UPLIFT COMPUTATIONS FOR MASONRY DAMS

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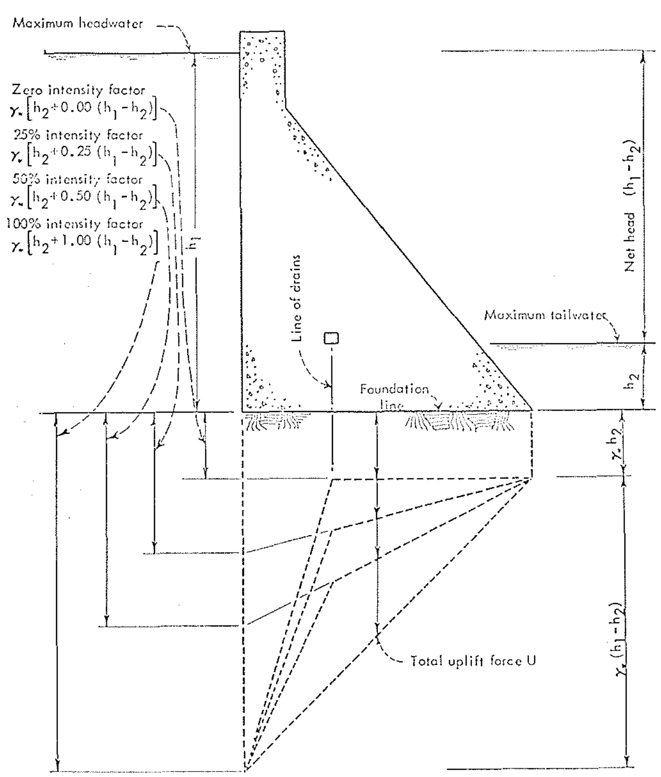
Introduction

In designing concrete dams, particularly of the gravity type shown in Figure 1, it is important to allow for uplift pressures. Such pressures are developed by water seeping through or under the dam, e.g., along preferential percolation planes which may correspond to interfaces between imperfectly bonded "lifts" or along the base of the structure. In such cases it may be assumed that the percolation planes are horizontal, even though within a perfectly homogenous, permeable structure of the same shape, seepage lines would actually trend downwards to some extent [1]*.

The importance of uplift in design is such that widespread measurements have been made in the past of pore pressures at the base of gravity dams, and the findings have been incorporated in various, more or less empirical design assumptions, of the general type illustrated in Fig. 1 taken from a Subcommittee Report to the A.S.C.E. [2]. In fact, two aspects of the design problem must be considered:

a) The effective area, considered as a proportion of the total base area of the structure, over which the uplift force is considered to act. In modern practice, the pore pressure is considered to be effective over the full base area [3], and this is assumed to be true here.

b) The estimation of the pore pressure, either using empirical assumptions referred to above, or more sophisticated methods, e.g., making full allowance for the spacing and diameter of the drains which are invariably incor-



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(*) Numerals in parentheses refer to corresponding items in Appendix II -References.

porated into such structures. This is the problem considered here.

In the present state of the art, as resumed by Davis and Sorensen [3], most reliance seems to be placed on the empirical approach, though approximations based on infiltration theory [4], or on computer studies using flow nets [5] have also been put forward.

The aim of the present paper is to show firstly that for a system of equally spaced drains of uniform diameter, an exact analytical solution based on Darcy's law of seepage is available. Under the stated assumptions the dam may be considered to be composed of a number of juxtaposed rectangular blocks with one vertical drain, at least, per block; see Figure 2. The lateral boundaries, AB and DC in Figure 2, often correspond to the limits of different pours, and may incorporate construction joints. From the standpoint of the theory to be developed, these limits are supposed to be impermeable (as in the case of an isolated block) or, what amounts to the same thing when dealing with an equally-spaced drain pattern in an otherwise homogeneous structure, are not supposed to constitute preferential drainage planes. If the permeability of the joints is greater than elsewhere, the present approach will give results on the safe side for uplift.

The theory developed on this basis takes into account the diameter and spacing of the drains. The analytical device used to obtain the solution is the well-known method of images, in which the desired boundary conditions, i.e. constant head or potential along the upstream and downstream seepage faces AB and BC in Figure 2, are obtained by setting up multiple mirror images of the real line of drains with respect to these faces. The only restriction on the method of images is that the drains themselves must be small in relation to the whole drainage area, a condition that is obviously satisfied in practice. A study, illustrated in Figure 3, has nevertheless been undertaken to check the validity of the approximation inherent in the theory.

Analysis of uplift pressure for a single drain

The analysis will be concerned with a single vertical drain, as typified by Figure 1 (in which the uplift pressure is plotted downwards below the base of the dam) or Figure 2 (showing uplift vertically upwards, on the assumption of zero tailwater pressure). Naturally, if there were additional drains on the axis of the rectangular dam section ABCD shown in Figure 2 the effect of these could be found by superposition.

Both in Figures 1 and 2, the mean uplift pressure acting across the dam has been drawn in as a linear function of distance measured from the upstream dam face, with a change in slope at the drain. More specifically, as stated in [3, 6]:

"The largest builders of dams in the United States, the U.S. Corps of Engineers, the U.S. Bureau of Reclamation and the Tennessee Valley Authority use similar assumptions... These are:

(*i*) a straight-line drop from reservoir level at the heel of the dam to a fraction of the difference in head between reservoir and tail water along the line of the drains, and

(*ii*) from there another straight-line drop to tailwater at the downstream toe.

The pressure is assumed to be effective over 100 % of the area of the base.

The value of that fraction now (1968) used by the Tennessee Valley Authority is 1/4 and that used by the Bureau of Reclamation is 1/3".

This convenient assumption of linearity requires justification. It may in fact be shown that, within the limitations already discussed of the method of images, the uplift distribution does obey conditions (*i*) and (*ii*) above rigorously.

In effect, the discharge $\mathfrak{q}$ per unit thickness infiltrating upwards across a given section in Figure 2 not intersecting the drain may be written in terms of Φ, the flow potential, as follows:

$$\mathfrak{q} = \int_{-n/2}^{+n/2} \frac{\partial \Phi}{\partial y}\, dx$$

$$= \frac{d}{dy} \left\{ \int_{-n/2}^{+n/2} \Phi\,(x,\ y)\, dx \right\}$$

Hence:

$$\int_{-n/2}^{+n/2} \Phi\,(x,\ y)\, dx = \mathfrak{q}y + \text{const.}$$

The mean value of Φ, given by $1/n$ times the integral on the left, is related to the average uplift pressure, as recalled below. The demonstration just given is a special case of Charny's theorem (see, for example, reference [7]).

The analysis now proceeds as follows:

Using Harr's sign convention [1], the complex potential ω due to a single drain in a semi-infinite domain equivalent to Figure 2 but with BC removed to infinity is:

$$\omega = \frac{\mathfrak{q}}{2\pi}\, ln\, \frac{\sin \pi\,(\mathbf{z} - is)/n}{\sin \pi\,(\mathbf{z} + is)/n} + \text{constant} \quad (1)$$

where, as shown on Figure 2, n denotes the width AD and s the distance between the centreline of the drain and AD. The discharge through unit height of the drain is $\mathfrak{q}$ and $\mathbf{z}$, the complex two-dimensional space variable, is equal to $(x + iy)$. The centre of coordinates for x, the abscissa, and y, the ordinate parallel to AB, is at the midpoint of AD. The complex potential:

$$\omega = \Phi + i\Psi$$

is such that, under the sign convention adopted:

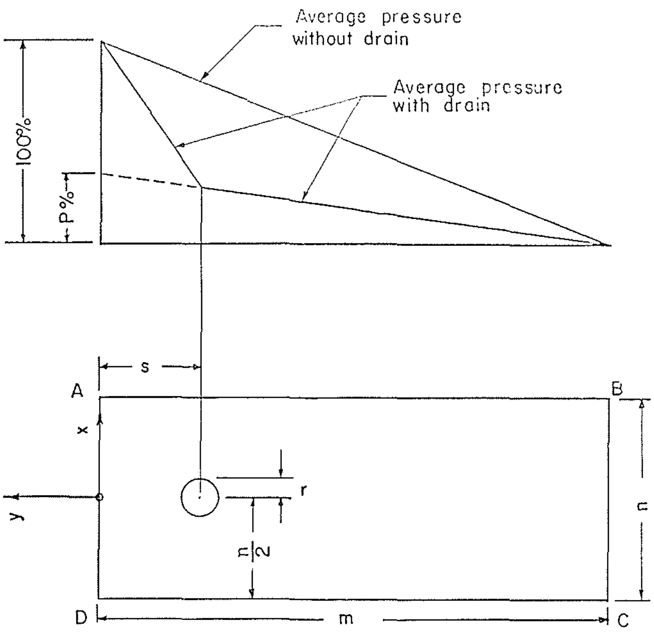
$$\Phi = -kh + \text{constant}$$

where k denotes the coefficient of permeability and h, the total head, is equal to the sum of:

$$\frac{p}{\gamma_w}$$

(where p in pounds per sq. ft. is the pressure at the given point) and of the height of the point above some arbitrary reference plane; the constant in the equation is also arbitrary, as are all similar constants in this analysis. This definition of total head assumes a negligible kinetic energy of flow. Finally, γ_w is the unit weight of water, in pounds per cu. ft.

The values of Ψ, the stream function, and of Φ, the potential function, are related through the formulae giving respectively



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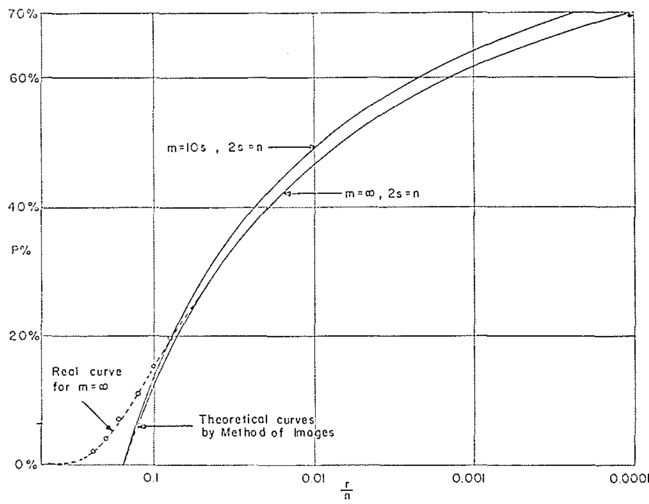

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(*i*) the x-component of seepage velocity as:

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \Psi}{\partial y}$$

(*ii*) the y-component of seepage velocity as:

$$\frac{\partial \Phi}{\partial y} = - \frac{\partial \Psi}{\partial x}$$

The method of images now enables the complex potential corresponding to the finite rectangle ABCD containing one drain as shown in Figure 2 to be readily found. The result is:

$$\omega = \frac{\mathbf{q}}{2\pi} ln \left| \frac{\ldots \sin \frac{\pi}{n} (\mathbf{z} - 2\,im - is) \sin \frac{\pi}{n} (\mathbf{z} - is) \sin \frac{\pi}{n} (\mathbf{z} + 2\,im - is) \ldots}{\ldots \sin \frac{\pi}{n} (\mathbf{z} - 2\,im + is) \sin \frac{\pi}{n} (\mathbf{z} + is) \sin \frac{\pi}{n} (\mathbf{z} + 2\,im + is) \ldots} \right| + \text{constant} \qquad (2)$$

where $m = \text{AB}$ or DC.

This expression ensures that:

a) AD and BC are lines of constant potential Φ, i.e., lines of constant pressure;

b) AB and DC are streamlines.

Without upsetting these conditions, a uniform flow may be added to (2), giving:

$$\omega = \frac{\mathbf{q}}{2\pi} ln \left| \frac{\ldots \sin \frac{\pi}{n} (\mathbf{z} - 2\,im - is) \sin \frac{\pi}{n} (\mathbf{z} - is) \sin \frac{\pi}{n} (\mathbf{z} + 2\,im - is) \ldots}{\ldots \sin \frac{\pi}{n} (\mathbf{z} - 2\,im + is) \sin \frac{\pi}{n} (\mathbf{z} + is) \sin \frac{\pi}{n} (\mathbf{z} + 2\,im + is) \ldots} \right| + i \frac{\mathbf{q}^*}{n} \mathbf{z} + \text{constant} \qquad (3)$$

where $\mathbf{q}^*$ is the added arbitrary flow entering AD and leaving BC, per unit height of block.

The further condition to be fulfilled relates to the pressure in the drain; this, for the particular problem with which we are concerned, is supposed to be equal to that at the downstream face of the dam BC. The pressure at the upstream face, AD, is of course equal to the hydrostatic pressure on the dam at that level.

Equation (3) gives in unsimplified form the pressures at the following points:

a) At the upstream face, e.g. at $\mathbf{z} = 0$:

$$\omega = \frac{\mathbf{q}}{2\pi} \, ln \left| \frac{\dots \sin\frac{\pi}{n}(-2im - is)\sin\frac{\pi}{n}(-is)\sin\frac{\pi}{n}(2im - is)\dots}{\dots \sin\frac{\pi}{n}(-2im + is)\sin\frac{\pi}{n}(is)\sin\frac{\pi}{n}(2im + is)\dots} \right| \qquad (4)$$

b) At the surface of the drain, supposing, as is required in the method of images, that r, the radius of the drain, is much smaller than m or s:

$$\omega = \frac{\mathbf{q}}{2\pi} \, ln \left| \frac{\dots \sin\frac{\pi}{n}(-2im - 2is)\sin\frac{\pi}{n}(-2is)\sin\frac{\pi}{n}(2im - 2is)\dots}{\dots \sin\frac{\pi}{n}(-2im)\sin\frac{\pi}{n}(ir)\sin\frac{\pi}{n}(2im)\dots} \right| + \frac{\mathbf{q}^*}{n} s \qquad (5)$$

c) At the downstream face, e.g., at $\mathbf{z} = -im$:

$$\omega = \frac{\mathbf{q}}{2\pi} \, ln \left| \frac{\dots \sin\frac{\pi}{n}(-3im - is)\sin\frac{\pi}{n}(-im - is)\sin\frac{\pi}{n}(im - is)\dots}{\dots \sin\frac{\pi}{n}(-3im + is)\sin\frac{\pi}{n}(-im + is)\sin\frac{\pi}{n}(im + is)\dots} \right| + \frac{\mathbf{q}^*}{n} m \qquad (6)$$

(In Equations (4), (5) and (6), the arbitrary constant appearing previously has been set equal to zero without loss of generality as far as the subsequent analysis is concerned).

The equality of pressures found from Equations (5) and (6) enables a relationship to be found giving $\mathbf{q}^*$ in terms of $\mathbf{q}$. In order to do this, it will be remembered that

$$\sin ia = i \sinh a \qquad (a \text{ any real number})$$

It is also convenient to write:

$$(\alpha - 1)\, ln \left| \frac{\sinh\frac{\pi}{n}(2s)}{\sinh\frac{\pi}{n} r} \right| = ln \left| \frac{\left(\cosh\frac{\pi}{n}4m - \cosh\frac{\pi}{n}4s\right)\left(\cosh\frac{\pi}{n}8m - \cosh\frac{\pi}{n}4s\right)\left(\cosh\frac{\pi}{n}12m - \cosh\frac{\pi}{n}4s\right)\dots}{\left(\cosh\frac{\pi}{n}4m - 1\right)\left(\cosh\frac{\pi}{n}8m - 1\right)\left(\cosh\frac{\pi}{n}12m - 1\right)\dots} \right| \qquad (7)$$

(in practice, especially for $m \gg s$, α will differ little from unity).

The pressure relationship just mentioned is satisfied by equating the real parts of the two expressions for ω, i.e., by equating the potential functions Φ which are, as previously explained, related directly to the pressures. When this is done, it is found that:

$$\mathbf{q}^* = \frac{n}{m - s}\left[\frac{\alpha}{2\pi} \, ln \left| \frac{\sinh\frac{\pi}{n}2s}{\sinh\frac{\pi}{n} r} \right| - \frac{s}{n}\right] \mathbf{q} \qquad (8)$$

When Charny's theorem is applied in the present context, the following results are found:

a) Between the upstream face, $y = 0$, and the drain, $y = -s$, the mean value of the potential Φ measured across the rectangle is equal to;

$$-(\mathbf{q} + \mathbf{q}^*)\frac{y}{n} + \text{constant} \qquad (9)$$

b) Between the drain at $y = -s$ and the downstream face at $y = -m$, the mean value of the potential Φ is given by:

$$\mathbf{q}\frac{s}{n} - \mathbf{q}^*\frac{y}{n} + \text{(same) constant} \qquad (10)$$

It will be observed how simple the expression for the mean potential is, contrasting in this respect with the expression for the potential at any point, which involves as has been seen the logarithm of the ratio of two infinite products of hyperbolic functions.

The mean pressures may be derived directly from the potential functions given above; if the arbitrary constant in Equations (9) and (10) is chosen so that the pressure at the downstream face is nil (in actual fact, a tailwater pressure may have to be added as in Fig. 1), the results are as follows:

a) Between the upstream face at $y = 0$ and the drain at $y = -s$, the mean uplift pressure at any section y is given by:

$$\frac{\gamma_w}{k}\left\{\mathbf{q}\frac{s + y}{n} + \mathbf{q}^*\frac{m + y}{n}\right\} \qquad (11)$$

b) Between the drain at $y = -s$ and the downstream face at $y = -m$, the mean uplift pressure is given by:

$$\frac{\gamma_w}{k}\mathbf{q}^*\frac{m + y}{n} \qquad (12)$$

Though $\mathbf{q}^*$ can be replaced in equations (11) and (12) using expression (8), and although the problem is then solved knowing that the uplift pressure at $y = 0$, i.e.:

$$\frac{\gamma_w}{k} \left\{ q \frac{s}{n} + q^* \frac{m}{n} \right\}$$

is equal to the depth of water at that point of the upstream face of the dam, it is nevertheless far more convenient to develop formulae giving the intensity factor, P, defined on Figure 2 and illustrated on Figure 1 for various values. P is a measure of the relative efficiency of the drain in reducing uplift, and is measured by extending the lower 'leg' of the average uplift pressure line to the upstream face of the dam.

Expression for the intensity factor, P

From Equation (11), replacing y by 0 and $-s$ successively, it may be readily seen that:

$$P = \frac{q^* m/n}{q\, s/n + q^* m/n}$$

Making use of Equation (8), it is found that:

$$P = \frac{\alpha \dfrac{1}{2\pi} \ln \left| \dfrac{\sinh \frac{\pi}{n} 2s}{\sinh \frac{\pi}{n} r} \right| - \dfrac{s}{n}}{\alpha \dfrac{1}{2\pi} \ln \left| \dfrac{\sinh \frac{\pi}{n} 2s}{\sinh \frac{\pi}{n} r} \right| - \dfrac{s^2}{mn}} \tag{13}$$

Expression (13) is the basic result sought for. An important special case may be noted; this is the one for an infinitely distant downstream face, or what in practice amounts to the same thing, an impermeable downstream face well removed from the drain. In this case, $\alpha = 1$ and:

$$P = 1 - \frac{\dfrac{s}{n}}{\dfrac{1}{2\pi} \ln \left| \dfrac{\sinh \frac{\pi}{n} 2s}{\sinh \frac{\pi}{n} r} \right|} \tag{13 a}$$

On the assumption that $\alpha = 1$ (only rigorously satisfied for infinite m), values of P have been plotted on Figure 3 for one typical case, in which the drain lies on the bisector of each corner angle ($2s = n$). Whereas in actual fact the intensity factor will drop to zero only when the walls of the drain touch the sides of the rectangle, i.e., for $2r = n$, the method of images suggests a zero value for a lower value of r than this known limit; this is of course because of the approximate nature of the method of images for large drains, as explained earlier. The real curve which has been traced on Figure 3 for the case of infinite m was found by using an electrical analogy.

The value of α defined by Equation (7) will generally be close to 1 and this has been assumed in computing the curve for $m = 10s$ on Figure 3. A departure of any consequence from $\alpha = 1$ should occur only for relatively small values of the ratio m/s, which are hardly relevant in the present context. Moreover, values of uplift computed with $\alpha = 1$ rather than the real value (which will be smaller than 1, except for infinite m) will be slightly conservative; the real value of P will be slightly lower than the estimate.

It is interesting to compare the theoretical result for this particular ratio of $s/n = 0.5$ to simplified assumptions of the type illustrated on Figure 1; the 25 % intensity factor is used in fact by the TVA [6] (*). The results of the comparison are as follows:

For $s = 0.5\, n$:

$P = 25\ \%$ corresponds to $r = 0.055\, n$
$P = 50\ \%$ corresponds to $r = 0.009\, n$

The minimum drain diameter required to suit any particular uplift assumption is thus precisely determined, at least for the given position of the drain.

Approximate analysis of the effect of a displacement of the drain

Though Formula (13) giving P is valid for any geometry, provided of course the drain is reasonably small in size, it is not very helpful in assessing without bother the effect of changing the location of the drain, i.e., of increasing or decreasing the distance s in a drained rectangular block ABCD of given size.

On the assumption that m is sufficiently large for Equation (13 a) to be a valid approximation this can be done by differentiating this equation with respect to s. Assuming in addition that the ratio s/n is sufficiently large for:

$$\tanh \frac{\pi}{n} (2s) \simeq 1$$

and also that $s \gg r$, it is found that:

$$\ln \left| \frac{\sinh \frac{\pi}{n} (2s)}{\sinh \frac{\pi}{n} r} \right| \left(\frac{\partial P}{\partial s} \right)_{r,n} \simeq -\frac{2\pi}{n} P$$

By a reverse procedure, this can be integrated to yield the following approximate result:

$$P \left(\frac{2\pi s}{n} - \ln \frac{2\pi}{n} r \right) = \text{constant} \tag{14}$$

once substitutions such as:

$$\ln \left| \sinh \frac{\pi}{n} r \right| \simeq \ln \frac{\pi}{n} r$$

have been made. Clearly, as s is increased, P is reduced, other things being equal (the efficiency of the drain is improved).

As an illustration of the use of the formula, the case $n = 2s = 100r$ has been selected, and an estimate made of P for $n = s = 100r$.

(*) Actually 25 % × $m/(m - s)$ using P as defined here.

The exact value of P for the reference case, which is of course illustrated on Figure 3, is $P = 46.8\ \%$. Upon substitution into Equation (14), the value of $P = 30.6\ \%$ is found with the drain shifted downstream to $s = 100\, r$; this also happens to be the exact value, found from Equation (13).

Although the efficiency of the drain does increase as the distance s between it and the upstream face is made greater, the stability of the structure as a whole will not necessarily be improved; there is the added uplift on the enlarged section of concrete between the dam face and the drain to be considered. There is bound to be an optimum distance s for any given sizes of drain and dam block.

Conclusion

A formula, Equation (13), has been derived for the intensity factor P giving the mean uplift pressure across the base of a gravity dam at the location of the line of drains. Knowing the value of P, the mean uplift pressure can be computed using the linear relationship for mean pressures between the line of drains and heel and toe of the dam respectively.

The analysis could be readily extended by superposition to multiple drain systems, or to the case in which the pressure within the drains happens to differ from that at the toe of the dam. An extension to the case of a impermeable downstream face, e.g., as the result of a special finish applied to this face, is also immediate.

It will be noted that the expression found for the intensity factor P takes account of the diameter of the drain, $2\, r$, as well as of the other geometrical data relating to drain position within the dam. It has been shown that the value of P may vary considerably as these parameters are changed. It therefore seems preferable to use the exact theory rather than blanket rules-of-thumb which, despite their wide acceptance amongst dam designers, do not take account of such parameters. The relative simplicity of the formula for P will enable this to be done readily in any practical case.

Acknowledgements

This problem was first brought to the writer's attention by Raoul Sabljak and Jean-Paul Beaudry, at that time on the staff of the consulting firm of Geo. Demers, Montreal; their solution involved either the use of a digital computer [6], or an electrical analogy designed and used by J.-P. Beaudry to demonstrate the linearity of the mean pressure curve empirically. Without this empirical demonstration, or friendly prodding on their part, the present writer's investigation would not have been undertaken. The electrical analogy measurements for Figure 3 were undertaken by G. Neveu, a research assistant at McGill University.

Appendix 1 — Notation

The following symbols are used in this paper:

h = total head in ft;
$i = \sqrt{-1}$;
k = coefficient of permeability in ft. per sec;
m, n = dimensions of dam block (see Fig. 2);
p = pressure at a point in lb. per sq. ft.;
P = intensity factor (see Figs. 1 and 2);
$\mathbf{q}$ = discharge per unit height of drain, in cfs per ft.;
$\mathbf{q}^*$ = added seepage discharge, in cfs per ft.;
r = radius of drain in ft.;
x, y = coordinates within dam block (see Fig. 2);
$\mathbf{z} = x + iy$;
α = factor ($\alpha \leqslant 1$) defined by Equation (7);
γ_w = unit weight of water in lb. per cu. ft.;
Φ = potential function;
Ψ = stream function;
$\omega = \Phi + i\Psi$.

Mean value of potential function evaluated in the article:

$$\int_{-n/2}^{+n/2} \Phi(x, y)\, dx$$

Appendix 2 — References

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Résumé

Calcul des souspressions dans les barrages-poids

Lorsqu'on projette un barrage-poids (fig. 1), le calcul des souspressions est primordial. Ces souspressions résultent des infiltrations ayant lieu dans l'ouvrage; on peut admettre que ces infiltrations aient lieu dans des plans horizontaux, correspondant par exemple à des chemins préférentiels créés par des coulées successives lors de la construction, bien que dans un massif parfaitement homogène, ces chemins doivent s'incliner quelque peu vers le bas [1].

Deux procédés sont habituellement mis en œuvre pour soulager un barrage-poids des souspressions:

1) On injecte du ciment dans les fondations. Ce procédé ne s'applique évidemment qu'aux fondations et non pas au barrage même, et d'ailleurs on ne s'y fie pas trop, d'où l'importance du deuxième procédé :

2) On aménage un réseau de drains espacés régulièrement, près du parement amont. On s'attache ici à étudier l'efficacité d'une telle rangée de drains, schématisée à la figure 2 qui représente un plot typique d'un barrage-poids :

$2r$ désigne le diamètre du drain situé à une distance s du parement amont;

n et m désignent la largeur et la longueur respectivement du plot.

Les souspressions, qui sont censées agir, comme d'habitude, sur la pleine section horizontale, décroissent de 100 % de la pression amont (ou plus précisément de la différence entre pressions amont et aval si l'aval est noyé), à la face amont du plot AD, à 0 % à la face aval BC. Il s'agit de déterminer la variation de la souspression moyenne

$$\overline{p}(y) = \int_{-n/2}^{+n/2} p(x, y)\, dx$$

entre ces limites, où $p(x, y)$ désigne la souspression locale en un point (x, y).

Dans le passé on s'est contenté surtout de règles empiriques de calcul. Par exemple, dans deux grandes administrations aux Etats-Unis, on a supposé [3, 6].

— une diminution linéaire de la souspression moyenne entre le parement amont et la ligne des drains, où la valeur moyenne atteinte a été estimée soit à 1/3, soit à 1/4, de la valeur au parement amont, ceci indépendamment du diamètre des drains et de la position de ceux-ci;

— une diminution linéaire, mais évidemment bien atténuée, entre la ligne des drains et le parement aval.

C'est l'objet de cette publication de démontrer :

a) que l'hypothèse de l'atténuation linéaire, soit en amont, soit en aval de la ligne des drains, est justifiée;

b) que la valeur de la souspression moyenne au droit des drains est justiciable d'un calcul exact; ainsi la souspression moyenne peut être évaluée en toute section du barrage.

L'analyse théorique est basée sur le théorème de Tcharnyi, bien connue dans l'étude des puits en milieu perméable; voir par exemple A. Vibert, « Sur une démonstration rigoureuse des formules de Dupuit », Le *Génie Civil*, le 1er janvier 1954. Dans le cas présent, on applique le théorème de Tcharnyi dans un plan horizontal, et on trouve sans difficulté que

$$\frac{d\overline{p}}{dy} = \frac{\gamma_w}{kn}\, q(y)$$

où :

γ_w désigne le poids spécifique de l'eau;

k le coefficient de perméabilité;

$q(y)$ le débit d'infiltration par unité de hauteur du plot, évaluée sur toute la largeur de celui-ci (soit entre AB, DC dans la figure 2, à l'ordonnée y).

Or $q(y)$ ne varie pas, sauf au droit du drain, et il en découle la propriété de linéarité de $\overline{p}(y)$ en fonction de y.

L'expression donnant la valeur de $\overline{p}$ au droit du drain, est déduite ensuite à l'aide de la méthode des images. La définition donnée dans le texte fait appel à la notion d'efficacité du drain, $1 - P$; s'il n'y a pas de drain, $P = 1$, et pour un drain pouvant capter tout le débit dans un barrage d'épaisseur finie, $P = 0$ (un tel drain impliquerait une pression statique dans le drain inférieure à celle régnant au parement aval; or, dans la théorie présentée, on suppose comme d'habitude que le drain communique librement avec l'aval, et que les deux pressions, celle dans le drain et celle au droit du parement aval, sont égales). Dans ces conditions P sera comme l'indique la figure 2 (on prolonge la droite représentative des souspressions moyennes entre le parement aval et le drain jusqu'au parement amont), et à toutes fins utiles on peut écrire :

$$P = 1 - \frac{\dfrac{s}{n}}{\dfrac{1}{2\pi}\, ln \left| \dfrac{sh\, \dfrac{\pi}{n}\, 2s}{sh\, \dfrac{\pi}{n}\, r} \right|} \qquad (13\,a)$$

Dans cette expression, on a supposé que m est grand, soit qu'à toutes fins utiles la face aval est imperméable, soit que le suintement par ce parement est faible. Comme l'indique la figure 3, le fait de prendre m infini pêche légèrement dans le sens de la sécurité. D'autre part sur la même figure, qui donne la valeur de P % pour $2s = n$ en fonction de r/n, on a indiqué, d'après une analogie électrique, la manière dont la vraie valeur de P s'écarte de la formule (13). En effet pour un diamètre $2r$ du drain très important par rapport aux dimensions du plot, la théorie des images cesse d'être valable. Cependant, l'application pratique des formules (13) ou (13 *a*) n'en est pas affectée, car le diamètre véritable des drains qu'on utilise tombe bien dans le domaine de validité de la méthode des images.