Foreword

The purpose of this series of articles is to present to artists, scientists and engineers a critical comparison of the ways in which their predecessors have represented water graphically, especially water in motion. For this I will begin with a brief historical review of how and when water was represented by each group. Rarely has the person depicting water been both artist and scientist, like Leonardo da Vinci and Dürer were. I will attempt to explain why artists and scientists took, or had to take, very different basic approaches, and present my thoughts about the reasons for the differences and also for some surprising similarities. For instance, scientists need to describe inner body motions because it is impossible to fully investigate water flow by paying attention only to what happens on a given surface, as painters do. But both artists and scientists have recognized the essential role of sinusoidal waves, which are at the root of every movement observed at the air-water interface. Scientists may arrive at a description of a rough sea using a Fourier series, for which they determine enough sine functions of different period, amplitude, and orientation that when added produce the desired effect [Pier- son 1952]. Artists may reach such synthesis by some other summations less easy to identify. For example, David Hockney (fig. 1) has shed light into this process by representing the complex surface of his swimming pool on the canvass with all kinds of sinusoidal waves in all orientations [Hockney 1988]. I wonder if the artist who painted the agitated waters at the ancient port of Cumae did not have something similar in mind, when he spread over his depiction of water what seem to be symbols of chaotic motion. An art analyst, instead, has seen in such symbols a calligraphic quality while recognizing their association with waves [Feder 1978].

It is interesting to explore what artists know, and what part knowledge plays in their depictions, and what part poetic inspiration. Why, for instance, did Egyptian artists like the triangular wave (fig. 2), which was also used by Mesopotamian astronomers [Neugebauer 1962] as a first approximation for a sine wave? Was it simply a shorthand way of indicating water? Why did Mayans choose a somewhat irregular sine wave? (fig. 3). Why did Chinese artists seem to have preferred for a long time certain wave patterns disregarding others? Why did many of them use very flat monocolored water surfaces? By comparison, why have scientists and engineers generally proposed very abstract representations? Usually they pay very little attention to secondary flows, and to details. In exploring questions like these, I hope this series will appeal equally to engineers, scientists and artists, and leave each group with the feeling of having seen things through the eyes of the other.

Introduction

The great diversity of shapes and movements manifested by water is known to all. We only need to follow the course of a river to see all kinds of phenomena, from the complex turbulent motions in falls, cascades or rapids, to the very still waters when we come to a lake. Or, if we are close to the seashore, or travel by sea from one continent to another, we know how many different faces water can show us, from a perfectly calm surface disturbed only by boat s, playful dolphins, or birds diving into or rising up from it, to violent awesome stormy seas. Water sweeps the whole gamut from perfectly regular to fully chaotic motions. The water surface goes from being the perfect mirror that lured Narcissus to the ugly monster that terrorized the Argonauts.

Art being more ancient than science, we find artistic depictions of water that go back several millennia, while descriptions by scientists are much more recent. The gap could perhaps be partially filled if we had some of the representations of water that must have been made in ancient times by men who were instrumental in advancing aspects of technology relating to water, like hydraulicians, and designers of ships or fountains, but so far I have been unable to find any.
old representations of water flows that can be considered as studies of such phenomena by ancient scientists or engineers.

In studying the depiction of water by artists and scientists, we soon discover that both groups have arrived at some abstractions that reveal that any water movement can be decomposed into and re-composed from very few basic motions. The most important are the primordial wave and the elementary vortex. In a simplified view, the superposition of waves can represent a wide spectrum of wavy surfaces. For instance, simple superposition of a few elementary waves with the right parameters can explain the sometimes surprising arrival of a big wave upon a beach where all had been rather quiet for some time. Similarly, by superposition of vortices, one can represent from very orderly flows to quite complex ones.

In these articles, we will see how these elementary or primordial flows recur in depictions from all ages. Other elementary flows have been discovered, but waves and vortices play the most important roles, so much so that, as we shall see, the wave and the vortex became the most common symbols to indicate water in several civilizations. No matter which aspects of art, science or technology we consider in this study, we will always find that the artists are essentially qualitative while engineers and scientists are, and need to be, quantitative.

This series of articles, which is the result of several decades of formal research work as an applied mathematician and of informal but very exciting study of representational art, begins with a general view of early depictions of water; hence it will necessarily include artistic rather than scientific representations. In the following articles, I will discuss extensively representations of water waves beginning with simple patterns and ending with very complex ones. My choice of topics is selective rather than global, since the entire subject of water depiction is too vast to be covered in a few articles.
Early depictions of water

It seems reasonable to assume that since humans began to represent the physical world around them, they did not omit water from such depictions. Unfortunately, not enough examples of such representations by primitive people have been discovered, mainly cave drawings and paintings. In some of them there are wavy lines which have been described as meanders by some art analysts [Huyghe 1962] but, hydrodynamically, they cannot be linked to water with any certainty.

Since historical times, most cultures have shown water in their graphic arts. The only culture that seems to have looked at water with some reluctance and even with fear is the pre-Columbian in Mexico [Decrosse 1991], and this may account for the scarcity of their representations of water. In our times, artists rarely represent water realistically, offering generally abstract representations that are often difficult to comprehend.

Water per se was not the main subject of early depictions, but rather as a part of a landscape or a composite picture. It is an important element in paintings describing old legends and myths, and also historical events involving water, but it is generally not the central theme. This is how one finds water in religious art. We have to examine much more recent work to find water as the main or even the only subject of the depiction. The same is true of technological drawings. Drawings of water-wheel mills, for instance [Ramelli 1588],

Waves

The most common symbol for water is a sinusoidal wave. In science, sinusoidal functions were studied quite early but most probably were never represented graphically [Neugebauer 1962]. Water waves are almost always present in pools, lakes, or seas. For artists, there appear to have existed two options for representing a water expanse in a picture: either to cover this area with a more or less uniform shade of some color (green, turquoise or blue, for example) or to cover it with lines indicative of the observed waviness of the surface. The second option obviously was quite popular and led to the lines we observe in early Egyptian art (fig. 2), which tend to be undulated lines with rather sharp changes in direction of the brush or other instrument used. My suggestion of a remarkable analogy between the way a wavy motion was seen by the Egyptian artist and by the Mesopotamian astronomer may now be clarified. Mesopotamian astronomers using numerical depiction represented initially what appeared to them as the sinusoidal motion of a planet by means of triangular waves made of pieces of linear functions. This, according to Neugebauer [1962] was followed by a second-order curve fitting involving segments of parabolas. What I see is a common way of abstracting, one with the brush on a papyrus and the other with numbers inscribed on a tablet.

Triangular, and even rectangular, waves are also found in Roman mosaics, although in some cases the representation may be influenced by the medium used to produce such serrated images as one can see in the Louvre’s Roman Mosaics or in those at Villa del Casale in Sicily (fig. 4). We also see them as relatively short lines placed here and there among aquatic plants, animals, boats, or human figures. Quite different are the depictions of water seen in the twelfth-century mosaics of the cathedral of Monreale, Sicily [Giordano 1982]. In one case, the mosaic pieces have been made to follow smooth sinusoidal lines of different colors in order to make the waves readily distinguishable. It is interesting that it was apparently felt that the viewers needed to be told that the scene was that of the sea (See Mare in fig. 7). In another case, the generation of waves by the wind is suggested. This scene will be discussed in another article of this series, but I felt that it should be shown here to delight the reader. It shows the work of a creative and talented artist, even if it may be criticized by a hydrodynamicist (fig. 8). In another scene, we see the stormy sea in which Saint Peter is sinking (fig. 7). Since then, the rough seas have been the object of many paintings. In contrast, only in this century have they called the attention of a number of hydrodynamists (fig. 1).

It is hard to find extant depictions of water by artists of the amazingly varied cultures that flourished in the Americas before the discovery and the conquest, but what little I have found is quite interesting. Water was certainly of great importance to the Mayas and to the Mochicas, but the few depictions of water at my reach are in some ways similar and in many other ways different from those of the civilizations of the old world, with which no contact had existed for a very long time. The common trait is that the wavy line seems to be universal, but we will discuss, in this series, some striking differences.
4. Water waves in Roman mosaics. [Drawn by E. Macagno from mosaics in Villa dei Casale, Piazza Armerina (Sicily) and the Louvre.]

5. Regular water waves in a mosaic in Monreale's cathedral. [Drawn by E. Macagno]

6. Artistic view of generation of water waves. [Drawn by E. Macagno from mosaic in Monreale's cathedral.]

7. Saint Peter sinking in a stormy sea. Mosaic in Monreale's cathedral. [Drawn by E. Macagno.]

8. Detail from a photograph by V. Cornish, of a stormy sea. [Courtesy of Cambridge University Press (Fig. 3 in the book *Ocean Waves*, by V. Cornish, 1934)]

9. Egyptian representation of a jet of granular material. *Deceased and mother pouring incense*. [By permission from MMA-NY, 30.4.104]

10. Photograph of a water jet by E. Macagno at the Institut für Hydromechanik, Karlsruhe University.

Jets

Another water phenomenon that we find in early depictions is the jet. Jets are easily produced by tilting cups or containers. But in order to appear in early art, they must have been part of some ritual, or symbols of one kind or another. Of course, jets are one of the elements of fountains, which usually are a combination of jets and pools of water. In an interesting merging of two artistic manifestations, we have painters depicting water fountains; but this seems to have happened in later times than other representations. In Egyptian art, jets usually have undular lines either as contour lines or as the representation of the jet itself (fig. 9). Perhaps this was done to indicate some turbulence or instability which may easily be present in the pouring water, wine or other liquids from a pitcher. Or, perhaps, the undulation was incorporated to make clear that it was a liquid flowing. It should be noted that even a jet coming out from a steady nozzle may have inherent instabilities, as does the one from a recent experiment shown in fig. 10. It could also be that the way to ensure that the viewer would see water, or other liquids, would be to encode its symbol: the wavy line.

In some cases jets are represented by only a few lines. In one such case that of a Sumerian cylinder of the third millennium, we can see jets coming out from vases beginning upwards (as if under some pressure) and then falling down (fig. 11). The historians of art see in these jets meanings that are very interesting, but here we are trying to detect what may have been the influence of some knowledge about water flow phenomena in the depiction of water. We know that there are a great number of shapes among water jets, because they are governed by several forces. Surface tension is the one that tends to produce complicated, and sometimes beautiful effects. But the interplay of other forces may also produce effects which are condensed in our distinction between laminar and turbulent jets. Probably, artists captured these aspects without bothering to investigate the apparent or hidden causes.

Streams, torrents and rivers

In the representations of rivers we see a greater influence from mythology than in depictions of other water flows. Both for water and air currents, a personification of the causes was quite common. Thus in Monreale (fig. 8), the wind was represented by a mythical head blowing the air that generates the waves. Similarly, in many works of art, as in one of the famous fountains of Piazza Navona in Roma, for instance, we see great personages as representing four rivers. Another such figure is at the foot of Trajan's column, where the river is the Danube. The idea of a man, or a god, usually with a vessel pouring incessantly a stream of water, was pervasive, and it was abandoned only recently if we count time in centuries.
There is an immense variety of flow phenomena along rivers, much more than in the sea; however, river phenomena have been less frequently depicted in art. If we believe the art historians' interpretations, one of the old representations of rivers is that of the statue of Gudea in the Louvre, in which the governor of Lagash holds a vase with a double stream which could be entering the vase as well as coming out (fig. 12). We are told that these are the Tigris and the Eufrates, close to the confluence of which the city of Lagash was situated. Another representation of rivers is shown in fig. 13 from an illuminated manuscript of the 12th century. The streams pour out from a vessel which is held by a mythical figure. It is notable that the streams are very simplified and that as in many other depictions, the flow ends by winding around in spiral vortices.

Mountainous rivers, represented in Chinese art, usually included cascades and surges and were more realistic than depictions of the same time in the West. (See, for instance, Fan K'uan’s hanging scroll of the eleventh century, Travelling among streams and mountains, in Cahill 1977, p. 31).

A river that has been depicted many times in stereotyped representations is the Jordan, at the place where John baptized Jesus (fig. 14). We have to come down to the late Middle ages or early Renaissance to find more naturalistic representations of the Jordan, as that of Verrochio in the Uffizi Gallery in Firenze.

Vortices

Perhaps because the lower Nile was much less turbulent than the Tigris and the Eufrates, and other rivers of Mesopotamia, the vortex is represented much more rarely in one place than in the other. Also, in the other great civilizations, the vortex is less frequent. Intriguingly, the vortex is also less popular in science. Even in the teaching of fluid mechanics, one finds much less about vortices and vortical flow than about waves. This is probably because vortices are considerably more difficult to capture by the artist or to study by the scientist. In legends and myths, there is a similar lack as compared with waves and currents. But vortices are fascinating and challenging phenomena, and sometimes they are incorporated in a depiction without much apparent reason.

The Assyrians have left an abundant collection of vortices interspersed with waves (fig. 15), not in paintings, but in bas-reliefs or carvings in rock and in the walls of palaces. They loved depictions of battles, but some of the murals show less bellicose scenes. In all of them we see vortices with spiral lines, but once in a while there are strange configurations that surely seem amazing to any hydrodynamicist with an eye for such patterns. I will discuss these vortices in more detail in another contribution. In a much more recent depiction (fig. 16), there are configurations which seem somewhat "Assyrian", those...
of the Ramayana art in which we see streams forming large vortices, whereas the Assyrians instead appear to have represented waves with vortices.

Vortices can be seen mixed with waves in a number of depictions of different periods, like the turbulent sea around a medieval ship (fig. 17) and those in the middle of depressions among the strong waves that appear in many Japanese seas (fig. 16).

The pure vortex has a pattern of concentric circles [see, e.g., Lamb 1945, or Milne-Thomson 1968], and not spirals. The flow with spiral lines is that of the superposition of a sink and a vortex. However, almost universally artists have chosen the spiral to represent vortices. The exception seems to come from Mexican pre-Columbian art, where one finds concentric circles. Clearly further study is needed for an art that is much more cryptic than old world representative art and perhaps also more cryptic than modern art. I say this because modern art, being very subjective, it is often fruitless to look for what inspired the work in the outside world in order to find the sources of the inspiration. In Mayan art, however, we can be certain that art was linked to other things than aesthetic internal motivations. The soul of pre-Columbian art may be very difficult to understand for students with a Western cultural background. I personally feel much more at ease with Western art, and even with some manifestations of Asiatic art, while Indo-American art continually puzzles me. I hope my guess that they captured the pure elementary vortex turns out to be correct.

15. Vortices interspersed with waves in the walls of an Assyrian palace at Khorsabad. [Musée du Louvre, AO 19889, Photo R.M.N.]

16. Vortex dominated water surface in Ramayana art. [Drawing by E. Macagno.]

17. Waves interspersed with vortices around a ship. [Drawing by E. Macagno based on Medieval pictures.]

18. Hiroshige's stormy waves with vortices in their throughs. [Drawing by E. Macagno.]
Drops, tears, rain

Rain, being the most frequent visual and tactile contact with water for humans since time immemorial, more prevalent than river, lake or sea, why is there so little about rain in the art of all ages? Rain is made of drops; other water drops are tears, which are also very rarely represented, unless it is true that paisley is inspired in tears and not in the shape of a fruit [Shearer 1981]. It does not seem appropriate to count depictions of clouds or mist as representations of drops although Aristotle [Middleton 1965] already knew that clouds are formed by minute drops. There are depictions of drops of blood, found mainly in Indo-American and Christian art. I am including illustrations of rain and drops that are really not from antiquity (figs. 19 to 22), but they are certainly of interest in a comparison between the views of scientists and artists that will be part of one of the articles in this series. In connection with Tlaloc, a Mexican god of rain and water, there seems to be an interesting association between water and blood [Decrosse 1991], and depictions of Tlaloc may show this relationship (fig. 20). We also see drops depicted by Leonardo and Crivelli. They slide in one case, as part of an experiment, on an inclined plate, and in the other case along the face of Mary Magdalene as part of a painting (figs. 21 and 22).

It is interesting to note that falling drops have a stereotyped representation with a point in the rear; although falling drops of small size are usually quite spherical. In fact, the pointed shape seems to be typical of non-Newtonian liquids that are quite different from water and air.
Bibliography


Depictions of waves and jets by Hockney are much more appealing to the hydrodynamicist than other recent modern works. In this book, Hockney has included very interesting discussions concerning the surface of the water.

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