Aqua depicta

Representation of water in art and science

II

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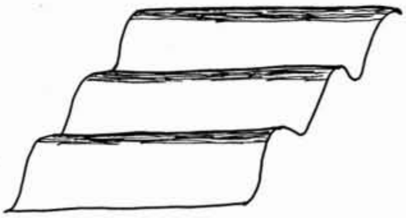
Foreword

This is the second paper concerned with representation of water through history by artists on one side and by scientists and engineers on the other. The emphasis here will be on the second group ; for this I must necessarily refer to the corresponding methodology, which becomes more and more dependent on physico-mathematical thinking as the historical narrative becomes more and more recent. The discussion of Leonardo da Vinci's or Agostino Ramelli's sketches of water flow has to be very different from that of later works, when the mathematics and the experimental observational techniques involved become more advanced and the studies increasingly quantitative rather than qualitative.

One must take into account that, even within the spirit of Leonardo's work, there is a great difference between the artist and the engineer and scientist. In his technico-scientific studies of water motion, he recognized the need for three-dimensionality, which is at the core of all analytical studies. In contrast, artists have remained almost unanimously on the surface, i.e., their approach has been two-dimensional. As we will see, there are very few exceptions to this statement, because artists observe and represent the water surface at a given instant and translate onto the canvas a perception and an abstraction of surface elevation, color and other properties. However complicated the artist's work may be, what we see in a painting are functions of two variables and not three (or four if time is also taken into account). For instance, the shape of the water surface depicted by an artist can be translated into the mathematic expression $h = f(x, y)$ in the usual Cartesian coordinates, where h is the elevation of each particle of the water surface. The engineer and the scientist consider that such elevation, as one part of their representation of water, must be shown as a time dependent variable.

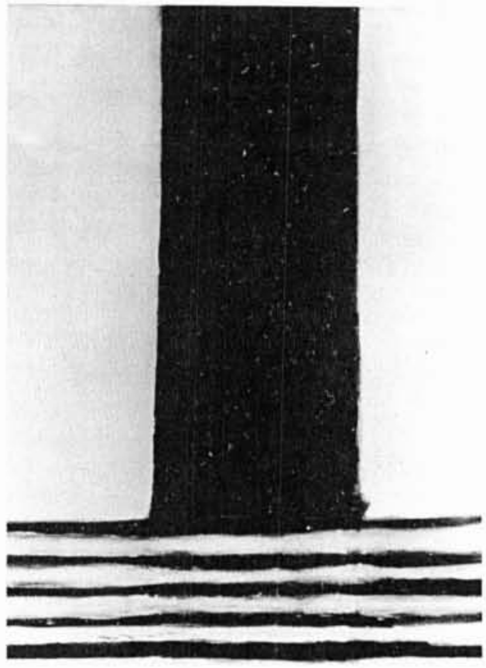
One could perhaps say that good artists can give a strong suggestion of motion in their depictions of water, and that they find a way of conveying a kinematical sense, a sense of dh/dt if not of $h(t)$. But very rarely do they add the other Cartesian coordinate, z, which scientists and angineers introduced early on because no study of water dynamics is possible without considering the elevation of all fluid particles, as well as any other property, as functions of x, y, z, t. Because discussion of this subject requires some technical language, readers acquainted with science and engineering will find interspersed, in this and following papers, material with which they are familiar, while others may find the same material rather unfamiliar. I will try to reduce technical material to a minimum and express my thoughts as simply as possible.

In depicting water, waves at the air-water interface are the most frequently represented by artists of all times ; no wonder that in many civilizations the most common symbol for water is the wave. Hence, we begin by describing the way in which scientists have dealt with water waves. Their views and methodology will be presented throughout this paper along with the views and abstractions (when extant) of artists. I will consider here mainly instances of regular systems of waves and leave more complex phenomena, such as the chaotic waves produced by storms and the intricate patterns due to the presence of walls and beaches, for the next articles.



1. Geometric representation of a water wave as a cylindrical surface.

2. Water waves that are nearly sino-cylindrical. Photograph taken by the author in the Iowa River.

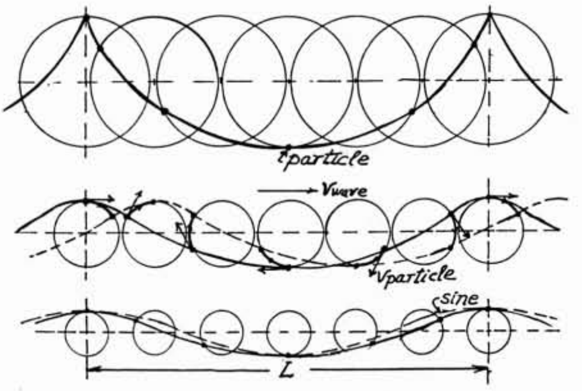


3. Modern artistic depiction of parallel waves. *Red River Valley, painting* by F. Stella. Courtesy of the Fogg Art Museum, Harvard University, Cambridge, MA.

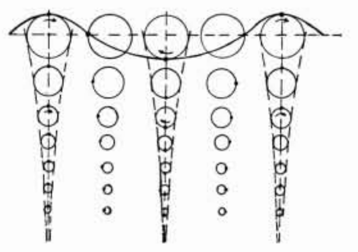
Elementary waves

In order to make the analysis of the forms of the water surface in a lake or in the sea accessible to a wide variety of readers, I will begin with some notions of hydrodynamics, in which we consider the simplest wave forms separate from any complicating factors, like the wind or obstacles and shores. In the middle of a big lake or at the sea, it is possible to find waves with crests and valleys arranged as parallel lines. As I discuss below, the surface can be classified mathematically as cylindrical, with generators that are straight and a directrix which is nearly a sine, or cosine, line, if the waves are not of large amplitude compared with the wavelength. Sometimes, it is useful to visualize surfaces by referring to common objects ; mathematicians often do this, for example when they explain a difficult point in the geometry of surfaces by referring to a « saddle » point because the surface they are speaking about resembles the saddle used in riding a horse. In this sense, we can refer to an object used in construction, the corrugated plates, to illustrate what is meant by a cylindrical surface with a sinusoidal directrix.

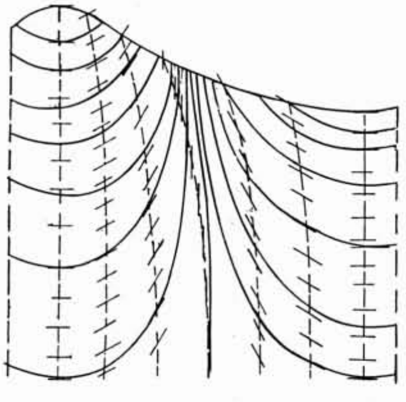
The notions I want to introduce can be better described if we refer to a cross-section of the water surface in the direction the waves are traveling, i.e., the direction perpendicular to the crests *(fig. 1)*. We should imagine the sine curve in figure 1 as the intersection, at a given instant, of the water surface (defined as cylindrical) with a vertical plane. In addition to this geometric sketch, I include photographic evidence *(fig. 2)* of water waves that are nearly sino-cylindrical together with an artistic depiction [by Stella 1958] of such waves showing the lines of parallel crests *(fig. 3)*. To illustrate the contrast between the views of a photographer or a painter and those of engineers and scientists I show in *fig. 4* some details of the motion of water particles, represented in a typical scientific way, which should be compared with sine curves depicted by artists that were presented in the previous contribution. It is not only that symbols and words are included in the present illustration : they were also added by artists, as in the calm sea in the mosaic at Monreale. We have in fig. 4 an attempt at describing the inner mechanism of this water motion, by indicating how the orbits of the water particles are compatible with the known fact that waves propagate rather quickly from one place to another. The added words and symbols (like the vectors) have a kinematic sense, the time is explicitly included in the orbits which are traversed by each particle at uniform speed. Time is a variable always absent from representations by artists, although some of them, in subtle ways, insinuate, if not the time, other kinematic features by strongly suggesting a sense of velocity and even of acceleration. We can see in any museum that many artistic depictions are not totally deprived of kinematics, and thus time is not entirely absent



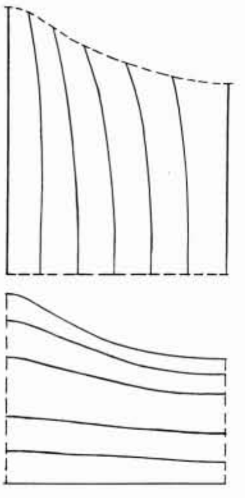
4. Wave profiles and orbits of particles for water waves of different amplitudes.



5. Orbits of particles in deep water. In shallow water, the orbits become elliptical with both axes decreasing with depth. At the bottom, the ellipses become completely flat.



6. Directions of the velocity field under a water wave. The solid lines are the streamlines.



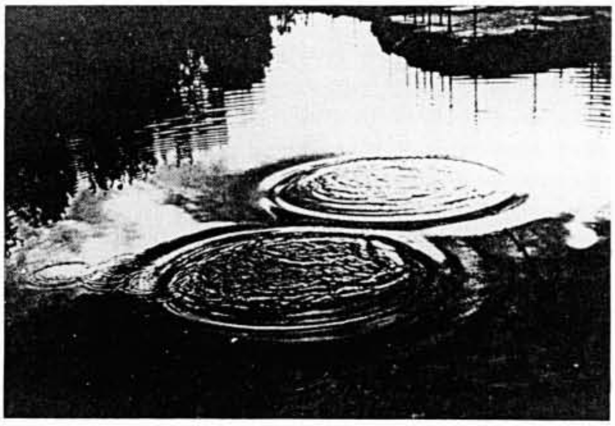
7. a-b Fluid lines. In 7a, the lines in the water at rest were vertical ; in 7b, they were horizontal.

from figurative art. But below the water surface there is also motion going on, and that is where a great difference in interest lies. I believe that the way any theme is treated depends strongly upon what one knows and what one does not know.

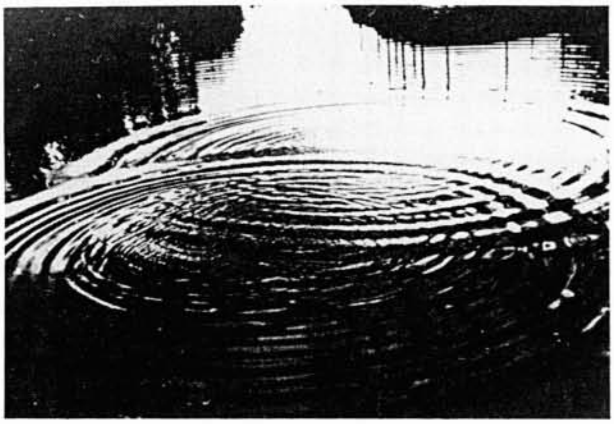
In *fig. 5,* I illustrate that in deep water, at the surface and below, particles have been found to move with constant speed along circular orbits. All the particles originally in the same vertical line move in phase, but the phase is different from one vertical line to another. Note also that the radii of the orbits decrease rapidly with depth. For shallow waters, meaning a depth relatively small compared with wavelength of the waves, the orbits become elliptical. Near the bottom the ellipses are very flat, and at the bottom they should theoretically become segments of straight lines. The discovery of the orbital motions of the water particles in water waves came, in fact, after the discovery of planetary orbits by Kepler.

We can observe that the water particles perform a motion quite different from that of the waves, without going to a hydrodynamics laboratory. Of course, particle and wave motions are related, but it is important to note that one is motion of matter and the other is motion of shape or form. We only need to throw a stick or any other floating body onto water in a place where waves are propagating in any given direction. By observing the motion of the floating object we will see that it moves up and down and back and forth, but that it stays on the average in the same place while waves pass by in rapid succession. A little cloud of dye may be still more convincing. All this may indeed be more detail than is needed presently, but I want to leave the reader with a strong sense that under waves much more is happening than what most casual observers are aware of.

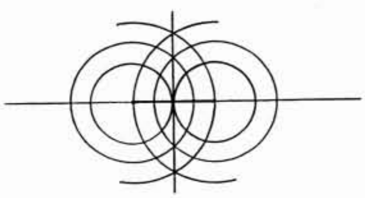
In trying to describe wave motion, scientists have developed several systems of lines, beside the pathlines sketched in *fig. 5*. The most commonly used are the streamlines, the streaklines and the fluid lines. Both the streamlines and the fluid lines will be illustrated here, not just because they are different and important in time-dependent motion, like that under waves, but also as a way to illustrate the multifaceted approach of science regarding motion. In *fig. 6,* we can see the direction-of-velocity field under a single wave length. The solid lines, whose tangent everywhere represents the velocity vector, are the so-called streamlines. In *fig. 7a* and *b,* I show fluid lines, which are those lines joining particles that were respectively vertical and horizontal lines when the water was at rest ; what we see is their shapes at a certain instant, as distorted by the passage of the waves. Like Leonardo da Vinci, we can compare the lines in *fig. 7a* with the stems of the plants in a wheat field as it is undulated by the wind. This is a famous and popular analogy but, in fact, the orbits of the water particles and those of the plant particles are not geometrically similar. Leonardo did not depict these waves, but they can be observed, e.g., in paintings and drawings by Vincent van Gogh [1889].



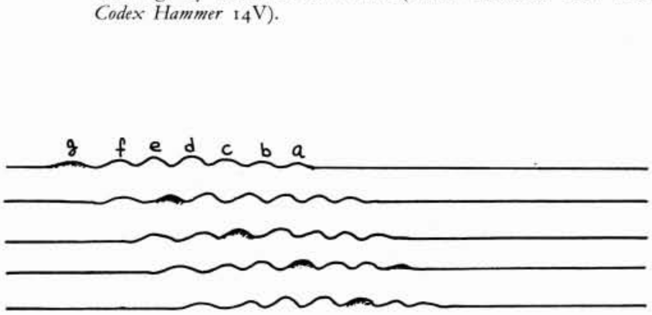
8. Circular water waves generated by throwing two stones into a pond.



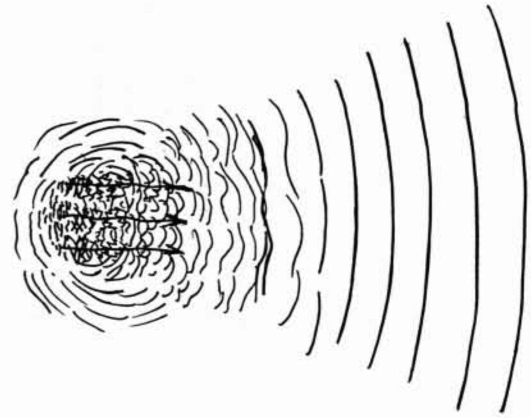
9. Superposition of circular wave trains shown in fig. 8 at an early stage.



10. Sketch for the superposition of circular waves based on drawings by Leonardo da Vinci. (*Codex Atlanticus* 266V and *Codex Hammer* 14V).



11. Evolution of a travelling group of waves. Drawing based on discussion and sketch by N. F. Barber [1969]. Waves disappear at the front as others form in the back of the group.



12. Qualitative schematic illustration of wind generated waves. Adaptation of a drawing by N. Bascom [1964].

13. Mechanical analogy at small scale of the wave generation by wind. The water falling down from a jet in a fountain produces a zone of turbulence in the center, from which waves emerge.

14. Depiction of a fountain with jet in the middle of a pool producing waves all around. V. Rysselberghe, *Springbrunnen in Sanssouci*. By permission from Bayerischen Staatsgemäldesammlungen, München, Germany.

Another seemingly simple water waves are those generated by throwing stones into a pool with otherwise still water *(fig. 8)*. These circular waves are similar to those produced by rain drops in the same pool. There is, however, an important difference ; if observed carefully one can see that the size of the object makes a difference, as we will find out soon when discussing waves controlled by different forces. But now let us look at the relatively larger waves due to the stone : they form what appears to be a perfect circular pattern which quickly propagates centrifugally. These waves are more complex, at least to the scientist, than the sinusoidal wave considered above. They have been much less frequently represented by either artists or scientists. They are very handy for the study of superposition of waves, because on only needs to throw two, or more, stones instead of one to see a beautiful intersecting pattern *(fig. 9)*. Already Leonardo da Vinci [see, e.g., Codex Atlanticus 266V, III-74 and Codex Hammer 14V, 23R] did some studies of superposition of wave patterns *(fig. 10)*. In this case, the scientist sees a few more things again, not only regarding internal motion, but the group of waves themselves ; they behave in a way that is approximately represented in *fig. 11*. Waves travel in this case also, but as they do, they not only change in shape constantly but some fade away and new ones form all the time. There is not only the speed at which any single wave travels, or phase velocity, but also other speeds, such as the group velocity and the modulation velocity. I mention these velocities without definition [the interested reader can see Kinsman 1965, Crapper 1984, Dean and Dalrymple 1984] only because I want to emphasize that in these so very common phenomena, each person may see many different things and arrive at different abstractions and representations, although something like a minimal common view is shared by all.

Generation of waves

To understand what different interpretations and depictions of water represent, it seems very helpful to have some elementary notions about the corresponding dynamics. In the case of waves, a common question is *how are they generated ?* Most waves are raised by the wind, although there are other causes which are usually as important in moderate or limited water expanses. When observing the water surface of rivers and canals we may notice waves that are very strong even without any wind present. They derive energy from that of the flow itself, or from a passing watercraft.

We should note here that from artistic studies of waves we generally can not deduce much about the mechanisms by which such waves come into existence, although some hints might be derived from the state of the water surface under different circumstances. I believe that the representations produced over several millennia would not reveal much about the generation of waves if we look only at those originating from the world of art. It is not usually the role of art to determine causes, but it is a role of science ; and this is one of the great differences between the two fields. There are a number of ways in which waves are known to come into existence. I know of only one explicit example of an artistic representation of one of the ways in which waves are generated [see in Article 1, fig. 6, Monreale].

In the Codex Hammer, when planning one of the books he never finally wrote, Leonardo da Vinci listed several different kinds of waves according to their causes :

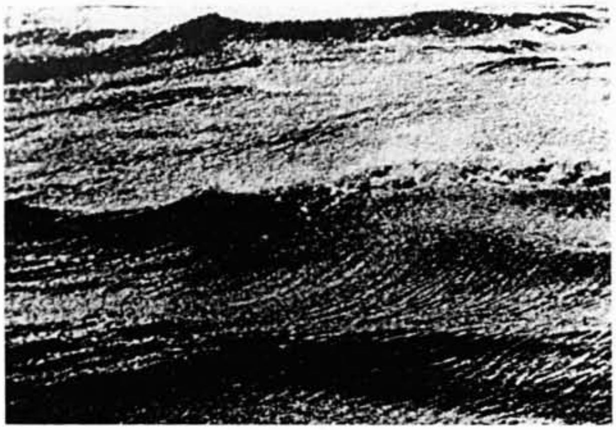
Book 2, of the diversity of water waves.

Water waves are produced by three different causes : the first is their own natural motion ; the second is the motion of the air which comes upon the water ; the third is impact of anything that hits the water. The wave caused by the own flow of the water can be of two kinds : the first has its length in the direction of the flow, and the second is transversal due to surelevation of the river or to any object opposing its flow [Codex Hammer 6R].

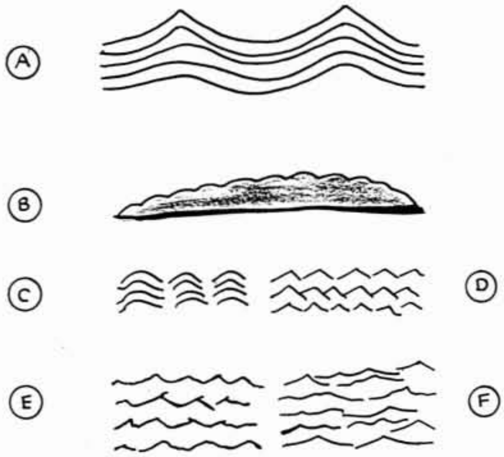
For large expanses of water, the main source of wave energy comes from the wind. On a perfectly smooth interface between air and water, the wind, if it were of uniform motion, would exert a tangential force, sometimes called friction drag. The flow thus generated in the water becomes wavy and turbulent very easily, due to instability phenomena ; once there is curvature of the interface, pressure can be exerted together with the friction drag and then waves can grow rapidly. But in the wind there are always uneven motions, and there are also vortices ; this tends to create different pressures on the water and the surface departs from being perfectly horizontal. Once the water surface has been disturbed more or less irregularly, we have undulations on which the wind can act, increasing their dimensions. In the region where this happens, the surface is far from exhibiting regular waves, but at some distance therefrom they emerge through a process in which certain irregular waves are filtered out. *Fig. 12* [adapted from Bascom 1964, fig. 16] is a representation of the whole picture of wave generation and should be compared with the one in the mosaic at Monreale ; we can see that there is a reversal of the sequence of stormy waves and regular waves. The Monreale artist had no doubt, a basic understanding of the hydrodynamic process implied and to say that there are errors in his representation maybe itself an error in judgement.

After leaving the region where they are generated, waves travel relatively fast and dissipate slowly. If big enough, they can reach far distant regions where the air is calm, where they appear to arise without any apparent cause. Cornish [see Bascom, p. 63] reported that a wave that came out of the South Atlantic was seen and measured on the Dover coast of England. According to Bascom, the size of wind waves depends on three factors : the velocity of the wind, the time period during which the wind blows, and the extent of open water over which the wind blows.

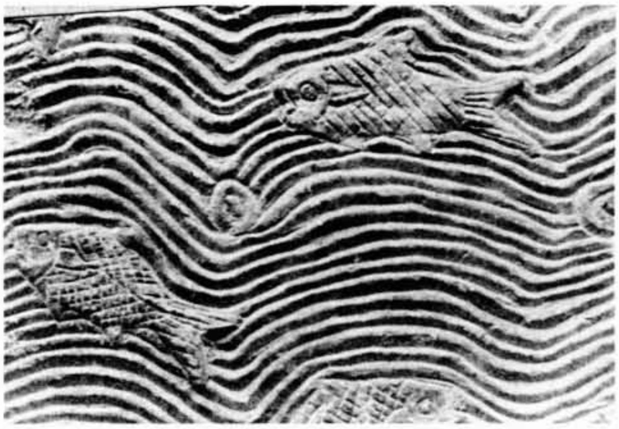
On a small scale, especially in some fountains with circular walls, one can observe a mechanical analogy of the generation of waves by the wind. There are fountains with a relatively large pool and a jet, or groups of jets, at the center ; the water falling down into the pool produces a turbulent chaotic effect nearby, but near the circular walls of the fountain one can observe rather regular waves that seem to have emerged through some kind of filtering process *(figs. 13 and 14)*.



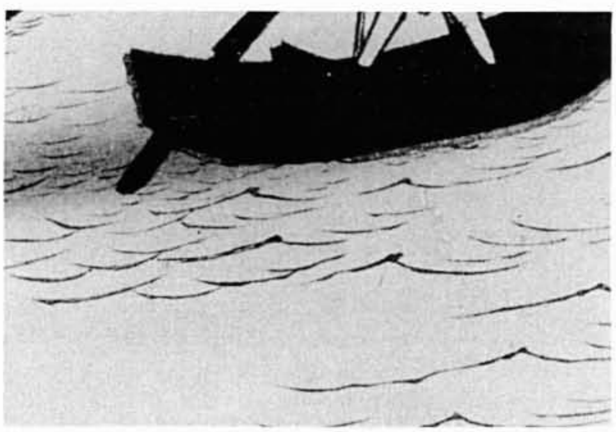
15. Capillary waves on the surface of gravity waves. From *Introduction to Water Waves* by G. D. Crapper. Compare with fig. 23, Courtesy of Ellis Horwood, Simon & Schuster International Group, Chichester, England.



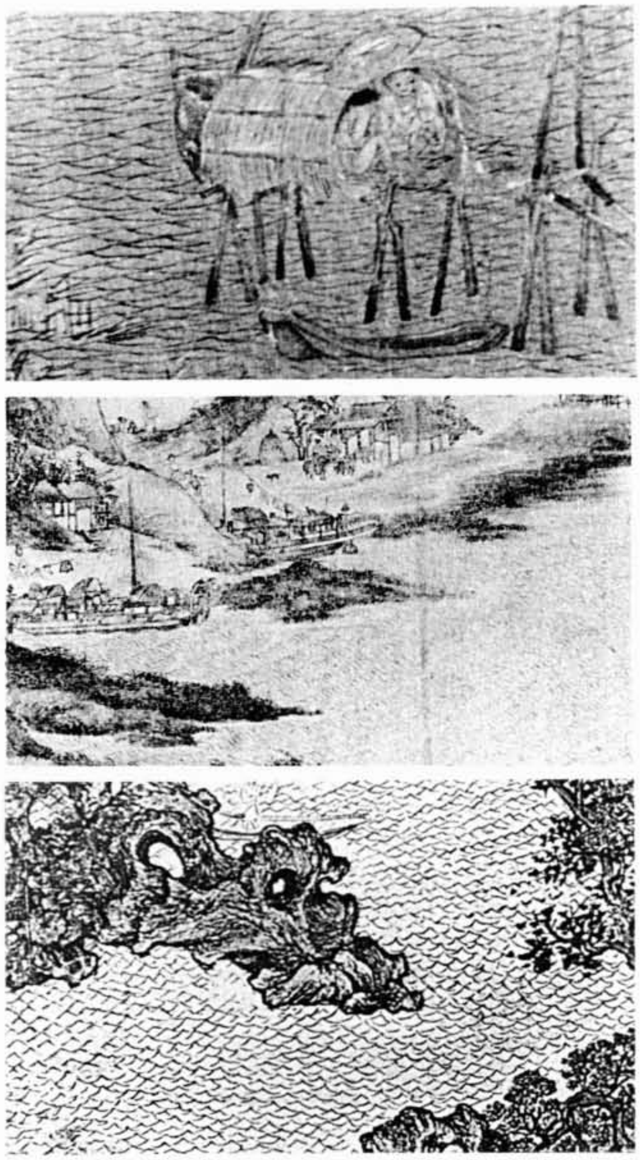
16. Typical wave profiles in art through the centuries. A. Assyrian, palace of Sennacherib. B. Greek, from "Tomba del Tuffatore", Paestum C-D. Chinese, from 10C to 14C. E. from the *Mustard Seed Garden Manual of Painting*. F. Japanese, from paintings by Hiroshighe.



17a. Trochoidal waves. Detail from an Assyrian mural in Sennacherib palace. Note that the usual profile is really upside down. The explanation may be that the artist was using routinely a shape learned correctly at the beginning. (See fig. 16a for the correct orientation). Courtesy of the Trustees of the British Museum.



17b. Trochoidal waves at the point of breaking in the painting *Voroi Ferry, Koami-Cho* by Hiroshige. Courtesy of G. Brazilier Inc. (See Plate 46 in *One Hundred Views of Edo*.).



18a-b-c. Three details of Chinese depictions of waves over five centuries. a) *A River journey at first snow fall,* Attr. to Chao Kan, 10C. b) *A market village by the river,* Anonymous handscroll (11-12C). c) *Forest Dwellings at Chü-ch'ü* by Wang Meng (14C). By permission of the National Palace Museum, Taichung, Taiwan.

Kinds of waves

From the point of view of the forces that govern their dynamic behavior, there are two kinds of waves : those dominated principally by the molecular forces which tend to maintain the water surface minimal (surface tension), and those dominated by the force of gravity. They are called, respectively, capillary waves and gravific waves. Rain drops, falling into a pool of water produce capillary waves and so does a rather small sudden breeze. A big stone falling into a pond causes gravific waves. There is also a range in which the waves have mixed behavior. Of course, capillary waves can play on the surface of larger gravific waves *(fig. 15)*. All this can be detected in paintings if one is aware of the existence of these phenomena.

From a more subtle point of view, which is a consequence of the kinematics of the particle orbits, there are the so-called sinusoidal, and trochoidal waves which were illustrated in *fig. 4*. This was already well understood in one of the earlier memoirs on the hydrodynamics of waves [Gerstner's Theorie der Wellen 1802]. Gerstner's approach did not become popular because it contained a particular vorticity distribution considered unrealistic. In art we find a much earlier recognition of the non-sinusoidal shape of most waves, especially those of amplitude large compared with its length. This is beautifully illustrated and described in the Mustard Seed Garden Manual of Painting [Sze 1959]. I have reproduced in *fig. 16* typical profiles of waves based on an illustration in that manual. In the same figures, I have included waves profiles obtained from a range of depictions extending over twenty five centuries and going from Assyrian waves to Japanese waves. In *Fig. 17a-b*, we have details of artistic depictions of trochoidal waves from those two extremes of the spectrum, while in *fig. 18*, we can see Chinese depictions of waves over several centuries.

Superposition of waves

Some aspects of the theory of water waves are simple and can afford to readers a general approach to basic information in a compact way. For instance, for single waves of relatively small amplitude, we can use a linearization of the differential equations of hydrodynamics which leads to the equation

$$\partial^2 h/\partial x^2 = (1/p^2)\ \partial^2 h/\partial t^2,$$

for the case of cylindrical water waves propagating in the x-direction [Kinsman 1984]. Herein, h is the height of a point on the water surface, t is the time and p is a parameter. To achieve this result, very important simplifications were introduced. If they are ignored the solution of the problem becomes extremely difficult, but not impossible, and much has been learnt about waves from non-linear theories [Stoker, 1957]. But for a discussion at an elementary level, we must rely mainly on the simplest theoretical approach.

The classical solution of the above partial differential equation, due to d'Alembert, takes a very general form :

$$z = F(x + pt) + G(x - pt),$$

but for the time being, we are going to consider a very special particular solution

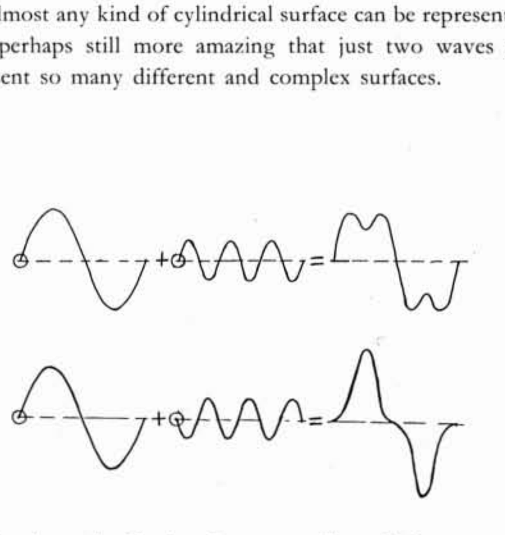
$$z = a \cos (kx - ct)$$

The great advantage of linear approximations resides in the possibility of adding simple particular solutions to obtain more general wave patterns. In more technical parlance, we can say that one can use Fourier's methods. Let us begin with the superposition of just two waves.

$$z = z_1 + z_2 = a_1 \cos (k_1 x - c_1 t) + a_2 \cos (k_2 x - c_2 t)$$

The wave amplitude is denoted by a ; k is related to the wave length by $L = 2\pi/k$, and c to the period by $T = 2\pi/c$. In *fig. 19*, I have illustrated two examples of superposition of two sinusoidal waves. Note the difference in the resultant waves.

If instead of using just two components, we assume an infinite number of waves, the theory shows the amazing fact that almost any kind of cylindrical surface can be represented. It is perhaps still more amazing that just two waves can represent so many different and complex surfaces.



19. As shown by Fourier, the superposition of sine waves can produce an almost infinite variety of profiles. In these two examples for water waves, the vertical scale has been exaggerated for the sake of a better illustration.

Wave profiles

From either a theoretical or empirical approach, simple waves can be classified by their profiles. This is the adequate criterion for waves depicted by artists since ancient times. But scientists can put this into equations, and when this is done in the form of a partial differential equation the power of the representations is indeed very great. The pure sinusoidal wave is difficult to find in nature, although it can be produced in the laboratory. It was surely accepted by artists much before scientists thought of this use of the sine function. The reason for its rarity in nature is because in natural settings a number of waves are usually present. But even if a wave were, so to speak, pure enough, whenever the amplitude is large relative to the wave length, the profile of the wave ceases to be sinusoidal. Water waves are essentially non-linear waves, and thus the shape of even the simplest waves of non-negligible amplitude is bound to show a non-symmetric profile. Larger waves have valleys or troughs that are wider than the crests. The larger the ratio of amplitude to length of the wave becomes, the stronger is this feature, until the crests become angular *(fig. 4)* in these cases, the wave is called trochoidal.

It is interesting, from a historical viewpoint, that before the developments of the modern theories of water waves, Gerstner offered a non-linear solution in which we can see that the profiles change with the relative value of the amplitude. Gerstner description is quite in accordance with observed facts though it is flawed by some fundamental subtle differences with the physics of the phenomenon. They can be discovered only if we investigate higher order kinematic properties of the motion under the surface.

A complete analysis of the superposition of two waves is unwarranted here, but at least we should examine a classic case in which the two amplitudes are the same and the differences in the values of k and c are relatively small compared respectively with k and c. We begin by noting that the expression for z can be changed into

$$z = 2a\cos(1/2)\,[(k_1 + k_2)x - (c_1 + c_2)t] \times$$
$$\times \cos(1/2)\,[(k_1 - k_2)x - (c_1 - c_2)t]\,.$$

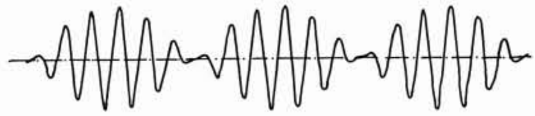
Because z is a function of two variables, x and t, to examine its behavior we may choose to observe it at a given x, as time passes, or to have a view at all positions at a given time. Let us make first $t = 0$, that yields

$$z = 2a\cos[(1/2)(\Delta k)x]\cos[kx]$$

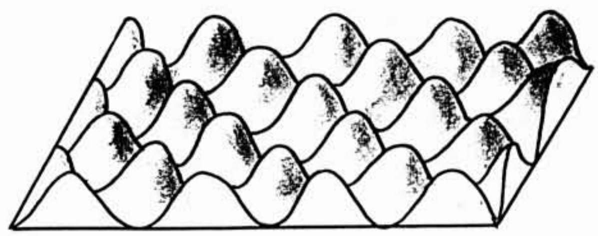
and then $x = 0$, that yields

$$z = 2a\cos[(1/2)(\Delta c)t]\cos[ct]\,.$$

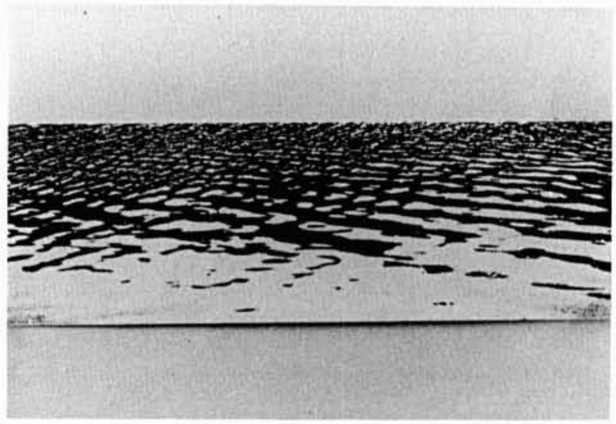
We have introduced $\Delta k = k_1 - k_2$ and $\Delta c = c_1 - c_2$. Note that because these differences are small we can set $k_1 \approx k_2 = k$ and $c_1 \approx c_2 = c$. This leads to waves that take the appearance of *fig. 20*.



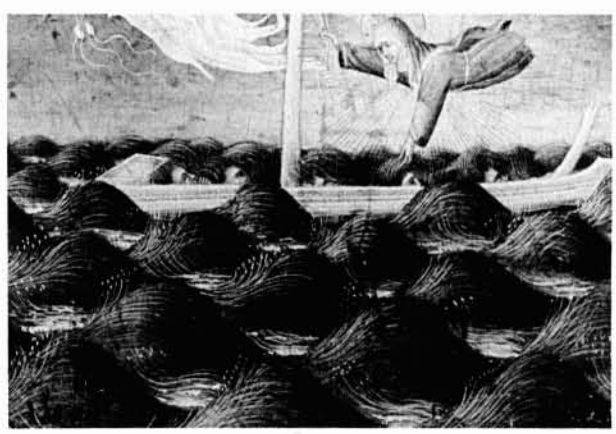
20. Another wave phenomenon described by superposition of simple sinusoidal waves. The vertical scale has been exaggerated as in some other figures.



21. Superposition of equal sinusoidal waves propagating in different directions.



22. Two trains of waves intersecting at an angle. Compare with figs. 21 and 23. Photograph taken by the author in the Iowa River.



23. Detail of *St Clare rescuing the shipwrecked* by Giovanni di Paolo (15C). Compare with fig. 7 in Article I, and also with figs. 15 and 21 in this article. By permission of the Staatliche Museum Preussischen Kulturbesitz, Berlin.

Wave trains and combinations of wave trains

Wave trains can be seen travelling in the sea and relatively large expanses of water at any time. If more than one travel in the same direction they always add up and result in a single, usually more complex, wave train. But waves propagating in opposite directions or at an angle produce quite different effects. Wave trains with the same characteristics, different only in having opposite direction of propagation, produce what is called standing waves or « clapotis ». I will discuss this case in the third article of this series because an interesting case of clapotis appears when waves arrive frontally to a vertical wall.

When wave trains travel in oblique direction the pattern that results is like a mesh of high patches or protuberances and hollows or depressions, and the sea looks rather agitated, and motion of form is difficult to follow, but an instantaneous view is relatively easy to draw or paint. One can also do some calculations and get a concrete idea of the properties of the resulting surface by adding the mathematical expressions for the two wave trains. I will consider only the case in which one direction of propagation is perpendicular to the other :

$$z = a_1 \sin(k_1 x - c_1 t) + a_2 \sin(k_2 y - c_2 t)\,.$$

In *fig. 21*, I show a drawing in perspective for which I did some calculations with the above equation for a fixed value of the time t. In *fig. 22* a detail of a photograph of intersecting waves is given. This figure should be useful to understand some representations of waves by artists of all times. When looking at the water surface under a relatively small angle, one easily develops the perception that the protuberances dominate the picture. This can be seen in depictions of stormy seas during a certain period. See, for example, fig. 7 of article I, where the agitated sea in which Saint Peter is sinking is taken from a mosaic at Monreale. See also *fig. 23*, that shows the stormy sea from which St. Clare rescues the shipwrecked [Giovanni di Paolo c. 1450].

Water as a mirror

Before closing this article, it seems warranted to describe briefly a study that will be included in another series in which I will discuss water as a reflecting surface capable of producing a never ending variety of images, most of them of great beauty. When water is perfectly still, it behaves very much as a mirror, and in fact some legends attest an early interest in this mirror before any other was made available to human beings.

The scene that is reflected in that perfect mirror is somewhat different from that that is observed directly. This was noted by Leonardo da Vinci who gave the simple explanation of different angles under which the object in question is actually seen. Indeed, it is much more interesting to study the images in water when it ceases to be at rest. As soon as small ripples appear, we begin to appreciate how many different modifications can be introduced even in very simple objects. We begin by seeing the contour full of undulations, and end up, when chaotic waves are present, by being unable to tell what is that is reflected. The succession of highly distorted images becomes so quick that we can sense only a rapidly changing distribution of colors in the water surface. Compare, for example, two paintings of the same scene by Monet : *The Ducal Palace* [Venice 1908] and *The Doge's Palace* seen from San Giorgio Maggiore [Venice 1908].

For the case of regular waves, we find in Leonardo's manuscripts an interesting discussion of the way in which the image of the Sun is seen in a train of waves. He uses in this case a knowledge of the law of reflection and some simple geometrical considerations [Codex Atlanticus 555V, VII-10]. Of course, Leonardo's studies can be extended to include the different kinds of waves and laws of physical optics as well as parameters like the quality of water and the state of the air. More interesting from the viewpoint of this series is perhaps the historical fact that reflections appear in paintings of ancient times rather rarely. There seem to be civilizations that never depicted a reflection in water. Early examples I have been able to find are given by Roman mosaics, (see e.g. reflections of aquatic animals in *Fauna of the Nile* National Museum of Naples, or *The Nile* in the Palestrina Museum). Of course, in recent times, if we speak in terms of centuries, there has been a great number of depictions of all kinds of reflections in wavy waters, predominantly in the Western world. There are certainly cases in which one, before some paintings, does not know what to admire more, the treatment of the scene depicted or its image in the wavy water surface.

Conclusion

At this juncture, it seems appropriate to end this contribution with some comments reflecting thoughts inspired by the topics here considered. It is often said that art is the beginning of knowledge for mankind. This seems true especially when one reflects that the split of fine and practical arts is a relatively recent event. However, knowledge may have had its beginning as much in technology as in art. From old times, in many depictions, figurative art reflected a keen understanding of motion. Knowledge originated in art tends to be of a qualitative nature, but we should not forget among other examples the studies and practice of perspective as an exact science by a number of painters. Quantitative studies of the human figure were part of serious concerns about proportions with the belief that in that case beauty responded, like music, to simple exact laws. More recently, we can mention the studies of the impressionists concerning the perception of color. It is only in relatively recent times that science has made of qualification one of its distinctive, if not exclusive, attributes. The difference lies not so much on quality versus quantity as in the approach and in the goals. In art, when it is figurative, description seems to be the essential feature, while analysis and synthesis characterize science while, for technology, prediction and function are typical.

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Credit for fee-authorizations is given with each figure. The participating institutions are the following: Fogg Art Museum, Harvard University, Cambridge (*fig. 3*, by F. Stella), Bayerischen Staatsgemäldesammlungen, München (*fig. 14*, by V. Rysselberghe), National Palace Museum, Taichung, Taiwan (*fig. 18*, Attr. to Chao Kan, 10C, Anonymous handscroll (11-12C), Wang Meng (14C)), Staatliche Museum Preussischen Kulturbesitz, Berlin (*fig. 23*, by Giovanni di Paolo (15C)).

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References

These references are divided in two sections. This is done because paintings and drawings are as important as papers, and because more than reading other papers, my readers should look at depictions of water by artists, and drawings by engineers and scientists if they want to pursue this initial study of *Aqua Depicta*. Thus the first section is the usual listing of books and papers. The second is a list of works of art that the reader may find interesting.

Books and Papers

BARATTE François (1978). — *Catalogue des mosaïques romaines et paléochrétiennes du Musée du Louvre*. Réunion des Musées Nationaux. Paris.

This catalogue may be useful to study the treatment of water waves in Roman mosaics. (See Fig. 4 of Article I).

BARBER N. F. (1969). — *Water Waves*. Wykeham Publ., London. A subsidiary of Taylor & Francis Ltd. London and Winchester.

BASCOM Willard (1964). — *Waves and Beaches. The Dynamics of the Ocean Surface*. Doubleday and Co. Garden City, New York.

CORNISH Vaughan (1934). — *Ocean Waves and Kindred Geophysical Phenomena*. Cambridge University Press. Additional notes by Harold Jeffreys (p. 121-159).

CRAPPER G. D. (1984). — *Introduction to Water Waves*. Ellis Horwood Lys. Publ., Chichester, a div. of John Wiley and Sons. New York.

DEAN Robert G. and DALRYMPLE Robert A. (1984). — *Water Wave Mechanics for Engineers and Scientists*. Prentice Hall, Englewood Cliffs, NJ.

GERSTNER (1802). — *Theorie der Wellen*. Abhand d. k. böm. Ges. d. Wissen.

KINSMAN Blair (1984). — *Wind Waves. Their Generation and Propagation on the Ocean Surface*. Dover, New York. (First publication in 1965).

LEBLOND Paul H. and MYSAK Lorenz A. (1978). — *Waves in the Ocean*. Elsevier Scientific Publ. Co. Amsterdam.

STOKER J. J. (1957). — *Water Waves. The Mathematical Theory with Applications*. Interscience Publ. Inc., New York.

SZE Mai-Mai (1959). — *The Way of Chinese Painting. Its Ideas and Techniques*. Random House Inc., New York.

TRICKER R. A. R. (1965). — *Bores, Breakers, Waves and Wakes. An introduction of waves on water*. American Elsevier Publ. Co., New York.

Paintings and drawings

DI PAOLO Giovanni c. (1450). — *Saint Clare Rescuing the Shipwrecked*. Staatliche Museum, Berlin-Dahlem., 1978 (2nd revised edition).

The catalogue description of this small part of a predella referes to *"the tossing storm-lashed sea"*. Both, Di Paolo and his commentator half a millenium afterward, understood that the stormy sea impresses us mainly by its protuberances. Of course, they are in fact chaotic and not as regular as depicted by Di Paolo. Compare with the stormy sea in Monreale's mosaics (fig. 7, Article I).

HIROSHIGE (1857). — *Yoroi Ferry, Koami-Cho*. Brooklyn Museum, New York.

This painting and others with similar waves have been reproduced in a book entitled *One Hundred Famous Views of Edo,* George Brazilier, 1986. The original prints are in the Brooklyn Museum.

LEONARDO da VINCI, c. 1500. Codices and Manuscripts. Hundreds of Leonardo's drawings represent water flow. The two codices listed below contain passages mentioned in the text of this article.

Il Codice Atlantico, published by Commissione Vinciana, Giunti-Barbèra, Firenze, Italy, 1975-80. There is another publication by Reale Accademia dei Lincei, Hoepli, Milano, 1894-1904. See also Macagno E., 1989. *Leonardian Fluid Mechanics in the Codex Atlanticus, I-III,* IIHR Monograph No. 105, The University of Iowa, Iowa City, IA.

Il Codice Leicester, publ. by R. Istituto Lombardo di Scienze ee Lettere, Giunti-Barbèra, Firenze, Italy, 1980. Now this codex is called Codex Hammer. See also Macagno E., 1989, *Leonardian Fluid Mechanics in the Codex Hammer,* IIHR Monograph No. 101, The University of Iowa, Iowa City, IA. For circular waves on water see Codex Hammer 12R, 23R.

STELLA Frank (1958). — *Red River Valley*. Fogg Art Museum, Harvard University, Cambridge, MA.

When I first looked at this painting I did not think that I could use it in my study. Years later, a more considered analysis makes me think that it is a good example of abstracting a view of some contour lines of the wavy water surface. Contour lines are used by scientists to represent water waves of more complex shapes, but not for sinusoidal waves.

VAN GOGH Vincent (1889). — *Mountain Landscape Seen across the Walls*. Ny Carlsberg Glyptotek, Copenhagen.

VAN GOGH Vincent (1889). — *Wheat field with cypresses*. National Gallery of London.

Van Gogh, who surely knew less about flow and waves than Leonardo, reflected magnificently motion and flow phenomena in his pictures.

Démolition de la machine de Marly.