

Flow analysis with radio-isotopes

APPLIED TO A HYDRO-ELECTRIC PLANT

L'analyse de l'écoulement au moyen d'isotopes radio-actifs

APPLICATION A UNE INSTALLATION HYDROÉLECTRIQUE

BY

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The relative dilution method combined with a radioactive γ —emitting tracer, has been used to determine the flow through a power plant with low head and short water ways. To counteract the incomplete mixing measurements were performed at 20 different points for each of which two determinations were made. The accuracy of the final value in this first trial was estimated at about 4%. It is pointed out that the accuracy attainable with this method is mainly a question of enlarging the number of measurements (injection number multiplied by detector number). The concentration records made at the mouth of the draft tube, using "in situ" scintillation detectors as probes, were also used to localize flow disturbances, e.g. separation from the ceiling of the draft tube. The constructor of the power plant is hereby given a possibility to check the quality of the conduit shape more exhaustively than is possible in model studies. An analysis of flow parameters has been performed for a straight hypothetical tube giving the same dispersion as the actual water way. The quality of the water way shape has been expressed as the friction coefficient of this tube. A trial has been made to calculate parameters for small scale as well as large scale eddies.

Combinée avec un indicateur à émissions γ , la méthode de dilution relative a été employée pour déterminer l'écoulement au travers d'une installation hydroélectrique de basse chute et ayant de courts chemins d'eau. Afin de tenir compte d'erreurs possibles dues à un mélange incomplet, les mesures ont porté sur une vingtaine de points différents pour chacun desquels on fit deux déterminations. La précision de la valeur définitive découlant de ce premier essai est évaluée à 4%. Les auteurs font remarquer que la précision obtenue avec cette méthode croît avec le nombre des mesures (nombre des injections multiplié par le nombre des détecteurs). Les mesures de concentration faites au débouché de l'aspirateur (pour lesquelles des détecteurs de scintillation placés à l'endroit même ont été employés comme capteurs) ont été également employées pour situer les perturbations de l'écoulement, telles que les phénomènes de décollement du plafond de l'aspirateur, par exemple. Cette méthode permet au constructeur des groupes d'effectuer un contrôle de la qualité du profil de la conduite plus complètement que cela ne serait possible par une étude sur modèle. Cette étude présente également une analyse des paramètres d'écoulement pour un tube fictif rectiligne présentant la même dispersion que la conduite « nature ». La qualité du profil de la conduite a été exprimée par le coefficient de frottement du tube considéré et un essai de calcul a été fait en vue de déterminer des paramètres valables tant pour les petits tourbillons que pour ceux de plus grande étendue.

INTRODUCTION

Since the end of the nineteenth century dilution methods combined with chemical or colorimetric technique to determine the concentration of the tracers, have been used for flow measurements in hydro-electric plants and natural streams [1]. Two types of dilution methods may be distinguished: one using a continuous injection of tracer and measuring the concentration in the homogenous region, the other introducing the tracer instantaneously and measuring the mean concentration in the tracer wave. The latter is often named the relative dilution method and has been used since 1926 [2, 3, 4, 5]. The relative dilution method was in 1958 combined with radio isotopes and was called the total-count method [6]. This technique was used at the Swedish Meteorological and Hydrological Institute for the first time in 1958 in a flow measurement in a natural stream and in 1959 also for turbine testing [7]. Lately the method [8] and its application to hydro-electric plants [9] has been discussed in this journal.

In the relative dilution method the flow is determined from the expression :

$$Q = \frac{M}{\int_0^T c dt} \quad (1)$$

where:

- Q means the flow;
- M the amount of tracer injected;
- c the concentration of the tracer at the time t at the gauging section;
- T the time used by the tracer cloud to pass the gauging section.

The relation above is valid only when the mixing is complete i.e. the integral has to stay constant over the gauging section. As will be shown by the present investigation, however, the method can be applied even when this condition is not fulfilled as, for instance, in a power plant with a low head of water and short water ways. Furthermore, the data collected can be used not only to evaluate the flow but also to determine disturbances in the flow as well as some parameters of the turbulence.

I. — EVALUATION OF FLOW AND EFFICIENCY

The power plant, equipped with Kaplan turbines, has a head of water of about 12 m. During the investigation the flow was about 200 m³/s. The shape of the water ways is shown in Figures 1 and 2.

The radio isotope, a few tenths of a gram ²⁴Na₂CO₃ or about 15 millicuries, was dissolved in water and injected instantly in one point near the intake. Two scintillation detectors, combined with recorders, measured the concentration *in situ* in a section about 75 m downstream at the mouth of the draft tube. This means a distance between the point of injection and the gauging section of only about seven diameters as calculated for a circular tube of the same mean area as that of the draft tube. Measurements were made in 10 different verticals with two measuring points in each. As a rule, two injections were made for every vertical. Thus totally 40 recordings of the concentrations as a function of time were made.

As mentioned above, the flow is given by the expression (1) when the mixing is complete. Preliminary investigations proved that the mix-

ing was most incomplete in the power plant investigated. Therefore the integral in the denominator in expression (1) was replaced by its mean value from k individual measurements at points distributed over the gauging section :

$$Q = \frac{M}{\frac{1}{k} \sum_0^T c dt} \quad (2a)$$

This is an approximation which is good if the points are suitably chosen and if the correlation is low between the velocity and the tracer concentration at each part of the gauging section. When the gauging section is split up in a large number k of small areas A/k the flow can in fact be written [18]

$$Q = \frac{M}{C} = \frac{1}{C} \frac{A}{k} \sum_1^k \int_0^T dt (u_i - \bar{u}) (c_i - \bar{c}) \quad (2b)$$

where A is the area of the gauging section; u_i and c_i are the velocity and concentration at a

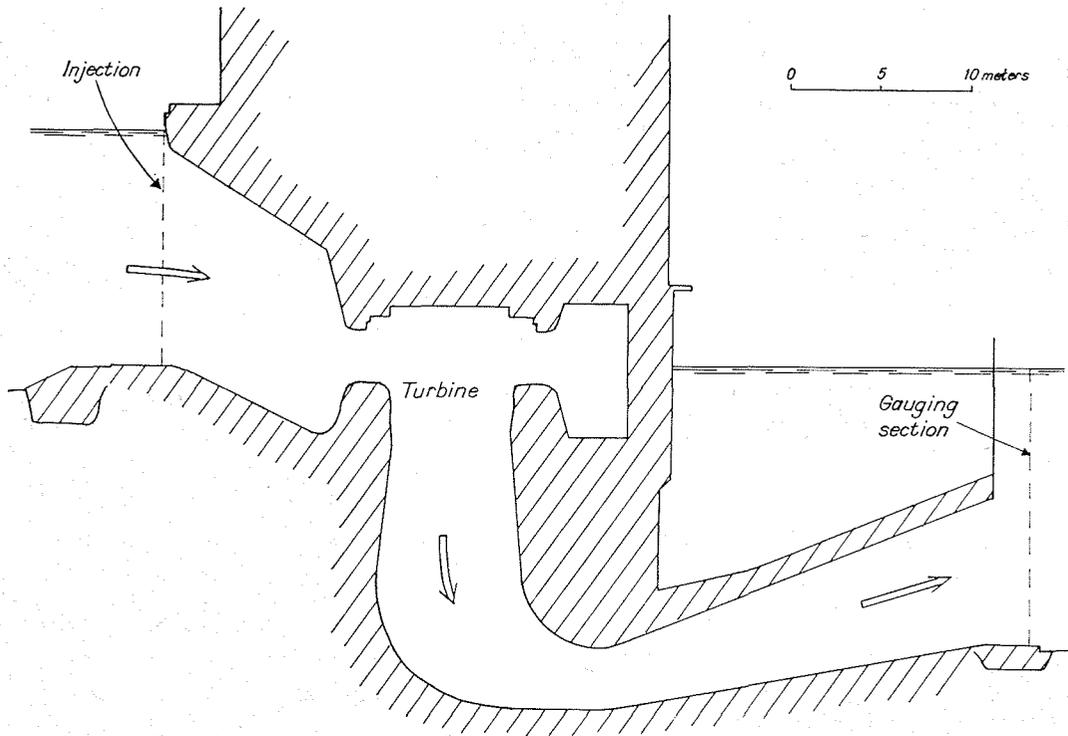


FIG. 1

Vertical section of the power plant, showing the water ways.

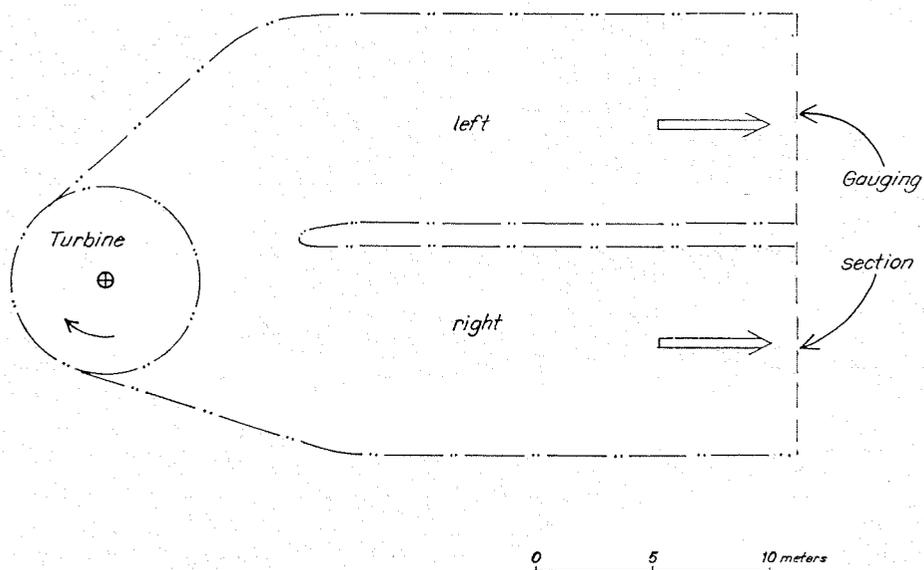


FIG. 2

Horizontal section of the draft tube.

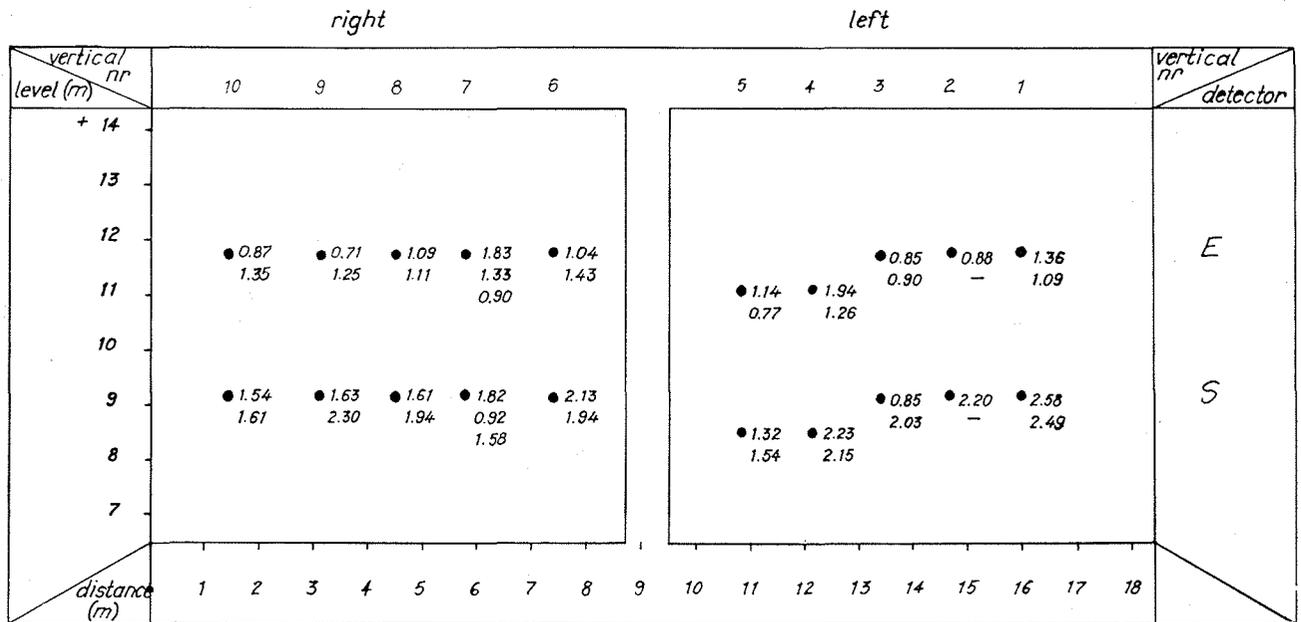


FIG. 3
Variations of the relative concentration over the two parts of the gauging section at the downstream end of the draft tube. Flow direction out from the paper.

point in one of the small areas; \bar{u} and \bar{c} their mean values over the section:

$$\bar{u} = (\sum u_i)/k = Q/A, \quad \bar{c} = (\sum c_i)/k;$$

and $C = \int_0^T \bar{c} dt$.

Since the velocity and the tracer concentration at the various areas should be essentially uncorrelated, the second term can be neglected and the expression (1) replaced by expression (2 a).

At each measurement a certain volume v of the radioactive solution was injected. The calibration of the instrument was made in a large volume V_0 of water, in which a volume v_0 from the same mother solution was uniformly dissolved. The expression (2), transformed into suitable quantities, is then:

$$Q = \frac{n_0 V_0 k e^{\alpha \Delta t}}{v_0 \sum \frac{1}{v} \int_0^T n(t) dt} \quad (3)$$

where:

- n_0 is the net counting rate of the detector at the calibration;
- $n(t)$ the counting rate at the time t ;
- $e^{\alpha \Delta t}$ a correction for the radioactive decay between the measurement and the calibration.

The efficiency η , the determination of which was the primary aim of this investigation, is obtained from the expression:

$$\eta = \frac{P}{9,8 \Delta H Q} \quad (4)$$

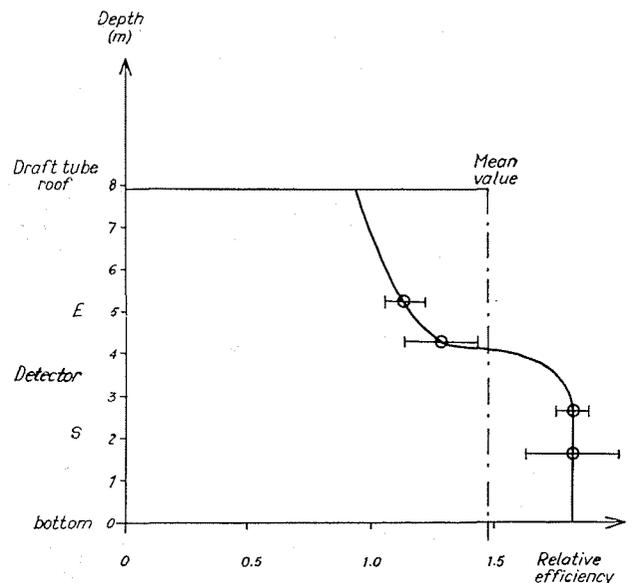


FIG. 4
The relative efficiency as a function of the depth.

where P is the electric power in kW and ΔH is the head of water in meters and Q is measured in m³/s. A combination of the expressions (3) and (4) gives:

$$\eta = \frac{v_0 e^{-a\Delta t}}{9,8 n_0 V_0} \frac{1}{k} \sum \frac{P}{\Delta H v} \int_0^T n(t) dt \quad (5)$$

which can also be looked upon as a mean of k different efficiency values. The numerical values of the individual integrals were evaluated from the recorder papers.

The variation of the relative concentrations

over the gauging section is shown in Figure 3. No trend is observed in horizontal direction. In vertical direction however there is a pronounced variation with the depth. The standard deviation observed at the two instruments levels, named the E- and S-levels, was 15 and 20 % respectively. To determine the mean efficiency for the whole gauging section a graphical procedure was used (Fig. 4). The efficiency thus evaluated coincided, within the estimated errors, with the value determined from conventional current meter measurements. The accuracy in this first trial was estimated at about 4 % (standard deviation of the mean value).

II. — FLOW DISTURBANCES AND TURBULENCE PARAMETERS

1. Theory of dispersion in water ways.

The present data, primarily intended for the determination of the efficiency, can also be used for a more detailed study of the flow through the conduits of the power station. Already from the very shape of the individual concentration-time distributions (c-t curves) some information about the turbulence generated in the water ways can be gained. To get a deeper knowledge about the turbulence, it has seemed desirable to make a comparison with the theory available at present, which is concerned with dispersion in straight tubes and open water ways. Some experiences from this field have also been used.

Turbulent dispersion of a tracer material in water is described by the following expression, if effects due to molecular diffusion, density variations, chemical and biological factors can be neglected:

$$\frac{\delta c}{\delta t} + \bar{u} \frac{\delta c}{\delta x} = \frac{\delta}{\delta x} \left(k_x \frac{\delta c}{\delta x} \right) + \frac{\delta}{\delta y} \left(k_y \frac{\delta c}{\delta y} \right) + \frac{\delta}{\delta z} \left(k_z \frac{\delta c}{\delta z} \right) \quad (6)$$

where k_x, k_y, k_z are diffusion coefficients in the directions x, y, z respectively; \bar{u} is the mean velocity and c the concentration at a point (x, y, z) . The x -axis represents the direction of flow.

Taylor [10] discusses this equation applied to dispersion in a straight uniform tube. He makes use of a reference system, moving with the mean velocity of the flow U in the direction

x_1 , for which case the differential equation is transformed to:

$$\frac{\delta c}{\delta t} = K \frac{\delta^2 c}{\delta x_1^2} \quad (7)$$

where $x_1 = x - Ut$ and c represents the mean concentration over a tube section. K represents a virtual diffusion coefficient including turbulent dispersion effects due to variations of velocity over the tube section (radial diffusion) and secondly also effects due to longitudinal diffusion. K can also be interpreted as a quantity that reflects the wall friction in the tube. Taylor has shown that for flow through both smooth and rough pipes the following formula holds:

$$K = \beta \sqrt{\gamma a U} \quad (8)$$

where a is the radius of the pipe, β a numerical coefficient, and γ the friction coefficient. For a straight pipe the theoretical value of β , borne out by experiment, is 7.14, but for curved pipes the value is found to be greater.

In the case of a plane, instantaneous injection, the solution can be written [11, 12]

$$c(x, t) = \frac{M}{A \sqrt{4 \pi K t}} e^{-(x-Ut)^2/4Kt} \quad (9)$$

where $c(x, t)$ is the concentration at the time t at the distance x from the injection, A the cross section.

Experimentally plane injections are often hard to attain. The experiment can therefore be simplified by making the injection in a single point. In this case the dispersion initially occurs in both radial and longitudinal direction. Through the reflection from the tube walls, however, a smoothing out of the concentration variations in radial direction is rapidly attained. This leads to a concentration distribution, approximately corresponding to the one received from a plane injection. Until such complete mixing in radial direction has been reached, i.e. until this approximation is good enough, the mean value of the concentration \bar{c} over the section can replace $c(x, t)$ in the expression (9). Thomas [11] has used the same approach to two-dimensional dispersion in natural water ways. The mean value of the concentration might be still more improved through the increasing of the number of injections.

The concentration distribution in expression (9) is for a constant value of t a normal distribution, symmetrical around $x = Ut$. The maximum has the value:

$$c_{\max} = \frac{M}{A\sqrt{4\pi Kt}} \quad (10)$$

The standard deviation σ_x of the tracer cloud at the same time t is given by:

$$\sigma_x^2 = 2Kt \quad (11)$$

or:

$$\sigma_x^2 = \frac{x_{1/2}^2}{8 \ln 2} \quad (12)$$

where $x_{1/2}$ stands for the distance, over which half the maximum concentration is exceeded. The concentration distribution for a given value of x has a maximum which exceeds the maximum given by the expression (10) and more the larger the ratio K/Ux is. The maximum will pass the section at a time t_{\max} , which is earlier than the time of arrival of the centre of mass, $t_{\text{cm}} = (x/U)$.

The standard deviation σ_t of the $c-t$ distribution is given by

$$\sigma_t^2 = \frac{2Kt_{\text{cm}}}{U^2} \left(1 + \frac{6K}{Ux} \right) \quad (13)$$

The ratio t_{\max}/t_{cm} as well as the ratio $\sigma_x^2/\sigma_t^2 U^2$ is a function of the factor K/Ux . When, for

instance, this factor is 10^{-2} the difference between the maximum concentrations of the two distributions is 0.5 %, and between the two corresponding times 1 %. The difference between σ_x^2 and $\sigma_t^2 U^2$ is then 6 % [11].

The purpose of the present calculations was, as mentioned above, not only to determine the flow, but also to try to evaluate parameters which could be used to describe the turbulence in the water ways of the plant. The statistical theory of turbulence [11, 13, 14] offers some possibilities. In this theory it is generally assumed that the motion can be separated into a mean flow and a superposed turbulent flow, the mean value of which is zero. The instantaneous velocity in the direction of flow in a certain point is then given by $u = \bar{u} + u_1$, where u_1 is the additional velocity from the mean value \bar{u} in the same point (the turbulent fluctuation).

The intensity of the turbulence can be expressed by the standard deviation of the instantaneous velocities. In the isotropic case, where the mean values in the three coordinate directions of the additional velocities squared are equal, the relative intensity of turbulence is given by:

$$\sqrt{\frac{u_1^2}{\bar{u}^2}} \quad (14)$$

Taylor [10] proved that in the case of flow at a constant speed through a long uniform pipe the virtual diffusion coefficient (dispersion coefficient) for sufficiently large times can be written as:

$$K = \overline{u'^2} \int_0^\infty R(\tau) d\tau \quad (15)$$

where u' is the difference between the local velocity u and the mean velocity U of the flow, $u' = u - U$, and $\overline{u'^2}$ is the mean square velocity deviation. $R(\tau)$ is the correlation between the velocity in the direction x of a particle at one instant of time and that of the same particle a definite time τ later. The integral is a parameter which represents the time scale of the turbulence; for sufficiently large values of τ the function $R(\tau)$ falls to zero and the integral becomes finite:

$$\int_0^\infty R(\tau) d\tau = t_0 \quad (16)$$

In this case the dispersion coefficient can be written as:

$$K = \overline{u'^2} t_0 \quad (17)$$

The expressions (11) and (15) combine to:

$$\sigma_x^2 = 2 \overline{u^2} t \int_0^\infty R(\tau) d\tau \quad (18)$$

Before the integral has reached its final value, the correlation function $R(\tau)$ can be evaluated as:

$$R(\tau) = \frac{1}{2 \overline{u^2}} \frac{d^2(\sigma_x^2)}{d\tau^2} \quad (19)$$

Thus, if the standard deviation σ_x is known as a function of time as well as $\overline{u^2}$, the relation between the correlation coefficient $R(\tau)$ and the time τ can be determined. As far as diffusion is concerned the length

$$l = \sqrt{\overline{u^2}} \cdot t_0 \quad (20 a)$$

is analogous to a mixing length. If the turbulence is superposed on a mean velocity \bar{u} , the length scale can also be expressed by the parameter:

$$L = \bar{u} t_0 \quad (20 b)$$

In the following calculations, an experimental work by Kalinske and Pien [15] will be used to determine the function $R(\tau)$. In a model these authors studied the dispersion of materia by turbulence in a straight open channel. The tracer was injected in one point at mid-depth. Their investigation showed that the distribution in vertical direction downstream the point of injection was a Gaussian, which is in agreement with an expression of the same type as expression (9). They were able to evaluate local values of the coefficient of dispersion from the relation between the vertical tracer spread and the distance from the section of injection. This dispersion coefficient was also evaluated in vertical sections near the point of injection, where the integral in equation (16) has not reached the final value t_0 . Using expression (19) they showed that the correlation coefficient R , as had been pointed out by for instance Dryden [16], varied as (*):

$$R = e^{-t/t_0} \quad (21)$$

(*) Kalinske and Pien used x as a variable instead of t by replacing t by x/U .

which is consistent with the definition (16) of t_0 . It follows from (19) and (21) that:

$$\sigma_x^2 = 2 \overline{u^2} t_0^2 \left[\frac{t}{t_0} - (1 - e^{-t/t_0}) \right] \quad (22)$$

For $t \gg t_0$ this becomes :

$$\sigma_x^2 = 2 \overline{u^2} t_0 \cdot [t - t_0] \quad (23)$$

which should be compared with the expression (18) in the form :

$$\sigma_x^2 = 2 \overline{u^2} t_0 t.$$

2. Analysis and results.

a) FLOW DISTURBANCES:

As already mentioned, a total of 40 measurements were performed over the gauging section in order to compensate for the incomplete mixing in this power plant. An idea about the state of mixing has already been given when Figure 3 was discussed. From this it is evident, that there is a significant difference in the relative mean concentrations at the two instrument levels, the average of which are 1.16 for the E- and 1.82 for the S-level. This indicates a different character of the flow observed by the two detectors.

On the other hand, no trend can be observed in the concentration values at a single level. Nor do the times of arrival of the individual tracer clouds exhibit a certain trend. It should therefore be justified to add the $c-t$ curves over the E- and S-levels respectively. This sum-curve for the S-level is represented in Figure 5. The corresponding E-curve, normalized to the maximum of the S-curve, is shown in the same figure. When compared, the shapes of the two curves show to be practically congruent for times less than about three minutes.

Thus, during this time interval, the mean concentration in the two levels varies in the same way. From Figure 6 it is obvious, that the activity in the E-level 10 minutes after the injection still exceeds the natural background contrary to the situation at the S-level. Thus, the tails must have considerably different lengths at the two levels. Furthermore strong fluctuations in the concentration are visible in the E-tails in Figure 5. The primary data showed that this was true even 10 minutes after injection.

Thus, on one hand the shape of the sum-curves for the two layers agreed and on the other hand a long-lasting fluctuating activity was observed in the E-level. From this it has been concluded, that the E-detector simultaneously registered the concentration in two different layers: a lower layer, the flow of which was much like the flow observed by the S-detector and an upper layer with strong turbulence and slow water exchange with the lower layer. The position of the boundary zone between the two layers must be near the middle of the draft tube, as can be seen from the η -depth curve in Figure 4. Another estimate can be made from the definite η -value combined with the η -value for the S-layer, under the assumption that the upper layer did not contain any activity; this indicates a boundary to be about 1 m above the

E-detector. The boundary zone if extrapolated upstream would touch the ceiling of the draft tube at an angle of 5-10 degrees. Thus, it is a fairly large part of the draft tube in this power plant that is not effectively used.

It is also possible to study the influence on the flow from the vertical wall, which divides the draft tube in two parts (Fig. 2). The clockwise rotating turbine axis is displaced from the axis of symmetry towards the right part of the draft tube. Differences in the flow through the two parts are evident already from the activity concentrations in Figure 6. The activity in the E-level of the left part of the draft tube about 10 minutes after the injection is considerably higher near the dividing wall than elsewhere. As might be expected from Figure 2, no such disturbance can be observed on the right side of the wall. At this time, the activity in the S-level has decreased to the background level, due to the more active water exchange in the bottom layer. If, however, the sum curves for the right and the left part in this level (Fig. 7) are considered it is clear that the activity level from about 1½ minutes after injection is higher in the left than in the right part. This indicates that the disturbance from the dividing wall observed in the E-level also occurs in the bottom layer.

From this interpretation of the data it is obvious, that the shape of the ceiling as well as of the dividing wall generates intensive turbulence.

b) FLOW PARAMETERS:

In the following an attempt will be made to describe the flow in the power plant conduits by evaluating some parameters from a fictitious pipe.

From hydraulic considerations it can be expected that the main turbulence is generated where the acceleration of the flow passes into a retardation [17]. The geometrical dimensions of the water way show, that this takes place in the turbine 46 m upstream the gauging section. As a matter of fact, the point of injection was chosen as to let the tracer pass into the turbine with a minimum of spread. Under these circumstances, the spread of the tracer upstream the turbine can be neglected and the injection thought of as taking place at the turbine. The average time required for the transportation of a tracer cloud from this place to the gauging section has, from the flow and the mean area of the water way been calculated at 18 sec.

When calculating the flow parameters the conduit on the downstream side of the turbine

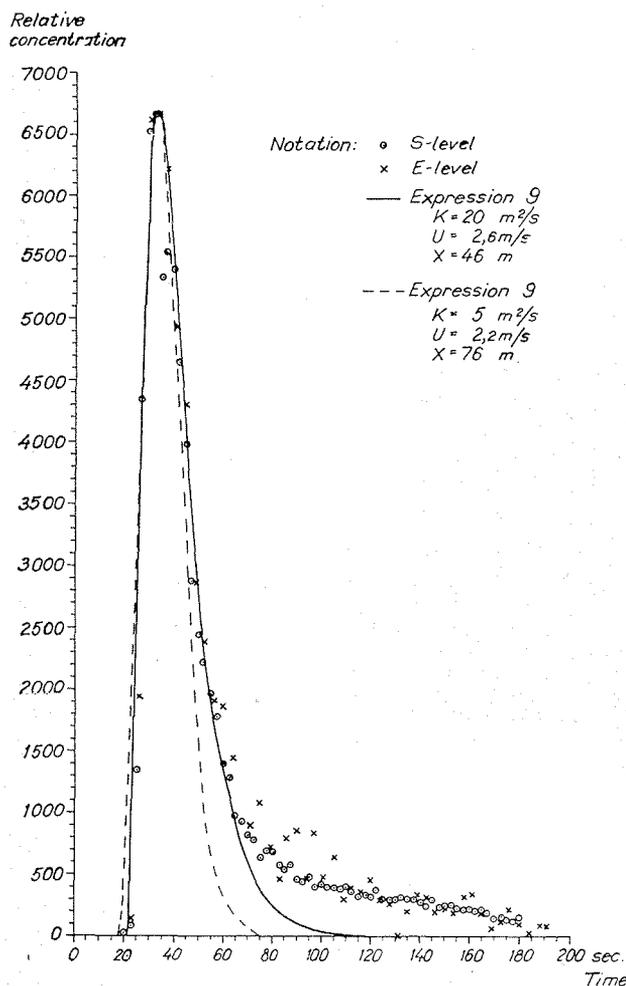


FIG. 5

Mean concentration as a function of the time after the injection for both E- and S-layer. The E-values have been normalized on the maximum of the S-values. The curves represent fitted Taylor distributions.

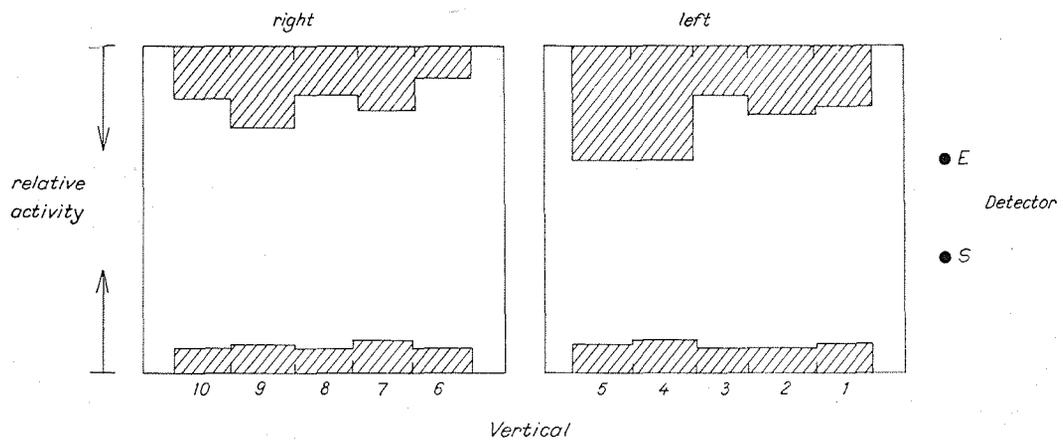


FIG. 6

Concentration of the radioactivity at the mouth of the draft tube 10 min:s after the injections. The effect of the natural radioactivity is included.

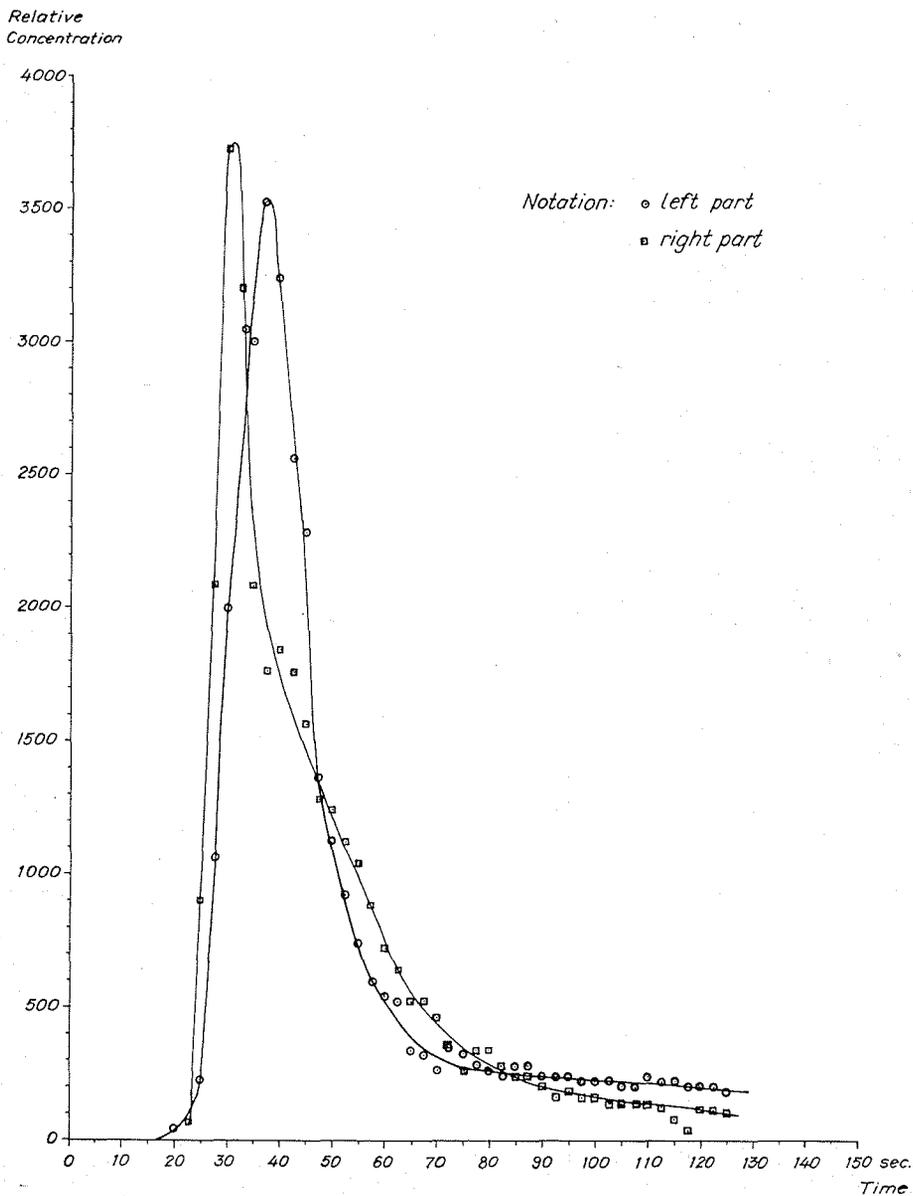


FIG. 7

Mean concentration as a function of the time after the injection for right and left part of the draft tube.

is thought of as replaced by a fictitious tube, the length of which agrees with the distance from an assumed "point of injection" in the turbine to the gauging section.

Turbulence parameters can be determined both from the individual tracer clouds and from the sum cloud.

The turbulence in the pipe consists of eddies of all dimensions from the scale of the pipe downwards, but the spread of an individual cloud can be expected to be caused mainly by eddies of sizes up to that of the cloud itself (much larger eddies only carrying the whole cloud along). With this fact in mind the dispersion coefficient obtained from individual tracer clouds can be described as corresponding to "small-scale eddies":

$$K_s = \overline{u'_s{}^2} t_{0,s} \tag{24}$$

Here $u'_s = u - u_{\text{cloud}}$ is the velocity deviation from the mean velocity $u_{\text{cloud}} = X/t$ of the cloud and $t_{0,s}$ is the corresponding time scale [cf.

equations (16) and (17)]; X is the distance (= length of the fictitious pipe) and t the time of travel. Owing to larger eddies t and u_{cloud} will vary from one cloud to another. The dispersion coefficient of the sum cloud, with mean velocity U , can then be written [18]:

$$K = K_s + K_1 = K_s + \overline{u'_1{}^2} t_{0,1} \tag{25}$$

where $u'_1 = u_{\text{cloud}} - U$, the index 1 referring to large-scale eddies. Of course the terms "small-scale eddies" and "large-scale eddies" are here used in a rather loose sense.

In order to determine the turbulence parameters for the *small scale eddies*, K_s and $t_{0,s}$ the relative spread values of the individual tracer clouds have been plotted against the times of arrival (modal times) in Figure 8. σ_x^2 was thereby calculated from the expression (12) by means of the half values of the $c-t$ curves. This means, that the peaks of the $c-t$ curves were replaced by Gaussians, which gives the same result as the expression (13) when the second term of the

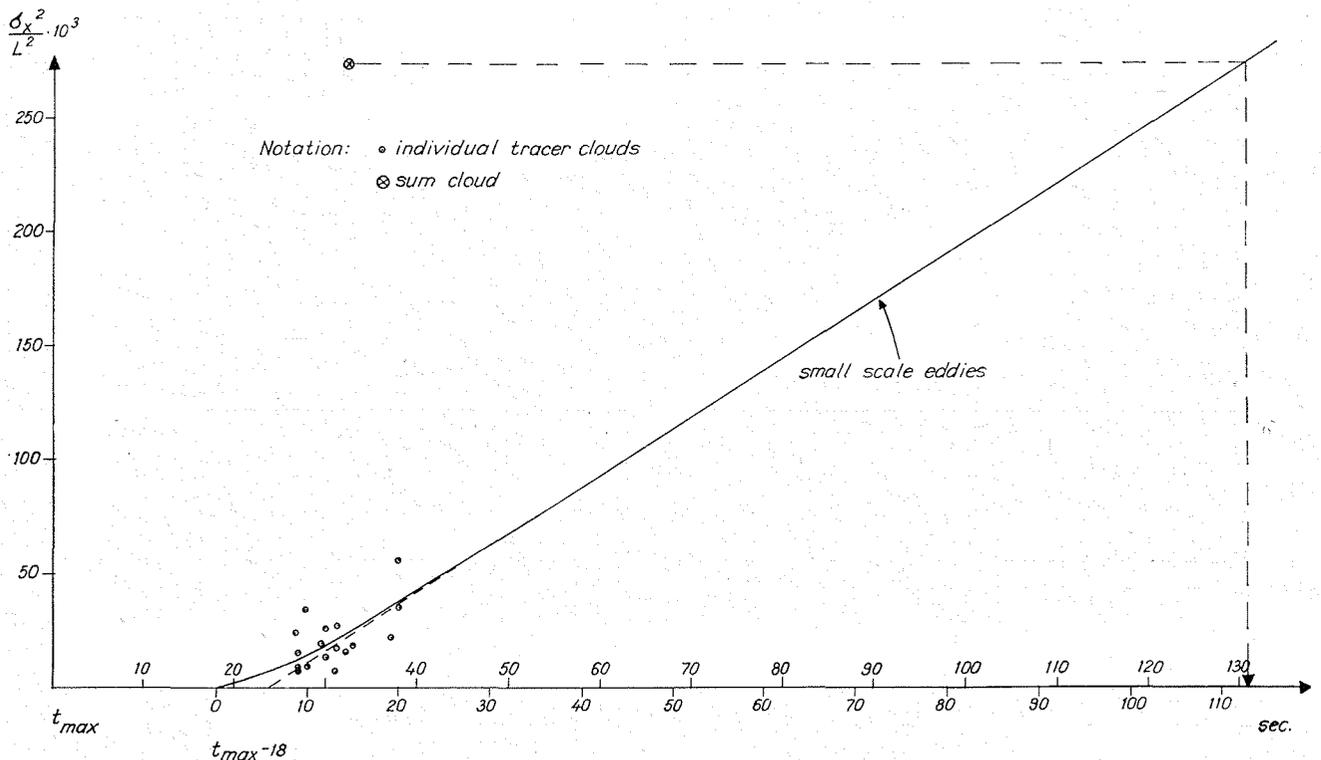


FIG. 8

Variance of x , as determined in the different concentration distributions measured in the S layer, as a function of the mode. The standard deviation has been measured in the scale of the distance from the turbine to the gauging section. The curve represents the expression 22. Concerning the dashed lines cf. the text.

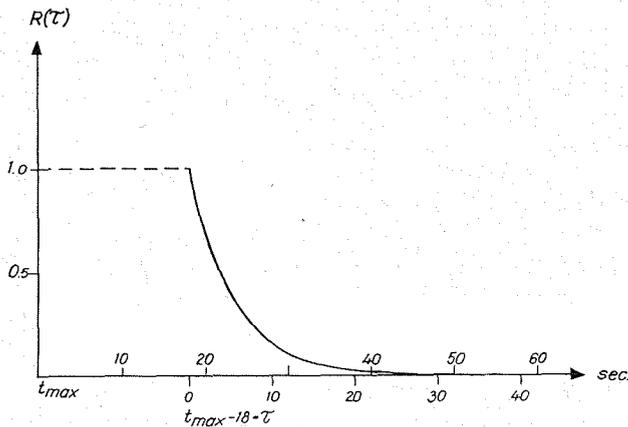


Fig. 9

The correlation $R(\tau)$ of the small scale eddies in the fictitious pipe describing the draft tube.

correction factor is neglected. The influence of the reflexion from the walls of the draft tube on the spread is by this approach minimised. The two parameters could then be evaluated by fitting the spread function in the expression (22) to

the individual tracer spread values. Because of the spread of these data this has been performed by dividing the cluster of points in two parts, fitting a spread function to the means. The time scale $t_{0,s}$ then being known, the length scales, the velocity variance $u_s'^2$ and the relative intensity of the flow can be determined from the expressions (20), (17) and (14) respectively. The parameter values are given in table I, whereas the correlation function corresponding to the small scale eddies is shown in Figure 9.

Information concerning the *large scale eddies* can be gained from a study of the sum curve. A fitting of a Taylor distribution [expression (9)] to the peak of the experimental sum curve determines the dispersion coefficient K and the mean velocity U for the flow in the fictitious pipe. The values thus obtained are found in table 1 and the correspondence between the two curves is fairly good, as can be seen from Figure 5. The conformity does apparently not include the "tails," i.e. the extended later parts of the $c-t$ curves. These are therefore attributed to such boundary effects in the conduits as not allowed for in the deduction of the theoretical expression. The K -value just determined lends

TABLE I

Flow parameters of a fictitious pipe simulating the flow through the power plant

PARAMETER	SYMBOL	PIPE LENGTH=ACTUAL SPREAD DISTANCE= 46 m		PIPE LENGTH = LENGTH OF WHOLE water way=76 m
		SMALL SCALE EDDIES	LARGE SCALE EDDIES	
Mean velocity.....	U	2.6 m/s		2.2 m/s
Reynolds number.....	\mathcal{R}	$2.7 \cdot 10^7$		$2.3 \cdot 10^7$
Dispersion coefficient of the sum cloud = plane injection.....	K	20 m ² /s		5 m ² /s
Friction coefficient	γ	0.045		0.004
Standard deviation of the sum cloud.....	σ_x	24 m		
Dispersion coefficient....	K	3 m ² /s	17 m ² /s	
Time scale.....	t_0	5 s	17 s	
Length scale.....	l	4 m	17 m	
—	L	14 m	44 m	
Standard deviation of cloud.....	σ_x	7 m	—	
Velocity variance.....	$\frac{\sigma_x^2}{U^2}$	0.5 m ² /s ²	1.0 m ² /s ²	
Relative intensity.....	$\frac{\sqrt{u'^2}}{U}$	30 %	40 %	

itself to the determination of the friction coefficient of the straight fictitious pipe [expression (8)]. The value calculated is about 25 times the value that could be expected for smooth flow of the same Reynolds number. The variance $u'_i{}^2$ can be determined from the spread of the times of arrival of the individual clouds (modal times). The time scale $t_{0,i}$ can then be calculated from the expression (24) and the dispersion coefficient K_i from the expression (25). The values received are given in table I.

It is also of interest to compare the standard deviation σ_x with the largest dimension of the draft tube, in this case about 18 m. For a single cloud σ_x is about 1/3 of this width, whereas for the sum curve σ_x is about the same. It can be seen from Figure 8, that a dispersion time of 110 sec. would have been required in the case of a point injection to reach a spread as large as that of the sum curve if effects due to wall reflec-

tion are neglected. This corresponds to a flow distance of about 300 m. Even if the wall reflection would have been effective in reducing this distance, it is apparent that the use of several injections made it possible to simulate in a stretch of 46 m a mixing quite as complete as that obtained at a much larger distance after a single point injection.

For a comparison with other power plants it might be of interest to know the K-value of the sum curve and the friction coefficient for a fictitious pipe of the same length as the actual measuring distance, 76 m. These values are also shown in table I.

The friction coefficient for this longer pipe is only twice as large as it would have been if the flow had been smooth. The fit in this case is not as good as when the spread was looked upon as starting in the turbine as can be seen from Figure 5.

III. — DISCUSSION

A discussion about the relative dilution method is essentially a question concerning the mixing in the water ways. The results of the present investigation indicate, that in a horizontal layer at the end of the draft tube the mean concentration observed in a gauging point is subject to fluctuations of statistical nature. This is shown also by the smoothness of the sum curves for each level. As a consequence, the average of the local mean concentrations represents the mean value for the level in question. If the fluctuations are statistical, however, the same average value would be obtained by adding mean concentrations from several measurements in one single point. This means that the time necessary for determining a single flow value can be considerably brought down since the detectors, in the experimental arrangement used, can be moved from one vertical to another only when the turbine is unloaded.

In the actual plant, measurements performed in different levels give different mean concentrations. This might occur even in other plants. If so, the representative mean concentration has to be measured for a sufficient number of levels to guarantee knowledge about the depth dependence.

In the present investigation, where the mixing was most incomplete, the standard deviation of a single measurement was between 15 and 20 %. This includes, beside a statistical error in the counting rate, also errors caused

by the instrumentation. The total standard deviation of the calculated mean of the 40 measurements was estimated to 4 %.

It can be mentioned that a second investigation using the same technique was carried out recently in another power plant, equipped with Francis turbines and with a head of about 70 meters. The distance between the injection and the gauging section was about 20 diameters. In this case, the standard deviation was 6 % mainly due to the much better mixing conditions but also to the considerably improved apparatus used. With 9 injections and two detectors the inaccuracy of the average was brought down to 1.4 %. It might be added, that no vertical variation in the concentration was observed. This means, that if the number of injections is increased by a factor 2, the error should be about 1 %.

The measurement of all the quantities determining the Q-value in expression (3) can be improved so that the inaccuracy becomes of the order of one per mille or less. The accuracy attainable with this method for measuring the flow would therefore remain mainly a question of enlarging the number of measurements. This can be made by increasing the number of injections or that of the detectors.

In this connection it can be mentioned that the flow also is determined from the ratio between the volume of the effective water way and the mean time of flow. In the present investi-

gation this value agreed well with the value determined from the dilution method used.

The technique of using radio isotopes used in the present investigation gives, together with the flow value, considerable information about the character of the flow in the plant, which makes it possible to localize disturbances in the water ways. The constructor of the power plant is hereby given a possibility to check the quality of the conduit shapes more exhaustively than is possible in model studies. It should however be observed, that in investigations where the primary aim is to localize such disturbances, the points of injection and the gauging points should be chosen with special regard to this.

The analysis of flow parameters is performed by replacing the water ways through the power station by a straight tube, for which a dispersion coefficient is determined. This has been used to get a measure of the quality of the water way shapes, expressed as a friction coefficient. It might be mentioned that this coefficient in the plant in question, where the efficiency was extremely low, was much larger than that for smooth pipes. For the plant recently investigated the preliminary data obtained indi-

cate that the friction coefficient was essentially the same as for a smooth pipe. The efficiency was in this case extremely high.

From the spread of the individual tracer clouds as well as of the sum cloud it has been possible to evaluate some turbulence parameters representing small scale and large scale eddies respectively.

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RÉSUMÉ

Depuis la fin du XIX^e siècle, la méthode de dilution a été utilisée pour la détermination des débits des fleuves et des usines hydroélectriques. En 1958, des traceurs radioactifs furent utilisés et la méthode prit pour nom : méthode de comptage total.

Le débit est donné par l'expression :

$$Q = \frac{M}{\int_0^T c dt}$$

où Q est le débit cherché;

M, la masse du traceur injecté;

c, la concentration du traceur à l'instant t;

T, le temps pendant lequel, le nuage passe dans la section de mesure.

De tels essais sont faits par l'Institut Météorologique et Hydrologique de Suède depuis 1958, et un des essais dans une usine équipée de turbines Kaplan. La chute était de 12 m et le débit d'environ 200 m³/s (voir fig. 1 et 2); le traceur utilisé était du ²⁴Na₂CO₃. Les injections, de 15 milli-curies chacune, se faisaient dans les pertuis d'entrée et les concentrations à la sortie du diffuseur étaient déterminées par deux compteurs à scintillations et leurs évolutions enregistrées; 10 verticales ont été explorées en deux points chacune; chaque point étant doublé, il y eut en tout 40 injections.

A partir des courbes (concentration en fonction du temps) enregistrées, il a été possible de déterminer en chaque point le $\int_0^T c dt$ correspondant et, en faisant leur moyenne, de calculer le débit et le rendement correspondant (fig. 3, 4, 5, 6, 7).

L'écart-type des distributions gaussiennes des concentrations a été calculé. Il était de 4 %. On a pu constater une différence très importante entre la moyenne des concentrations relatives obtenues dans les deux parties du diffuseur : respectivement 1,16 et 1,82. De même, les temps de passage des nuages étaient très nettement différents.

Les valeurs des concentrations en des points d'un même plan horizontal ont montré un caractère statistique; leur moyenne représentait donc bien la valeur moyenne de la concentration par palier horizontal.

Les caractéristiques hydrauliques (voir tabl. I) ont été tirées d'expériences faites en remplaçant l'ensemble où s'opéraient les mesures par un tuyau circulaire fictif de même longueur et d'un diamètre moyen tenant compte de toutes les caractéristiques géométriques.

Selon un type de calcul donné par G.I. Taylor, les auteurs ont pu déterminer d'après leur mesure de concentration les valeurs de divers paramètres caractérisant la turbulence de l'écoulement (voir le tabl. I et les fig. 8 et 9).