An international comparison of integration techniques for traverse methods in flow measurement

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Contents

Notation

A, $a_0, a_1, \ldots a_k$, B	Constants
b	Intercept at pipe wall of gradient of graph of V against Z at the outermost measuring position (see Fig. 2)
C, c_1, c_2	Constants
D	Diameter of pipe
j	Number of measuring positions per radius
K	Constant
k	$j + 2$
p	Von Karman power law index
Q_1	Volume flowrate between pipe centre line and outermost measuring position
Q_2	Volume flowrate between outermost measuring position and pipe wall
q	Volume flowrate from an annulus within the pipe
R	Radius of pipe
$\overline{V}$	Mean velocity in the pipe
V_i	Velocity at i-th measuring position
V_i'	Gradient of graph of V against Z^2 at the i-th measuring position
$V_{m,i}$	Mean of the velocities at the boundaries of the i-th annulus in the pipe
V_0	Velocity at the pipe centre line
V_y	Velocity at a distance y from the nearest pipe wall
δV	Mean velocity in peripheral zone
$\Delta\overline{V}$	Correction to mean velocity in Renner's modification of the tangential rule

y	Distance from pipe wall
Z	Dimensionless radial position ($= 1 - (y/R)$)
ΔZ^2	Difference between the squares of successive values of Z

Introduction

A large number and variety of flowmeters exist for measuring the flowrate of fluids in circular pipes, but the head loss incurred by such flowmeters is often unacceptable, or perhaps a single series of flow measurements is required where it is impractical to instal a flowmeter (as in the site testing of a large pump or turbine). In these cases a traverse technique is commonly used in which the point velocity is measured at a series of positions along two or four diameters of the pipe and some numerical or graphical method is used to combine these individual measurements and give the mean velocity in the pipe.

For many years a few basic techniques such as planimetry or Simpson's rule were used, but gradually methods were specifically designed to deal with the velocity distribution which occurs in pipe flows approaching the fully developed flow situation.

As new techniques were developed various workers [1-5] carried out comparisons of these techniques with one or more of the conventional methods, but no-one has yet reviewed all of the methods currently used internationally or compared these methods when applied to the same experimental results.

In the work described here the flow of air in a pipe was examined at ten flowrates by traversing along four diameters equally spaced around the pipe circumference with a pitot-static tube. Along each diameter the velocity was measured at the pipe centre line and 42 other positions. An orifice plate, previously calibrated in the National Engineering Laboratory water flow primai calibration facility, was used to measure the flowrat

The individual velocity measurements were then ser to over 30 flow measurement experts in Europe an America with an invitation to apply their own chosei methods to the results for each flowrate and calculat the corresponding mean pipe velocities, a limitatioi being that no more than ten measuring positions pei diameter should be used.

In addition to the methods used by the authors contributions to the investigation were obtained from nine sources in five countries (Italy, Norway, Rumania, USA and West Germany) and details of those who participated are given in the Appendix. A total of fourteen numerical and graphical integration techniques were used including some modifications of 'standard' methods. All of the calculated mean velocities were compared with the corresponding mean velocities as measured by the orifice plate, and where the data were available the variations in mean velocity computed from each traverse diameter were also compared.

Experimental procedure

The experimental measurements were carried out in a 203 mm (8 in) diameter pipe using air, drawn through the rig by a 67 kW (90 hp) fan, as the test fluid. The traverse plane was some 25 pipe diameters downstream of the pipe inlet, and the orifice plate was a further 12 pipe diameters downstream. A plain pipe was used at the inlet since this results in a much shorter length of pipe being required before fully developed flow is established than with the more conventional bellmouth-shaped entries which are normally installed to reduce head losses. By observing the orifice plate differential pressure while the pitot-static tube was moved from the minimum to the maximum insertion position we confirmed that the traverse plane was sufficiently far upstream of the orifice plate to ensure that the probe did not influence the orifice plate reading.

The positions at which point velocity measurements were made are given in Table 1, together with the velocities measured at one of the medium flowrates on the four diameters traversed. The mean velocities measured by the orifice place for the ten flowrates are given in Table 2.

Measurement of such a large number of point velocities necessarily extended over an appreciable period – indeed different diameters were in some instances traversed on different days for a given flowrate. All of the measurements at a particular flowrate were 'norma-

TABLE 1

TRAVERSE POSITIONS AND SPECIMEN SET OF MEASURED VELOCITIES

Traverse position (1 - y/r)	Measured point velocity (m/s)			
	Diameter No 1	Diameter No 2	Diameter No 3	Diameter No 4
0.9630	28.837	29.052	28.416	28.335
0.9481	30.185	30.770	30.014	29.693
0.9432	30.697	31.084	30.361	30.230
0.9234	32.101	32.196	31.673	31.515
0.9037	33.069	33.135	32.698	32.394
0.8740	34.332	34.506	33.868	33.762
0.8469	35.109	35.233	34.806	34.593
0.8370	35.705	35.724	35.202	35.034
0.8000	36.712	36.702	36.211	36.074
0.7259	38.257	38.730	37.948	37.767
0.7086	38.596	39.149	38.362	38.126
0.6938	39.073	39.517	38.666	38.500
0.6493	39.493	40.079	39.463	39.277
0.5999	40.303	40.806	40.366	40.044
0.5653	40.732	41.254	40.878	40.715
0.5480	41.239	41.346	41.093	41.020
0.4986	41.471	42.244	41.828	41.438
0.3999	42.141	42.757	42.864	42.231
0.3159	42.836	43.226	43.541	42.870
0.2788	43.286	43.438	43.804	43.019
0.2245	43.625	43.733	44.021	43.348
0.0000	43.955	43.955	43.955	43.955
0.2250	43.496	43.566	42.812	44.049
0.2793	43.123	43.268	42.440	43.798
0.3164	42.891	43.201	42.354	43.642
0.4003	42.457	42.496	41.381	43.151
0.4991	41.330	41.493	40.640	42.208
0.5485	40.838	40.859	40.150	41.686
0.5658	40.931	40.683	39.920	41.530
0.6004	40.471	40.598	39.528	41.073
0.6498	39.557	39.521	38.734	40.409
0.6942	38.941	38.782	38.335	39.733
0.7091	38.619	38.481	38.168	39.322
0.7264	38.479	38.250	37.664	38.873
0.8004	36.745	36.707	36.704	37.239
0.8350	35.962	35.872	35.295	36.538
0.8449	35.645	35.345	34.914	36.319
0.8745	34.548	34.255	33.986	35.313
0.9042	33.516	33.182	32.885	33.946
0.9239	32.448	32.219	31.861	32.975
0.9437	31.201	30.736	30.758	31.799
0.9486	30.570	30.186	30.466	31.293
0.9610	29.548	29.047	29.487	30.301

TABLE 2

RESULTS OBTAINED FROM INTEGRATION TECHNIQUES USED

Method and contributor	Percentage difference between integration and orifice plate velocity									
	Test run number									
	1	2	3	4	5	6	7	8	9	10
Log-linear										
Cosma	-0.11	-0.06	0.27	-0.21	0.05	0.04	0.28	-0.03	-0.01	0.25
Renner	-0.16	-0.08	-0.11	0.24	-0.23	-0.10	0.22	-0.22	-0.15	0.19
Roth	0.11	0.004	0.15	0.34	0.08	-0.03	0.25	-0.09	-0.04	0.35
Modified log-linear										
Ruegg	-0.36	-0.34	-0.21	-0.03	-0.41	-0.34	-0.21	-0.46	-0.35	-0.05
Tangential rule										
Kinghorn	0.83	0.79	0.94	1.10	0.70	0.77	0.87	0.65	0.74	1.04
Renner	0.80	0.74	1.07	1.06	0.71	0.72	0.57	0.60	0.78	1.01
Modified tangential rule										
Kinghorn	0.24	0.23	0.37	0.58	0.18	0.22	0.35	0.09	0.21	0.48
Renner	-0.04	0.03	0.04	0.36	-0.11	0.02	-0.13	-0.10	0.08	0.31
Modified trapezoidal rule										
Johnson	-0.14	-0.17	-0.03	0.24	-0.19	-0.10	0.17	-0.31	-0.14	0.09
Tonini	0.06	-0.02	0.07	0.47	-0.01	0.03	0.23	-0.06	0.02	0.32
Geromiller	0.08	0.13	0.33	0.48	0.05	0.17	0.33	0.04	0.13	0.43
Modified Simpson rule										
Alming	-0.21	-0.14	-0.29	0.49	-0.06	0.14	-0.17	0.09	-0.04	0.39
Milanese	0.42	0.36	0.31	0.61	0.07	0.11	0.32	0.01	0.04	0.19
Tonini	0.16	0.05	0.10	0.49	-0.02	0.06	0.29	-0.01	0.06	0.33
Von Karman power law										
Ruegg	0.28	-0.12	0.40	0.56	0.11	0.24	0.35	0.12	0.22	0.51
Cubic method										
Kinghorn	0.26	0.19	0.28	0.58	0.13	0.19	0.39	0.07	0.14	0.39
Tonini	0.15	0.05	0.14	0.51	0.02	0.08	0.30	0.01	0.07	0.34
Log-Tchebychev method										
Kinghorn	-0.10	-0.13	0.03	0.22	-0.20	-0.11	-0.06	-0.25	-0.19	0.05
Spiral method										
Cosma	0.25	0.03	0.24	0.40	-0.14	-0.01	0.19	-0.17	-0.05	-0.06
Graphical method										
Renner	-0.16				-0.23					-0.04
Tonini	0.24	0.14	0.17	0.55	0.11	0.06	0.40	0.06	0.06	0.44
Mean velocity from orifice plate (m/s)	18.777	24.042	28.168	33.201	37.492	42.583	47.150	52.250	56.818	59.277

lized' against a single reference orifice plate measurement to compensate for any flowrate variations which were present. In addition, all of the velocity measurements were corrected for the effects of blockage, transverse velocity gradient, and so on as prescribed in the British Standard on the use of pitot tubes [6] ; since this Standard corrects for turbulence effects only by adjusting the mean velocity obtained when the ten points per diameter log-linear method is used, it was necessary to use Kinghorn's [7] results to correct the individual velocities for this effect. It will be noted from Table 1 that corresponding positions on the two opposite radii are not always at precisely the same radial positions ; this was a consequence of the accuracy with which the traversing gear could be positioned (0.125 mm (0.005 in)) but should not significantly affect the results.

Integration techniques considered

There are basically two types of integration technique for the calculation of a mean pipe velocity from the individual point velocities : numerical and graphical.

In numerical techniques the velocity distribution curve may be treated as a mathematical relation between the point velocities and the radial positions at which they are measured, and an integration technique applied without regard to the fluid mechanics of the flow. Alternatively a knowledge of the theoretical velocity distribution may be used in deriving an integration technique. In practice only the 'tangential rule' [8] may properly be classed in the first group, and only the log-linear method [2] in the second. All other numerical methods apply a purely mathematical technique to

TABLE 3

ACCURACY AND REPEATABILITY OF INTEGRATION TECHNIQUES

No	Method name	Contributor	No of points per radius	No of radii used	Mean percentage deviation from orifice plate result	Percentage standard deviation of technique relative to		Percentage standard deviation of orifice plate result
						Orifice plate	42-point cubics	
1	Log-linear	Cosma	5	8	0.047	0.141	0.114	0.08
		Renner	3	8	-0.040	0.174	0.075	0.16
		Roth	5	4	-0.102	0.159	0.052	0.15
1A	Modified log-linear	Ruegg	5	4	-0.276	0.139	0.055	0.13
2	Tangential rule	Kinghorn	5	8	0.843	0.138	0.039	0.13
		Renner	5	8	0.806	0.172	0.146	0.09
2A	AMCA	Kinghorn	5	8	0.296	0.146	0.040	0.14
2B	Modified tangential rule	Renner	5	8	0.046	0.159	0.123	0.10
3A	Modified trapezoidal	Johnson	5 + centre line	8	-0.098	0.174	0.052	0.17
3B	Modified trapezoidal	Tonini	5 + centre line	4	0.111	0.163	0.044	0.16
		Geromiller	5	4	0.217	0.153	0.077	0.13
4A	Modified Simpson's rule	Alming	5 + centre line	4	0.020	0.244	0.214	0.12
4B	Modified Simpson's rule	Milanese	5 + centre line	4	0.244	0.184	0.111	0.15
		Tonini	5 + centre line	4	0.151	0.158	0.099	0.12
5	Von Karman power law	Ruegg	5	4	0.310	0.147	0.046	0.14
6	Method of cubics	Kinghorn	21 + centre line	8	0.264	0.145	-	-
		Tonini	5 + centre line	4	0.167	0.156	0.036	0.15
7	Log-Tchebychev	Kinghorn	5	8	-0.08	0.127	0.057	0.11
8	Spiral	Cosma	5	8	0.068	0.180	0.116	0.14
9	Graphical	Renner	21 + centre line	8	-0.143	-	-	-
		Tonini	5	4	0.223	0.170	0.054	0.16

assess the flowrate contribution from the core of the pipe, but make use of known velocity distribution laws to calculate the flowrate in the region of the pipe wall.

In graphical techniques the area enclosed by the graph of velocity against the square of radial position, or by that of velocity times radial position against radial position, is evaluated directly (usually by planimetry) between the centre line and the position of the outermost measured velocity. The flowrate contribution of the peripheral area between the outermost measuring positions and the wall is, however, normally evaluated by integrating some theoretical velocity distribution in the wall region.

The methods used in this study are listed in Table 3, together with the names of the contributors who used them, and the numbering of the methods is as given in the following subsections.

Log-linear Method (Method 1)

Since the introduction of the log-linear method by Winternitz and Fischl [2] in 1957 it has become probably the most commonly used. These authors showed that the velocity distribution may be represented by the equation :

$$V_y = A + B \log \frac{y}{D} + C\left(\frac{y}{D}\right) \qquad (1)$$

and that it is possible, for any number of measuring positions per diameter, to determine positions at which measurements may be made such that the mean pipe velocity is the simple arithmetic average of the individual velocities. The location of the measurement positions for 1-15 measurements per radius have been tabulated in Reference 7.

This method was used by Cosma, Renner and Roth.

Modified Log-linear Method (Method 1 A)

For this study Ruegg has modified the log-linear function to the form :

$$V_y = \frac{c_1}{1 + c_2} \frac{y}{R} - c_1 \, ln \left(\frac{c_2 + y/R}{c_2}\right) \qquad (2)$$

where c_1 and c_2 are constants for a particular set of velocity measurements.

The method adopted was to use non-linear regression analysis to fit equation (2) to the measured velocities at the dimensionless radial positions closest to $\sqrt{0.1}$, $\sqrt{0.3}$, $\sqrt{0.5}$, $\sqrt{0.7}$ and $\sqrt{0.9}$. These values of $1 - (y/R)$ are at the area centres of five equal annular areas in the pipe, that is the same measuring positions as chosen for the tangential rule (see below). In this way the constants c_1 and c_2 were evaluated and the mean velocity in the pipe for each flowrate found by analytical integration of equation (2) for each diameter and averaging the four results.

Tangential Rule (Method 2)

Before the introduction of the log-linear rule, the tangential rule was the most widely used numerical integration technique for flow measurement. In it the cross-section of the pipe is considered to be divided into j concentric rings of equal area (the innermost being a circle). Velocity measurements are then made at the radial position on each ring which divides the ring into two equal areas, and the mean velocity for a given traverse is taken as the arithmetic mean of the j velocities. Thus for five measurements per radius, the measuring positions are $Z = 0.316$, 0.548, 0.707, 0.837 and 0.949, where $Z = 1 - (y/R)$.

Although this method has been seldom used in recent years we applied it to the results for comparative purposes. It was also used by Renner.

AMCA Method (Method 2A)

We used a modification of the tangential rule commonly referred to as the 'AMCA' method. The Air Moving and Conditioning Association Inc. (AMCA) of America [9] recommend that traverse measurements be made at five positions per radius, given by $Z = 0.316$, 0.548, 0.707, 0.837 and 0.961. The first four of these positions are the same as the first four of the 'tangential' method positions for five measurements per radius, and the last is almost identical to the outermost position in the five points per radius log-linear method.

Modified Tangential Rule (Method 2B)

Renner modified the tangential rule by computing a correction to the mean calculated velocity to compensate for the velocity rapidly reducing to zero between the outermost measuring position and the wall. Using the theoretical logarithmic velocity distribution close to the wall he calculated, for five measuring positions per radius, that the mean velocity should be reduced by an amount $\Delta\overline{V}$, given by :

$$\Delta\overline{V} = 0.052\,94\,(V_4 - V_5)$$

where V_4 and V_5 are the velocities measured at the two outermost positions.

Modified Trapezoidal Method (Method 3A)

The general formula for the application of the trapezoidal method is given by :

$$\overline{V} = \sum_{i=0}^{j} (V_m\ \Delta Z^2)_i \qquad (3)$$

where $V_{m,i}$ is the mean of the velocities at the boundaries of the i-th strip, $Z = 1 - (y/R)$ and

$$\Delta Z_i^2 = Z_{i+1}^2 - Z_i^2 .$$

Note that $Z_o = 0$ and $Z_{j+1} = 1$

This method was used by Johnson for five velocity measurements per radius together with the measured centre line velocity and the assumption of zero velocity at the wall. Since the outermost strip is bounded by the wall, evaluation of $V_{m,j}$ (the velocity closest to the wall to be used in the simplified equation (3)) would be subject to large errors because of the steep non-linear velocity gradient in this region. The relationship :

$$V_y = K\left(\frac{y}{R}\right)^{1/p} \qquad (4)$$

was therefore used by Johnson to evaluate the mean velocity in the outermost strip in Fig. 1. The constants K and p were evaluated by substituting in equation (4) the values of V_y and y/R at the two outermost positions at which velocities were measured, Z_{j-1} and Z_j.

To determine the values of $V_{m,i}$ for $i < j$, Johnson averaged the velocities at the values of Z_i limiting the i-th strip. That is, $V_{m,i} = (V_i + V_{i+1})/2$, and the values of V_i and V_{i+1} were in turn obtained by linear interpolation between the two experimentally obtained velocities closest to the positions Z_i and Z_{i+1}.

This procedure was applied to the mean velocity distribution, that is, the values of the V_i were calculated for each of the eight radii for which experimental data were available, and then the average of the velocities at each of the chosen radial positions was found, giving the averaged radial velocity distribution.

Johnson applied this method to three different sets of measuring positions for each of the ten flowrates for which data were supplied. In each case the outermost position was chosen at $Z_1 = 0.962$, and the remaining positions were given by :

a $Z_1 = 0.316, 0.649, 0.836$ and 0.924
b $Z_1 = 0.548, 0.694, 0.874$ and 0.943
c $Z_1 = 0.447, 0.632, 0.775$ and 0.894.

The results were virtually identical in each case, and so only those from set (a) have been used in the analysis here.

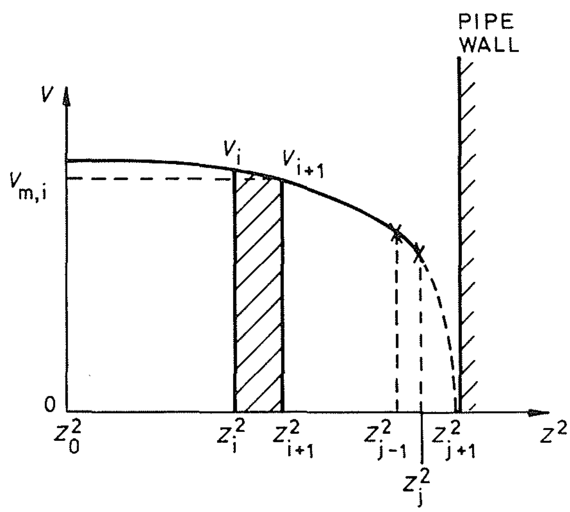

Figure 1 – Use of the modified tangential rule or Simpson rule

Modified Trapezoidal Method (Method 3 B)

A modification of the trapezoidal method was used in which the volume flow-rate represented by a segment of a curve of velocity against the square of radial position is given by :

$$q = \pi R^2 \left\{ (V_i + V_{i+1}) \frac{\Delta Z_i^2}{2} + R^2 (V_i' - V_{i+1}') \frac{\Delta Z_i^4}{2} \right\} \quad (5)$$

where V_i' is the gradient of the curve at Z_i^2.

Geromiller [10] modified a method originally used by Spielbauer, Faiss and Müller [11] for rectangular ducts, which was itself a modification of the trapezoidal method to take into account the region of the pipe between the outermost measuring point and the wall. The flow in this latter region is given by

$$Q_2 = \pi R^2 \frac{p}{p+1} V_j \Delta Z_j^2 . \quad (6)$$

Geromiller showed that the mean velocity in the pipe was then given by

$$\bar{V} = \frac{2p+1}{2(p+1)} V_j \Delta Z_j^2 - \frac{p}{2(p+1)} R^2 V_j \Delta Z_j^4$$

$$+ \sum_{i=0}^{j-1} (V_i + V_{i+1}) \frac{\Delta Z_i^2}{2} + \sum_{i=0}^{j-1} (V_{i+1} - V_i) \frac{\Delta Z_i^4}{2} R^2 \quad (7)$$

This method was applied by Geromiller in the current study to velocity measurements at dimensionless radial positions of 0.316, 0.548, 0.709, 0.846 and 0.962, and by Tonini at positions of 0.400, 0.600, 0.726, 0.874 and 0.962.

Modified Simpson Rule (Method 4 A)

The area under a curve is evaluated using the Simpson rule by dividing the area between the limits of the curve into an even number of equal widths and computing the value of

$$\left(\frac{\text{width of each strip}}{3} \right) \times \left(\begin{array}{l} \text{sum of extreme ordinates} \\ + 2 \times \text{sum of odd ordinates} \\ + 4 \times \text{sum of even ordinates} \end{array} \right).$$

Alming [12] used the Simpson rule between the pipe centre line and the outermost measuring position, and analytic integration of the relation in equation [4] between the outermost measuring position and the wall. Thus (see Fig. 1) the values of Z_1 to Z_5 (Z_j in this case) were calculated such that :

$$Z_5^2 - Z_4^2 = Z_4^2 - Z_3^2 = Z_3^2 - Z_2^2 = Z_2^2 - Z_1^2 = Z_1^2 - Z_0^2$$

Z_5^2 is chosen as 0.9.

From the experimental data supplied, the point velocities at each of these values of Z_i were read off from a plot of point velocity against radial position for each of four radii (forming two mutually perpendicular diameters) and the mean flow between the

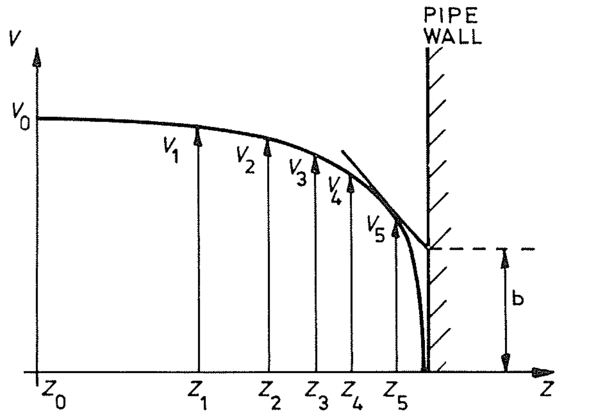

Figure 2 – Evaluation of peripheral flowrate by the Alming method

centre line and Z_5 for each radius calculated from :

$$Q_1 = \pi \frac{Z_{i+1}^2 - Z_i^2}{3} \times (V_0 + 4V_1 + 2V_2 + 4V_3 + 2V_4 + V_5) R. \quad (8)$$

In order to calculate the contribution to the flow from the annulus between Z_5 and the wall, the tangent at Z_5 to the graph of local velocity against Z is first drawn by eye and the value of b (see Fig. 2) found. Alming [12] has shown that the flowrate Q_2 in this outer annulus is given by :

$$Q_2 = 2\pi V_5 Z_5 R^2 \left(\frac{1}{2 - \frac{b}{V_5}} - \frac{Z_5}{3 - \frac{b}{V_5}} \right) \quad (9)$$

The mean velocity in the pipe based on the results from one radius is then given by :

$$\bar{V} = \frac{Q_1 + Q_2}{\pi R^2} \quad (10)$$

and the final mean velocity is obtained by averaging the velocities obtained from the four radii under consideration.

Polynomial Method (Method 4 B)

Milanese fitted a polynomial of the form :

$$V = a_0 + a_1 Z^2 + a_2 Z^4 + a_3 Z^6 + \ldots a_i Z^{2i} + \ldots + a_k Z^{2k} \quad (11)$$

to the relation between point velocity and radial position for the region $0 \leqslant Z \leqslant Z_j$, Z_j being the position closest to the wall at which a measured velocity is used. With j velocities per radius being measured then $k = j + 2$.

In order to evaluate the coefficients $a_0, a_1, \ldots a_k$, a set of k simultaneous equations are generated by substituting the centre line velocity and j other measured velocities into equation (11) with the appropriate values of Z, by putting the derivative of equation (11) equal to zero at the centre line, and by putting the derivative of equation (11) equal to the derivative of the relation :

$$\frac{V_y}{V_j} = \left(\frac{1 - Z}{1 - Z_j}\right)^{1/p} \tag{12}$$

at $Z = Z_j$.

Equation (12) is again the von Karman power law for the velocity distribution at the wall.

The flowrate between the outermost measured velocity and the wall is obtained by analytic integration of equation (12), and that between the centre line and outermost measuring position by applying the Simpson rule to the function in equation (11). The value used for p was 7 for all ten flowrates, and the velocities used were those measured at $Z^2 = 0.160$, 0.320, 0.481, 0.640 and 0.817 (together with the centre line velocity).

Tonini also used this method, but applied it to the velocities measured at $Z^2 = 0.160$, 0.360, 0.527, 0.764 and 0.925.

Von Karman Power Law (Method 5)

The von Karman Power Law, relating the velocity at a point in a pipe to the distance of that point from the nearest wall, is given by equation (4). Ruegg used the measured velocities at the positions closest to $r/R = \sqrt{0.1}$, $\sqrt{0.3}$, $\sqrt{0.5}$, $\sqrt{0.7}$ and $\sqrt{0.9}$, and found the best fit of equation (4) to these data by non-linear regression analysis.

Integration of equation (4) over the pipe radius yields :

$$\frac{\bar{V}}{V_o} = \frac{2p^2}{(p + 1)(2p + 1)} \tag{13}$$

where V_o is the velocity at the pipe centre line. As both V_o and p are evaluated in the regression equation, $\bar{V}$ is easily obtained.

Method of Cubics (Method 6)

The method of cubics was first applied to flow measurement by Coffin in an unpublished International Current Meter Group Report [13] and was subsequently discussed by us [5]. It essentially consists of fitting a series of cubic curves between adjacent pairs of measured velocities, splining the curves by the condition that the gradients of successive curves should be equal at their common boundaries. The region between the outermost measuring point and the wall is dealt with by incorporating a modification of the von Karman power law equation (equation (17)) into the overall formula, and the mean velocity in the pipe, $\bar{V}$, is given by :

$$\begin{aligned}
\bar{V} = {} & V_0\left[\frac{2}{3}Z_1^2 + \frac{1}{4}Z_1^2\left(\frac{Z_1}{3Z_2} - 1\right) - \frac{1}{12}Z_2^2\right] + \\
& + V_1\left(\frac{1}{6}Z_1^2 + \frac{2}{3}Z_2^2 - \frac{1}{12}Z_3^2\right) - \\
& - V_2\left(\frac{1}{12}\frac{Z_1^3}{Z_2}\right) + \\
& + \sum_{i=2}^{j-2} V_i\left(\frac{1}{12}Z_{i-2}^2 - \frac{2}{3}Z_{i-1}^2 + \frac{2}{3}Z_{i+1}^2 - \right. \\
& \left. - \frac{1}{12}Z_{i+2}^2\right) + \\
& + V_{j-1}\left(\frac{1}{12}Z_{j-3}^2 - \frac{2}{3}Z_{j-2}^2 + \frac{1}{12}Z_{j-1}^2 + \right. \\
& \left. + \frac{1}{2}Z_j^2\right) + \\
& + V_j\left[\frac{1}{12}Z_{j-2}^2 - \frac{2}{3}Z_{j-1}^2 + \frac{7}{12}Z_j^2 + \right. \\
& \left. + \frac{(Z_j^2 - Z_{j-1}^2)^2}{12p(1 - Z_j^2)R^2} + \frac{p}{p + 1}(1 - Z_j^2)\right] .
\end{aligned} \tag{14}$$

This method was used by Tonini for 11 measuring positions per diameter (including the centre line position), the radial positions being given by $z = 0.400$, 0.600, 0.726, 0.874, and 0.962, and we applied the method to all 43 measured velocities on each diameter.

Log-Tchebychev Method (Method 7)

The log-Tchebychev method consists of fitting Tchebychev polynomials to the central core of the flow, and using a logarithmic distribution close to the wall in the same way as the log-linear does for an odd number of measurements per radius. Thus for j velocity measurements per radius the pipe cross-sectional area is divided into two regions by a circle such that the outer annular region has an area of $\pi R^2/j$, and the velocity is measured at the position in this annulus at which the mean velocity in this region would occur according to the logarithmic velocity distribution law [2].

The inner region is in turn divided into two equal areas, and the remaining measuring positions are determined according to the Tchebychev quadrature rules [14] relative to the circle which does this. An alternative, but equivalent, method of computing the positions in the central zone is given by Richter [15].

The log-Tchebychev method was applied by us to velocities at the measuring positions given by $Z = 0.287$, 0.570, 0.689, 0.689, 0.847 and 0.962.

Spiral Method (Method 8)

Cosma [16] introduced the concept of considering the velocity measuring positions to be distributed along spirals rather than along radii. For practical reasons the points must, of course, lie along radii, but Fig. 3 shows how the individual velocity measurements would be

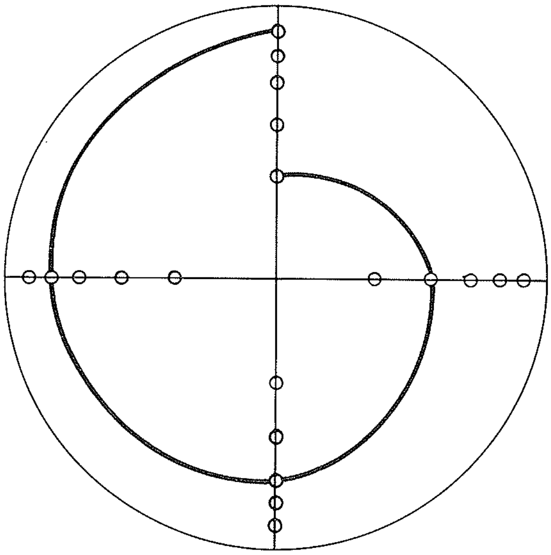

Figure 3 – The grouping of individual velocity measurements in Cosma's spiral method

grouped in order to calculate one of four estimates of the mean pipe velocity.

If, for example, the measuring positions are located at the log-linear positions on four radii, the overall mean pipe velocity is independent of how the individual measurements are grouped. Cosma has, however, shown that the variation among the four estimates of velocity using his method is less than the variation among estimates obtained by averaging along each radius. This holds provided that the velocity distribution is such that the variation in the radial direction is greater than the circumferential variation at a given radial position.

It therefore follows that if the number of possible point velocity measurements is limited (for example by the number of current meters available for use in a fixed array in a pipe) it would be more accurate to distribute them differently on each radius so that a single spiral can pass through all of the measuring positions.

Cosma, in applying the spiral method, plotted all of the available velocity measurements and interpolated on each graph to evaluate the velocities at the 40 points per radius log-linear positions for each of the eight radii at the ten flowrates. For each flowrate he then calculated the mean velocity corresponding to each of the eight possible spirals, each spiral passing through 40 points. When this procedure is compared with other methods, the result from any one spiral should be compared with the results of any method applied to a set of five points on eight radii.

Graphical Integration (Method 9)

The flowrate in a pipe may be derived graphically from individual point velocity measurements by plotting either these velocities against the square of their radial position, or the product of each velocity and its radial position against radial position, and by evaluating the area under the curve which is normally found with the aid of a planimeter.

In the former case the mean velocity is given by :

$$\bar{V} = \int_0^1 V_r \, dZ^2 \qquad (15)$$

and in the latter case by

$$\bar{V} = \int_0^1 V_r \, Z \, dZ \qquad (16)$$

Since the latter method clearly gives a discontinuity in the curve at the centre line the former method is normally used, and indeed is the one recommended by the International Electrotechnical Commission [17].

Because of the rapidly changing gradient of the velocity distribution close to the wall, direct evaluation of the area under the curve in this region is potentially inaccurate. The IEC recommends, therefore, that the curve between the outermost measuring position and the wall be drawn using the formula :

$$V_y = V_j \left(\frac{1 - Z_y^2}{1 - Z_j^2} \right)^{1/p} \qquad (17)$$

The exponent p is found by substituting into equation [17] the appropriate values for the two outermost measuring positions and velocities.

An alternative method recommended by the IEC is to find the mean velocity in the peripheral zone by analytic integration of equation [17], which results in :

$$\delta V = \frac{p}{p+1} V_j \qquad (18)$$

The mean velocity in the central core of the flow is found graphically by the use of equation [15], and the results combined to give the mean pipe velocity.

This method was used by Tonini with measuring positions given by $Z^2 = 0.160, 0.360, 0.527, 0.764$ and 0.925, and by Renner using all of the 21 measured velocities on each radius.

Comparison of results

The mean velocity calculated by each contributor for the various integration techniques was compared with the value obtained by the orifice plate measurement for each flowrate, and the percentage difference between the two calculated from

percentage difference =

$$= 100 \times \frac{\text{integration velocity} - \text{orifice plate velocity}}{\text{orifice plate velocity}}$$

The results are summarized in Table 2, a negative sign indicating that the integration technique underestimated the flowrate. From these results it is clear that there is very close agreement (within about 0.2 per cent) between many of the integration technique results and the orifice plate results ; the exceptions to this are the modified log-linear method, the tangential rule, the AMCA method and, to a lesser extent, the von Karman power law.

Of at least equal importance is the way in which the results for a given integration technique vary from test to test, since this gives a measure of the repeatability (as opposed to the accuracy) of the method. In order to examine the repeatability it was first necessary to normalize all of the integration technique results to some reference level, since there would be a degree of random scatter in the orifice plate results themselves, and straightforward comparison with them would not be sufficient. For each test run, therefore, the mean velocity obtained by each integration technique was subtracted from the mean velocity obtained using the 42 points per diameter cubic method, and the result expressed as a percentage of the velocity measured by the orifice plate. For each integration technique the standard deviation of this quantity was then calculated from the ten values available.

For each integration technique the mean of the ten deviations from the orifice plate result is given in Table 3, together with the percentage standard deviation of the results using that technique relative to both the orifice plate and 42-point cubic method results.

It is of interest to note in passing that these data also provide an estimate of the repeatability of the orifice plate results. The square root of the difference of the two variances already calculated (the squares of the values in columns 7 and 8 of Table 3) should have the same value in every case, equal to the standard deviation of the orifice plate results. These values are given in column 9 of Table 3 and are constant to within about ± 0.045 per cent, giving a mean value of 0.12 as the percentage standard deviation of the orifice plate results.

Finally, some contributors used data from only four radii, while others used the results from all eight radii which were traversed, and in some cases more or less than five velocity measurements per radius were used. Variations of this sort with the same integration technique being used allowed a comparison of the effect of varying the number of radii or points per radius.

Conclusions

It is clear that even in the worst case (the tangential rule) the contribution to the overall flowrate measurement error arising from the integration technique itself is less than about 1 per cent. The techniques which gave the poorer results in this study were either included for historical reasons (the tangential rule and AMCA method) or were methods developed for the current study (the modified log-linear rule and the Von Karman power law method).

The remaining techniques all agreed so closely with each other that it is impossible to pick out any one as being the most accurate, since the orifice plate results have been shown to have a standard deviation of about 0.13 per cent. In addition there is little to choose between the techniques from the point of view of repeatability, as most of them produced a standard deviation of results relative to the 42-point cubic method of less than about 0.1 per cent.

Although only a few comparisons were possible, varying the number of radii or number of points per radius on which results are based appears to make no significant difference.

Thus the main conclusion to be drawn is that, for good flow conditions, any of the more modern integration techniques may be used and can be expected to introduce an error of less than about 0.2 per cent. This exercise has therefore demonstrated beyond doubt that there is little or nothing to be gained by any further modification of integration techniques to allow more accurate flowrate determinations by traverse techniques in good flow conditions.

In practice traverse methods are often used when the flow is asymmetric or swirling, and future work should be orientated towards such applications. The test of integration techniques for flow measurement purposes should be their accuracy when applied to a variety of different velocity distributions, and not simply how well they can deal with a velocity distribution agreeing closely with the theoretical distribution for which the technique has been specifically developed.

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