Effluent and Influent Seepage on the Hydrodynamic Forces Acting on an Idealized Noncohesive Sediment Particle. Ph. D. Dissertation, Utah State University, Logan, Utah, (1969), p. 133.
- [4] ROUSE (H.). — Elementary Mechanics of Fluids New York: John Wiley and Sons, Inc., (1946), p. 376.



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