UPLIFT COMPUTATIONS FOR HOLLOW GRAVITY DAMS

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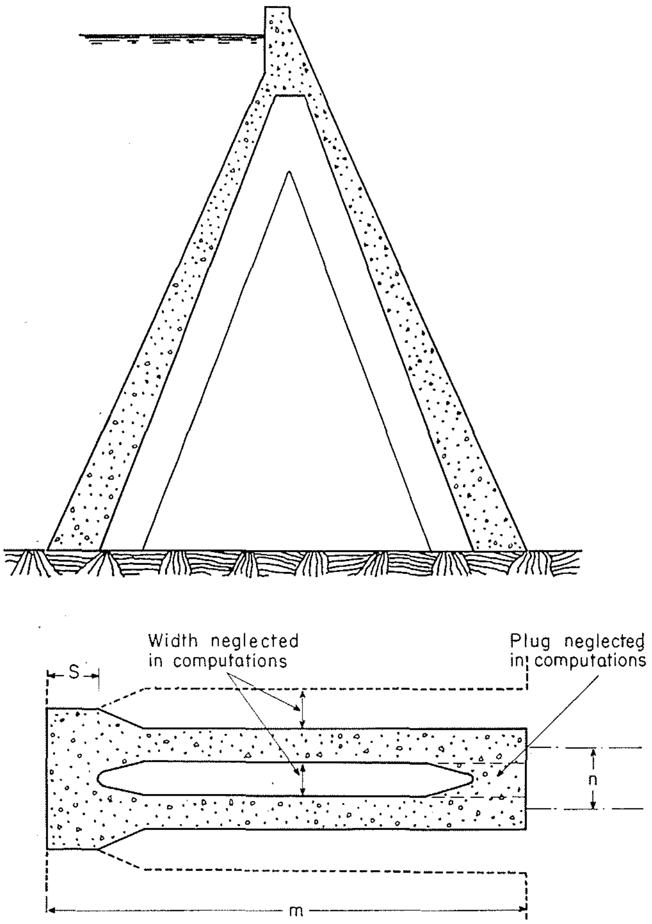
Introduction

Hollow gravity dams, of which Figure 1 shows an example reproduced schematically from a standard treatment of the subject [1] (*), have the advantage of greatly reducing uplift pressures due to water percolating through or under the structure. As is well known, such uplift pressures play a very important, adverse role in the stability of the more familiar, closed-joint type of gravity dam, and special means (drainage holes and galleries, and grouting) are employed to reduce these pressures. Naturally, by opening up the joints within the dam so that these joints serve in effect as highly efficient, elongated drains, the uplift can be reduced very markedly indeed. In many situations, the added complications in construction seem to be well repaid by so doing.

To date, however, no treatment of the uplift problem based on drainage theory has appeared; it is the purpose of the present article to fill this gap. In order to do so, certain simplifications in the somewhat complicated geometry of Figure 1 will be made. The first, obvious simplification is to assume that seepage takes place horizontally. Within the dam itself, this assumption is no doubt justified by the mode of construction by horizontal "lifts", with preferential percolation planes corresponding to imperfect bonding between lifts. A second simplification is in the geometry of the hollow joint; though presumed to be acting as a perfect drain despite its narrowness, the hollow joint will be assimilated to a line slit of negligible thickness, within which the pressure is assumed to be that of the downstream face with which the slit is connected directly. In other words, the basic geometry is assumed to be as shown in Figure 2, and the formulae will be derived for this layout, side BAH being the upstream face of the dam (subjected to full uplift pressure P_0) and CDEFG the downstream dam face and slit at zero uplift pressure. Should the downstream side of the dam be subjected in actual fact to tailwater pressure, this pressure may be taken as the reference datum, and the uplift pressures elsewhere are then evaluated with respect to this datum pressure in terms of the difference P_0/γ_w between upstream and downstream water levels.

The treatment is based on a theorem for mean uplift pressure used in a preceding publication [2], which assumed that the method of images was applicable, as indeed it is here. The method of images in the present instance would imply the setting up of an infinite two-dimensional array of slits throughout the entire z-plane (see Fig. 2) and the simulation of the relevant boundary conditions —either as lines of constant potential or as streamlines— over the rectangle BHGC by this means, but in actual fact this procedure does not have to be resorted to in order to make use of the theorem. The basic requirement is much simpler: the distribution of inflow by percolation through the rectangular block BHGC must be known as a function of the distance d measured down the slit. Providing that the slit is located on the centerline of the block, and also that the permeability k of the block is uniform

(*) Numerals in parentheses refer to corresponding items in Appendix 2: References.



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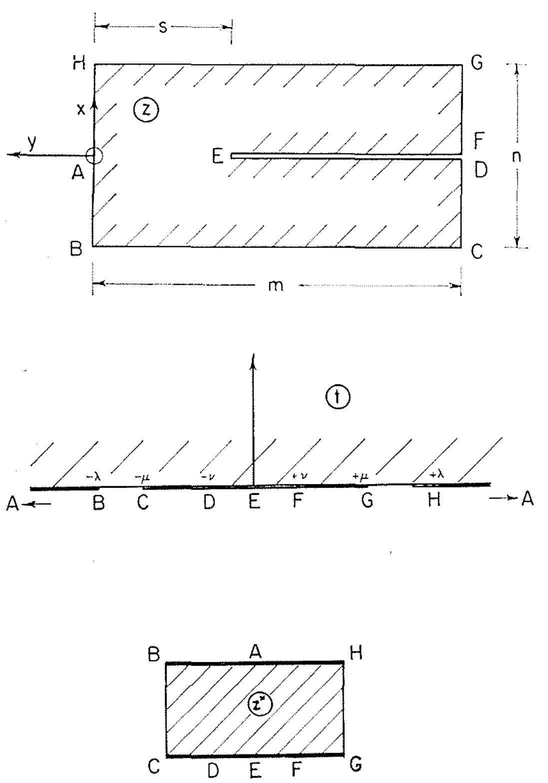

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(with Darcy's law of infiltration being obeyed throughout), the mean uplift pressure:

$$\bar{p}\,(y) = \frac{1}{n} \int_{-n/2}^{+n/2} p\,(x,\ y)\,dx$$

may then be determined, as will now be shown.

Uplift as a function of discharge through the slit

The mean uplift pressure $\bar{p}$ averaged across the width n of the rectangular dam block may be expressed very simply in terms of the discharge through a single, isolated drain [2]. The basic formula may be extended of course to the case of a slit, considered as a succession of such drains, and when put into differential form yields the following result :

$$\frac{d\bar{p}}{dy} = \frac{\gamma_w}{kn}\, q \qquad (1)$$

where, in this context, q is the total discharge percolating past the section considered, per unit height of the dam, k is the permeability, and γ_w is the unit weight of water. In writing this formula, it is to be understood that q is positive whereas y is negative (see Fig. 2). Thus p is a decreasing function of $(-y)$ as one goes from the upstream face BAH to the downstream face CDFG. The constants of integration required for determining $\bar{p}$ for any value of y are found:

(*i*) by setting $\bar{p} = 0$ on face CDFG, i.e. for $y = -m$;

(*ii*) by relating Q, the total discharge per unit height percolating through face BAH, i.e. for $y = 0$, to the known upstream pressure there, and thus eliminating the discharge from the uplift equation.

In the process, the determination of Q is made possible (though not of prime concern here). The problem thus reduces basically to one of evaluating q as a function of y at any section. In order to do this, two conformal transformations now to be described are required.

Conformal transformations for determining the discharge distribution through the slit

The use of flow nets for solving infiltration problems governed by Darcy's law is well established. Such flow nets may be described mathematically by functions of a complex variable, one flow net being transformed into another by the use of conformal transformations. If one or more conformal transformations may be found so as to yield a simple flow pattern, typically parallel flow (e.g. between boundaries BAH and CEG in the z^*-plane of Fig. 2), as the end product of their successive operation on an original, unknown flow pattern (such as that shown on the z-plane of Fig. 2 of concern here), then the latter has been in effect solved. In particular, if correspondence is established in this way between points on the z^*-plane, along EFG say, and points on the z-plane (down the slit EF and along the downstream face FG), then the flow distribution along EFG in the original z-plane has been determined. This follows from the observation that equal intercepts on EFG in the z^*-plane correspond to equal flows, or equal increments in the stream function to put it more precisely, in that plane and these equal increments are transferred under conformal mapping onto other planes (for brevity, only one half of the original figure, that of positive x, is referred to, but similar remarks apply to the symmetrical counterpart too).

Two conformal transformations are required to do this:

(*i*) The first brings the dam block and slit represented by ABCDEFGH in the z-plane of Figure 2 into correspondence with the upper half of the t-plane, on which the upstream face AH of the original figure is made to correspond to the real axis between infinity and $+\lambda$, and the slit EF and downstream face FG, to the real axis from 0 to $+\nu$, and from $+\nu$ to $+\mu$;

(*ii*) A second transformation brings the upper half of the t-plane into correspondence with the interior of the rectangle BHGC in the z^*-plane.

The link between the t-plane and the other two planes is provided by the well-known Schwartz-Christoffel transformation:

(I) The first transformation may be written:

$$\frac{dz}{dt} = \frac{a\sqrt{\lambda^2 - \nu^2}\, t}{\sqrt{(t^2 - \lambda^2)(t^2 - \mu^2)(t^2 - \nu^2)}} \qquad (2)$$

where $a\sqrt{\lambda^2 - \nu^2}$ is a convenient constant multiplier; λ, μ and ν are the abscissae corresponding to the transformed points H, G and F in the t-plane, as shown in Figure 2.

The integral of equation (2) is:

$$z = a\,\mathrm{F}\left(\mathrm{arsin}\sqrt{\frac{\lambda^2 - \nu^2}{t^2 - \nu^2}}, \sqrt{\frac{\mu^2 - \nu^2}{\lambda^2 - \nu^2}}\right)$$

for $t \geqslant \lambda$ (3 *a*)

or:

$$z = -ia\,\mathrm{F}\left(\mathrm{artan}\sqrt{\frac{\lambda^2 - \nu^2}{\nu^2 - t^2}}, \sqrt{\frac{\lambda^2 - \mu^2}{\lambda^2 - \nu^2}}\right)$$

for $t \leqslant \nu$ (3 *b*)

where F is the incomplete elliptic integral tabulated in standard texts, such as Jahnke and Emde [3] whose notation is also adopted here for the complete elliptic integral $\mathrm{K} = \mathrm{K}(k^*)$. In the present instance, it is convenient to write:

$$k^* = \sqrt{\frac{\mu^2 - \nu^2}{\lambda^2 - \nu^2}}$$

where k^* is the modulus; k', the complementary modulus, is given by:

$$k' = \sqrt{\frac{\lambda^2 - \mu^2}{\lambda^2 - \nu^2}}$$

It will be recalled that $\mathrm{K}' = \mathrm{K}'(k^*)$ is defined as follows:

$$\mathrm{K}'(k^*) = \mathrm{K}(k')$$

On putting $t = \lambda$ in equation (3 *a*), it is found at point H that:

$$n/2 = a\mathrm{K}(k^*) \qquad (4)$$

and, on putting $t = \nu$ in equation (3 *b*), that at point F:

$$m = a\mathrm{K}'(k^*) \qquad (5)$$

Moreover, on putting $t = 0$ in the same equation, it follows that, at point E:

$$s = a\,\mathrm{F}\left(\mathrm{artan}\sqrt{\frac{\lambda^2 - \nu^2}{\nu^2}}, k'\right) \qquad (6)$$

Knowing m, n and s from the geometry of the dam block slit, the ratios of the three parameters λ, μ and ν may readily be found from equations (4, 5 and 6). In practical applications, μ will generally be very nearly equal to ν, necessitating the use of an asymptotic formula for K′ in equation (5) rather than tabulated values.

(II) The second transformation may conveniently be expressed as:

$$\frac{dz^*}{dt} = \frac{b\lambda}{\sqrt{(t^2 - \mu^2)(t^2 - \lambda^2)}} \qquad (7)$$

where $b\lambda$ is a constant multiplier, giving:

$$z^* = b\,\mathrm{F}\left(\mathrm{arsin}\frac{t}{\mu}, \frac{\mu}{\lambda}\right)$$

for $t \leqslant \mu$ (8)

It will be noted that, at G:

$$(z^*)_{\mathrm{G}} = b\mathrm{K}\left(\frac{\mu}{\lambda}\right) \qquad (9)$$

and, at F:

$$(z^*)_{\mathrm{F}} = b\,\mathrm{F}\left(\mathrm{arsin}\frac{\nu}{\mu}, \frac{\mu}{\lambda}\right) \qquad (10)$$

As explained previously, the difference between z^*-plane abscissae actually determines the proportion of the total flow passing through the points concerned. For instance, the proportion of the total seepage flow reaching the downstream face FG as opposed to the slit EF is given by:

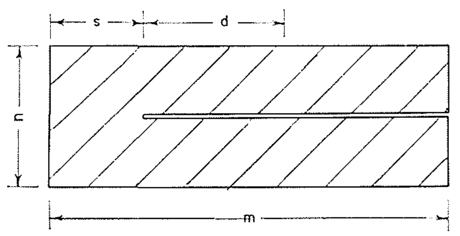
$$\frac{\mathrm{K}\left(\frac{\mu}{\lambda}\right) - \mathrm{F}\left(\mathrm{arsin}\frac{\nu}{\mu}, \frac{\mu}{\lambda}\right)}{\mathrm{K}\left(\frac{\mu}{\lambda}\right)} \qquad (11)$$

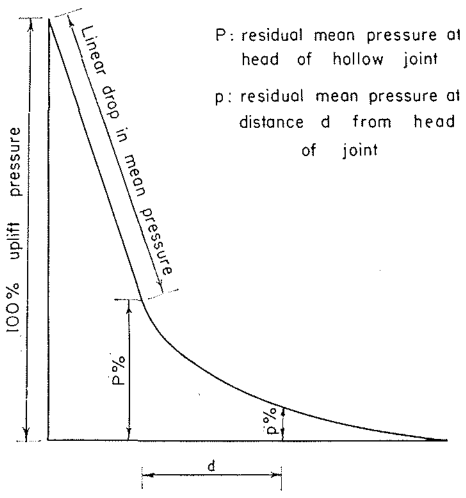
and has been listed in Table 1 for various slit arrangements, as determined by the ratios n/m and s/m. It should be mentioned in passing that many of the values chosen represent patently inadequate slit lengths and spacings; bearing this in mind, it is clear that for hollow joints of the general type represented in Figure 1, a very small proportion of the total percolating flow reaches the downstream face rather than the hollow joint. This, of course, was to be expected.

Table 1

Basic parameters for selected cases

$\frac{n}{m}$	$\frac{s}{m}$	$\frac{\lambda}{\nu}$	$\frac{\mu}{\nu}$	Residual flow through downstream face as a percentage of total infiltration flow, from Equation (11)
1.0	0.1	1.05056	1.00152	6.9 %
1.0	0.2	1.20805	1.00674	10.1 %
1.0	0.3	1.49126	1.01785	14.0 %
0.75	0.1	1.08921	1.00034	2.8 %
0.75	0.2	1.37304	1.00162	4.4 %
0.75	0.3	1.90366	1.00481	6.8 %
0.50	0.1	1.20398	1.00001	0.4 %
0.50	0.2	1.89916	1.00007	0.8 %
0.50	0.3	3.36943	1.00029	1.6 %
0.25	0.1	1.89909	1.00000	0.0 %
0.25	0.2	6.21313	1.00000	0.0 %
0.25	0.3	21.69954	1.00000	0.02 %





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Formula for the uplift pressure $\overline{p}$ in the general case

Expression 11 is only a special case of a general formula giving q, the discharge seeping past a given location, say $y = -r$, on the slit towards the downstream side, as already mentioned:

$$q = Q\left[1 - \frac{F\left(\arcsin\frac{t_r}{\mu}, \frac{\mu}{\lambda}\right)}{K\left(\frac{\mu}{\lambda}\right)}\right] \qquad (12)$$

where, of course, from equation 3 b:

$$r = a\,F\left(\operatorname{artan}\sqrt{\frac{\lambda^2 - \nu^2}{\nu^2 - t_r^2}}, k'\right) \qquad (13)$$

with $r = (s + d) \geqslant s$, as in Figure 3. It will be convenient to abridge the formulae by putting:

$$F(t)^* = F\left(\arcsin\frac{t}{\mu}, \frac{\mu}{\lambda}\right) \qquad (14)$$

and:

$$K^* = K\left(\frac{\mu}{\lambda}\right)$$

Then, according to equation (1):

$$\frac{d\overline{p}}{dy} = \frac{Q\gamma_w}{kn}\left[1 - \frac{F(t)^*}{K^*}\right] \qquad (15)$$

for $|y| \geqslant s$

giving:

$$\overline{p} = \frac{Q\gamma_w}{kn}\int_A^{\nu}\left[1 - \frac{F(t)^*}{K^*}\right] \times \frac{a\sqrt{\lambda^2 - \nu^2}\,t\,dt}{\sqrt{(\lambda^2 - t^2)(\mu^2 - t^2)(\nu^2 - t^2)}} \qquad (16)$$

seeing that:

$$\frac{dy}{dt} = \frac{1}{i}\frac{dz}{dt} = -\frac{a\sqrt{\lambda^2 - \nu^2}\,t}{\sqrt{(\lambda^2 - t^2)(\mu^2 - t^2)(\nu^2 - t^2)}}$$

and that ν, the upper bound of integration in equation (16), corresponds in fact to section FG, where $\overline{p} = 0$. The lower bound A is found from equation (13) giving r as a function of the corresponding value t_r, and involves the elliptic function sn (the inverse function of the incomplete elliptic integral F):

$$A = \sqrt{\lambda^2 - \frac{\lambda^2 - \nu^2}{sn^2(r/a, k')}} \qquad (17)$$

with $r = (s + d) \geqslant s$

The average uplift pressure $\overline{p} = \overline{P}$ at the cross-section through point E, at the head of the slit, is given by:

$$\overline{P} = \frac{Q\gamma_w}{kn}\int_0^{\nu}\left[1 - \frac{F(t)^*}{K^*}\right] \times \frac{a\sqrt{\lambda^2 - \nu^2}\;t\,dt}{\sqrt{(\lambda^2 - t^2)(\mu^2 - t^2)(\nu^2 - t^2)}} \qquad (18)$$

seeing that $t = 0$ there (see Fig. 3). It follows from equation (1) that the uplift pressure P_0 at the upstream face BAH is equal to:

$$P_0 = \frac{Q\gamma_w}{kn} s + \overline{P} \tag{19}$$

The mean uplift pressure $\overline{p}$ at any section:

$$y = -r = -(s + d)$$

with $d \geqslant 0$ may be found as a fraction of the pressure P_0 acting at the upstream face upon division of equation (16) by equation (19), with consequent elimination of Q. Recalling from equation (6) that:

$$\frac{s}{a} = F\left(\operatorname{artan}\sqrt{\frac{\lambda^2 - \nu^2}{\nu^2}},\ k'\right)$$

the final result may be expressed as:

$$\frac{\overline{p}}{P_0} = \frac{\displaystyle\int_A^\nu \left[1 - \frac{F(t)^*}{K^*}\right] \frac{t\,dt}{\sqrt{(\lambda^2 - t^2)(\mu^2 - t^2)(\nu^2 - t^2)}}}{\dfrac{F\left(\operatorname{artan}\sqrt{\dfrac{\lambda^2 - \nu^2}{\nu^2}},\ k\right)}{\sqrt{\lambda^2 - \nu^2}} + \displaystyle\int_0^\nu \left[1 - \frac{F(t)^*}{K^*}\right] \frac{t\,dt}{\sqrt{(\lambda^2 - t^2)(\mu^2 - t^2)(\nu^2 - t^2)}}} \tag{20}$$

valid for $d \geqslant 0$; see equation (17) for the definition of A.

For values of y between 0 and $-s$ (where $d = 0$), the uplift varies linearly with distance between P_0 at the upstream face given by equation (19) and the value $\overline{P}$ at $y = -s$, given by equation (18). As already explained, equations (4, 5 and 6) enable the ratios of the parameters λ, μ and ν to be found, together with the value of the constant a, from a knowledge of the dimensions m, n and s.

Formula for the mean uplift pressure $\overline{p}$ for the special case of infinite m

If the rectangular dam block is extended indefinitely in the $(-y)$ direction, it is clear that the resulting uplift pressures calculated within the range $0 < |y| < m$ will exceed those corresponding to the exact formula without this extension. The error thus made is likely to be small, though, in view of the rapid reduction in seepage discharge at appreciable distances from the head of the slit at $y = -s$; see Table 1. Hence if there is any advantage to be gained in doing so, there is hardly any practical objection to using such an approximation, rather than the exact formula found above.

In fact, this procedure is to be recommended on account of predictable difficulties in endeavouring to integrate the improper integrals in equation (20), with two zeroes of the denominator very close to one another in practice, at $t = \nu$ and $t = \mu$; Table 1 shows that these parameters may be so close as to be hardly distinguishable.

The case of infinite m is simply derived by putting $\nu = \mu$, say, giving:

$$\mu = \lambda \operatorname{sech}\frac{\pi s}{n} \tag{21}$$

This result follows from the observation that equation (3 a) is now replaced by:

$$z = a \operatorname{arcot}\sqrt{\frac{t^2 - \lambda^2}{\lambda^2 - \mu^2}}$$

valid for $t \geqslant \lambda$ (22 a)

and equation (3 b) by:

$$z = -ia \operatorname{arcoth}\sqrt{\frac{\lambda^2 - t^2}{\lambda^2 - \mu^2}}$$

valid for $t \leqslant \mu$ (22 b)

It follows too that the expression:

$$\frac{F\left(\operatorname{artan}\sqrt{\dfrac{\lambda^2 - \nu^2}{\nu^2}},\ k'\right)}{\sqrt{\lambda^2 - \nu^2}}$$

in the denominator of equation (20) may be replaced by:

$$\frac{1}{\mu} \cdot \frac{s\pi}{n} \operatorname{cosech}\frac{s\pi}{n},$$

seeing that now $k' = 1$.

The resulting expression for the mean uplift pressure at a section $y = -(s + d)$ is therefore as follows:

$$\frac{\bar{p}}{P_0} = \frac{\displaystyle\int_{A^*}^{\mu} \left\{ 1 - \frac{F(t)^*}{K^*} \right\} \frac{t\,dt}{(\mu^2 - t^2)\sqrt{\lambda^2 - t^2}}}{\displaystyle\frac{1}{\mu}\frac{s\pi}{n}\operatorname{cosech}\frac{s\pi}{n} + \int_{0}^{\mu} \left\{ 1 - \frac{F(t)^*}{K^*} \right\} \frac{t\,dt}{(\mu^2 - t^2)\sqrt{\lambda^2 - t^2}}} \qquad (23)$$

valid for $d \geqslant 0$

where $\mu/\lambda = \operatorname{sech} \pi s/n$ from equation (21) and:

$$A^* = \mu \frac{\sqrt{\operatorname{sech}^2 \frac{\pi s}{n} - \operatorname{sech}^2 \frac{\pi}{n}(s + d)}}{\operatorname{sech}\frac{\pi s}{n} \tanh \frac{\pi}{n}(s + d)}$$

For integration a more convenient form of equation (23) is:

$$\frac{\bar{p}}{P_0} = \frac{\displaystyle\int_{d}^{\infty} \left\{ 1 - \frac{F(v)^{**}}{K(\operatorname{sech} \pi s/n)} \right\} dv}{\displaystyle s + \int_{0}^{\infty} \left\{ 1 - \frac{F(v)^{**}}{K(\operatorname{sech} \pi s/n)} \right\} dv} \qquad (24)$$

valid for $d \geqslant 0$

where v is a dummy variable replacing d under the integral sign, and:

$$F(v)^{**} = F\left(\operatorname{arsin} \frac{\sqrt{\operatorname{sech}^2 \frac{\pi s}{n} - \operatorname{sech}^2 \frac{\pi}{n}(s + v)}}{\operatorname{sech}\frac{\pi s}{n} \tanh \frac{\pi}{n}(s + v)},\ \operatorname{sech}\frac{\pi s}{n} \right) \qquad (25)$$

This may be derived directly from equations (1, 12, 21 and 22 *b*). Substitution of:

$$u = \operatorname{sech} \frac{\pi}{n}(s + v) \qquad (26)$$

and integration by parts using arsin u as a temporary variable of integration is still required to convert the integrals in equation (24) into finite form suitable for numerical integration.

Mean uplift pressure distribution for case of infinite m, *given as a percentage of uplift at upstream face* **Table 2**

Ratio $\frac{s}{n}$	$\bar{P}$, uplift at head of slit (%)	$\bar{p}$, uplift down slit: $d = 0.5\frac{s}{n}$ (%)	$d = \frac{s}{n}$ (%)	$d = 1.5\frac{s}{n}$ (%)	$d = 2\frac{s}{n}$ (%)	$d = 2.5\frac{s}{n}$ (%)	$d = 3\frac{s}{n}$ (%)
0.1	67.5	55.3	46.3	39.0	33.1	28.1	23.9
0.2	52.1	36.0	25.8	18.7	13.6	9.9	7.2
0.3	42.3	24.8	15.2	9.5	5.9	3.7	2.3
0.4	35.5	17.7	9.3	5.0	2.6	1.4	0.8
0.5	30.6	13.0	5.9	2.7	1.2	0.55	0.25
0.75	22.7	6.5	2.0	0.6	0.2	0.05	0.01
1.0	18.1	3.5	0.7	0.15	0.03	—	—
1.5	11.9	1.1	0.1	0.01	—	—	—
2.0	9.2	0.4	0.02	—	—	—	—
3.0	6.3	0.06	—	—	—	—	—

Note. — Mean uplift pressure varies linearly between upstream face of dam and head of slit.

Results

The mean uplift pressure $\bar{p}$, or more strictly, $\bar{p}$ as a percentage of P_0, as found from equation (24) is tabulated in Table II for various values of the ratio (s/n) fixing the geometry of the slit.

Between the upstream face (with 100 % uplift pressure P_0) and the head of the slit at E, where the mean uplift is $\bar{P}$, the average uplift pressure varies linearly. Beyond this point, percentages of residual mean uplift $\bar{p}$ at sections:

$$d = s/2;\ s;\ 3/2\,s;\ 2\,s;\ 5/2\,s \text{ and } 3\,s$$

have been computed and are given in the Table. It will be noted that the mean uplift rapidly becomes negligible beyond the head of the slit.

Conclusion

Formulae have been developed giving the average uplift pressures required for design at any section of a hollow gravity dam. In order to do so, the real finite cavity in each dam block has been replaced by a narrow slit; however, the real pressures must be less than those so calculated.

It will be noted that:

(*a*) the mean uplift pressure varies linearly from the dam face to the head of the slit;

(*b*) at the head of the slit (supposedly connecting to zero tailwater pressure), the residual mean uplift will usually be a small fraction of the full value at the upstream dam face; see Table II;

(*c*) in practical cases, it can be assumed that the dam block extends to an unbounded distance beyond the head of the slit, seeing that uplift dies away rapidly beyond this point, thus simplifying the computations.

Acknowledgement

The computing required for the preparation of Tables I and II was done with the aid of funds from the Department of Civil Engineering and Applied Mechanics on the McGill computer.

Appendix 1: Notations

The following symbols are used in this paper:

a = constant of proportionality in equation (2), in ft. units;
A = lower bound of integration defined in equation (17);
A^* = lower bound of integration defined in equation (23);
b = constant of proportionality in equation (7), in ft. units;
d = distance measured from head of slit; see Figure 3;
$F(t)^*$ = F (arsin t/μ, μ/λ) where F denotes the incomplete elliptic integral of the first kind;
$F(v)^{**}$ is another version of the same integral defined in equation (25);
$i = \sqrt{-1}$;
k = coefficient of permeability in ft. per sec.;
$k' = \sqrt{\dfrac{\lambda^2 - \mu^2}{\lambda^2 - \nu^2}}$ complementary modulus of an elliptic integral;
$k^* = \sqrt{\dfrac{\mu^2 - \nu^2}{\lambda^2 - \nu^2}}$ modulus of an elliptic integral (written as k in standard texts);
$K(k^*)$ = complete elliptic integral of the first kind, with modulus k^*;
$K'(k^*)$ = complete elliptic integral of the first kind, with modulus k';
$K^* = K(\mu/\lambda)$;
m, n = overall dimensions of a dam block in ft; see Figure 2;
p = uplift pressure at a point in lb. per sq. ft.; function of x and y;
$\bar{p}$ = average uplift pressure at a section; function of y only;
$\bar{P}$ = value of $\bar{p}$ at head of slit in a dam block;
P_0 = value of uplift at upstream face of dam in lb per sq. ft.;
q = total discharge per dam block percolating past a given section per unit height of dam, in ft^2 per sec.;
Q = value of q at upstream face;
$r = s + d$;
s = distance between upstream face of dam and head of slit, in ft.;
t = complex variable relating to t-plane; see Figure 2;
u = substituted variable in equation (26);
v = dummy variable in equation (24);
x, y = coordinates in z-plane (plane of dam block), in ft.;
$z = x + iy$;
z^* = complex variable relating to z^*-plane; see Figure 2;
γ_w = unit weight of water in lb. per cu. ft.;
λ, μ, ν = abscissae in t-plane; see Figure 2.

Appendix 2: References

[1] DAVIS (C. V.) and SORENSEN (K. E.). — Handbook of Applied Hydraulics, *McGraw-Hill Book Co.*, New-York (1969).

[2] RANSFORD (G.D.). — Uplift Computations in Masonry Dams, *La Houille Blanche*, n° 1/1972, p. 65-71.

[3] JAHNKE, EMDE, LOSCH. — Tables of Higher Functions, 6th Edn., *McGraw-Hill Book Co.*, New-York (1960).

(*Voir le résumé en français à la page suivante*)

Geoffrey D. RANSFORD

Résumé

Calcul des souspressions dans les barrages-poids évidés

On projette des barrages-poids évidés de plus en plus souvent dans le but notamment de soulager l'ouvrage des souspressions associées avec des barrages-poids classiques, qui ont fait l'objet d'un article précédent [3].

Un barrage-poids évidé est illustré sur la figure 1; les évidements, espacés régulièrement le long du barrage, communiquent avec l'aval.

Dans les calculs de stabilité, on a besoin de connaître non pas la souspression $p(x, y)$ en chaque point d'un plot du barrage (fig. 2), mais plutôt la souspression moyenne :

$$\overline{p} = \int_{-n/2}^{+n/2} p(x, y)\, dx$$

où n désigne la largeur du plot (plus précisément, l'entr'axe des entailles successives dans le barrage (fig. 1). Dans le schéma de calcul adopté on convient de négliger la largeur de ces entailles, une hypothèse qui va dans le sens de la sécurité).

Les calculs sont basés sur la formule générale déjà mise en lumière dans l'article précédent :

$$\frac{d\overline{p}}{dy} = \frac{\gamma\omega}{kn}\, q(y)$$

où :

q désigne le débit total traversant la section du plot considéré, par unité de hauteur du barrage;

k la perméabilité;

$\gamma\omega$ le poids spécifique de l'eau.

Le calcul de la souspression moyenne se ramène donc à l'évaluation du débit q à chaque section du barrage.

Dans une première partie de l'article on démontre que le débit d'infiltration aboutissant au parement aval du barrage, CDFG dans la figure 2, est négligeable. En d'autres termes, dans tous les cas pratiques susceptibles de se présenter, on peut considérer que le parement aval est infiniment éloigné.

Sur ces bases, on peut calculer le régime des souspressions moyennes à l'aide du procédé bien connu de Schwartz-Christoffel; la figure 2 montre les diverses transformations requises.

Les résultats peuvent s'exprimer ainsi (fig. 3) :

1) Entre le parement amont et le point E (extrémité amont de l'entaille) $\overline{p}$ décroît suivant une loi linéaire de la pression amont (ou, de façon plus générale, si il existe une contre-pression à l'aval, de la différence entre la pression amont et la pression aval), la valeur $\overline{p}$ de la souspression moyenne en E étant donnée par le tableau II.

2) En aval de E, donc le long de l'entaille, la souspression moyenne diminue suivant une courbe, donnée également dans des cas types dans le tableau II. Dans les applications pratiques, on trouvera que les souspressions décroissent rapidement le long de l'entaille, pour devenir négligeables.